

Soil Structure Interaction
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Lecture-64
Soil Structure Interaction for Pile Foundation (Contd.)

In this class I will discuss how to determine the interaction factor for laterally loaded piles to determine the settlement of pile group considering the effect of all interaction factors due to all the piles in group or the interaction effect of other piles on a particular pile in a group.

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Elastic Analysis
Displacement of Pile Group under lateral load by Interaction Factor Approach

Interaction Factor (α_{ij}) for displacement = Additional displacement caused by the adjacent pile / displacement of pile under its own loading

$$\rho_k = \bar{\rho}_k \left[H_k + \sum_{\substack{j=1 \\ j \neq k}}^n H_j \alpha_{\rho H k j} \right]$$

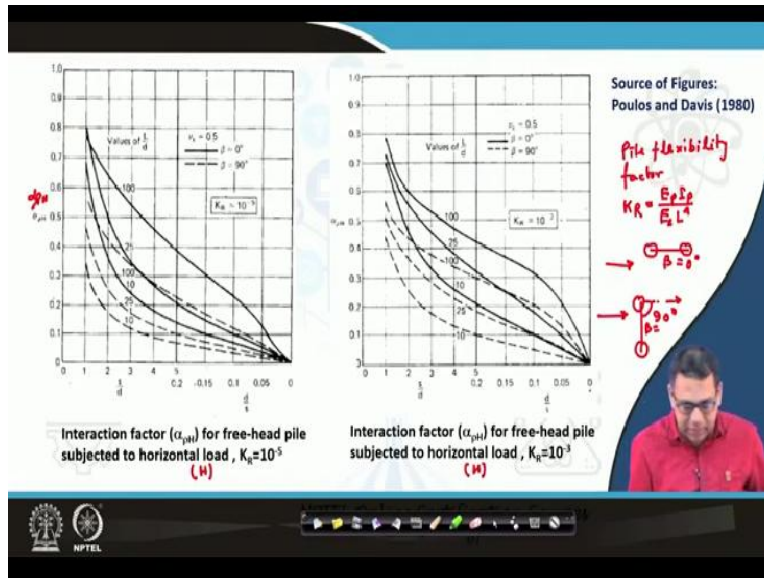
$$H_k = \sum H_j$$

ρ_{pH} is the unit displacement i.e the displacement of a single free-head pile under unit horizontal load
 H_j is the load on pile j
 $\alpha_{\rho H k j}$ is the value of $\alpha_{\rho H k j}$ (interaction factor for displacement under horizontal load) for two piles k and j and angle β is the angle between the direction of loading and the line joining the centers of piles k and j
 Poulos and Davis (1980)

Handwritten notes on the slide:
 $\alpha_{\rho H} \rightarrow$ Settlement under H
 $\alpha_{\rho M} \rightarrow$ displacement under M
 $\alpha_{\theta H} \rightarrow$ Rotation under H
 $\alpha_{\theta M} \rightarrow$ Rotation under M

This was the explanation given and it also shows different interaction factors that are to be considered. The interaction will be denoted by α with a suffix indicating the type of interaction, type of load and, in the expression, the number of pile. $\alpha_{\rho H}$ indicates that the interaction factor is for settlement (ρ) and the load is a horizontal point load (H). Similarly if M is in the suffix, it denotes that a concentrated moment is acting on the pile group and if θ is in the suffix, it denotes that the interaction is considered for rotation. Finally an interaction factor $\alpha_{\rho H k j}$ speaks of the effect the j^{th} pile has on the settlement (as ρ is in suffix) of k^{th} pile in a group subjected to a horizontal load of H .

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The above shown charts give the α_{pH} value of free-head piles for a μ value of 0.5, various values of L/d ratio and two β values of 0° and 90° depending upon the s/d or d/s ratio. The x axis reads the value of s/d ratio and if the s/d ratio is more than 5, d/s ratio should be considered which is also present on the x axis of the charts. The two charts are given for two K_R values (10^{-3} & 10^{-5}) where K_R is the pile flexibility factor and:

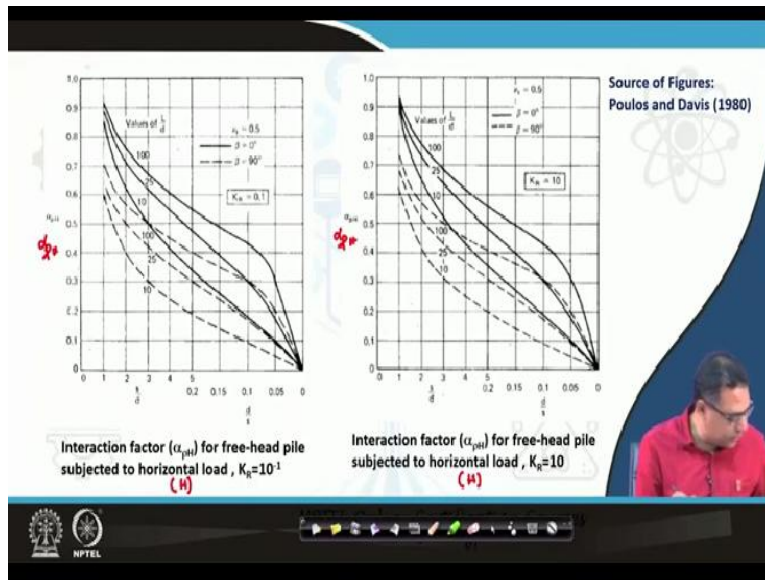
$$K_R = \frac{E_p I_p}{E_s L^4}$$

$E_p I_p$ is the flexible stiffness of the pile, L is the length of the pile and S is the elastic modulus of the soil.

The procedure to determine the spacing between each set of piles was already discussed in case of pile group under compression. The angle β was also mentioned to be the angle between the direction of load and the line joining two piles. In the first slide shown in this lecture, the β value for the k^{th} pile and the centre pile would be 45° as it is a pile group square pattern.

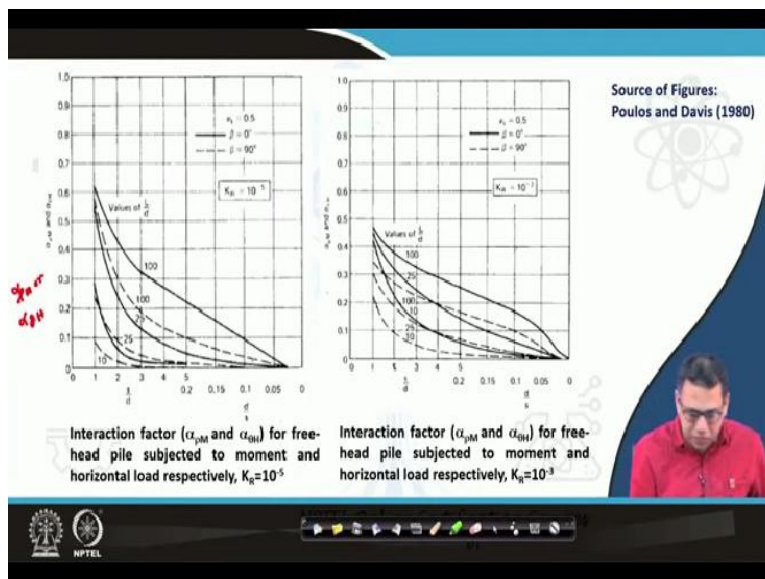
The above shown charts (for α_{pH}) are developed for a Poisson's ratio of 0.5, but no corrections are recommended if soil has any other μ value. This is because it was found that the variation due to the Poisson's ratio was found to be very less. So, though the charts are for $\mu = 0.5$, they can be used for any μ value.

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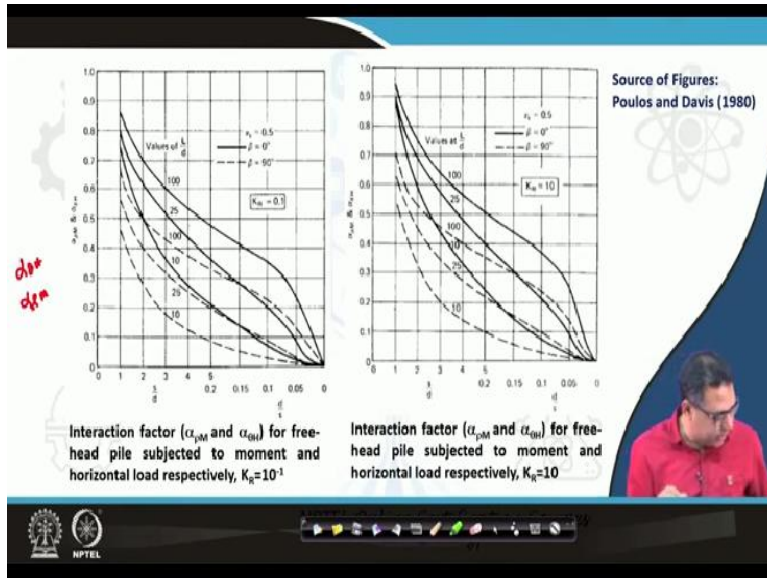
These are the charts for K_R values of 10 and 10^{-1} .

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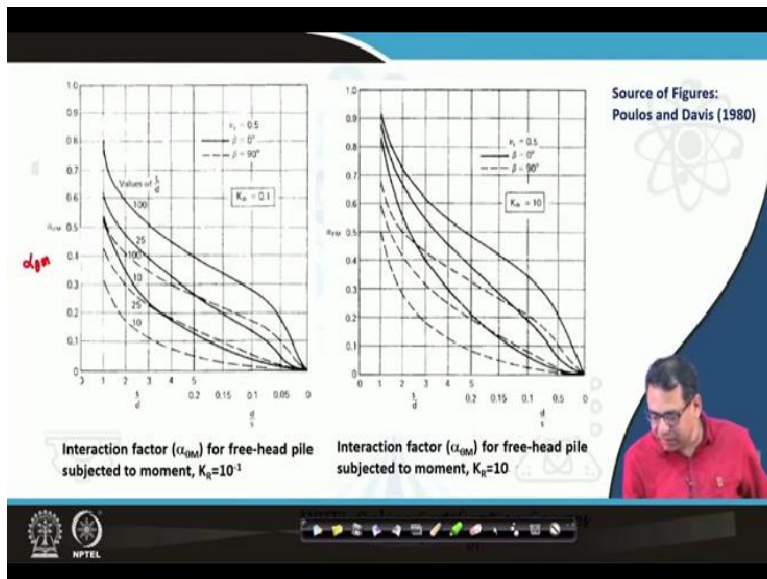


The above shown are the charts required to determine the $\alpha_{\rho M}$ and $\alpha_{\rho H}$ values. $\alpha_{\rho M}$ is the settlement interaction factor due to moment and $\alpha_{\rho H}$ is the rotation interaction factor due to horizontal load. Note that the chart is same for both the interaction factors. All other depending factors are similar to that of $\alpha_{\rho H}$.

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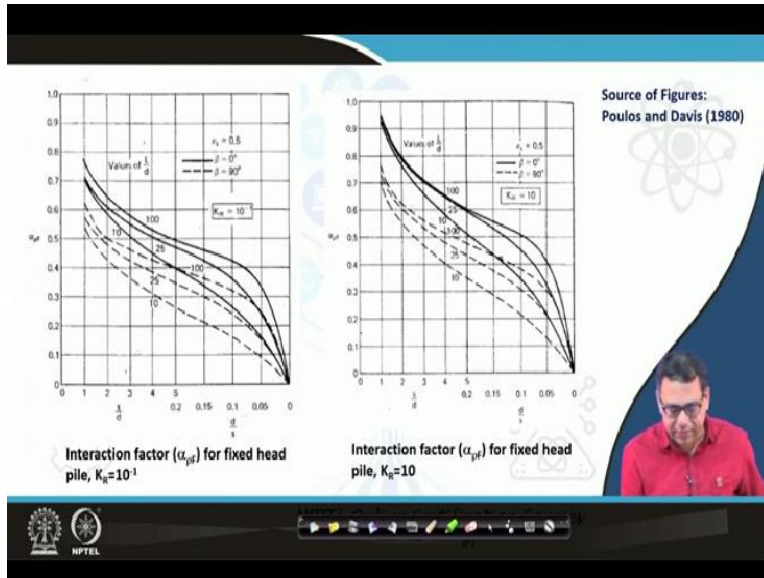


These are some other charts for α_{pM} and $\alpha_{\theta H}$ for K_R values of 10^{-1} and 10. (Refer Slide Time: 06:15)



Similar charts are proposed for $\alpha_{\theta M}$ which are shown above. This factor speaks about the influence of other piles in rotation when the group is subjected to a moment, M. The above shown are charts for K_R values of 10 and 10^{-1} . But, similar to other interaction factors, this is also given for four K_R values (10^{-5} , 10^{-3} , 10^{-1} and 10). The charts discussed till now are for the free-head piles.

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The above are the interaction factors of settlement in case of a fixed head pile, α_{pF} . There is a small difference in the nomenclature of the interaction factors in case of free-headed and fixed head piles. In the free head pile factors, the second letter of suffix denoted the type of load (H- point load & M- moment). But in fixed head piles, the second is 'F' which denotes fixed head pile. It was already mentioned that in fixed head pile groups moment cannot be applied and hence there is no need to indicate the loading type in the interaction factors.

Similar to the all other charts, the charts for α_{pF} were also developed for four values of K_R . Remember that all the values the charts so far show, are for the ground line level only. So, as the rotation for fixed head pile at the ground level (at the fixed head itself) is 0, there are no charts for $\alpha_{\theta F}$.

In the subgrade modulus approach, the quantities could be determined along the depth at different locations, but here the factors for rotation and deflection are for the ground level only. So, for the free headed pile, the deflection and rotation can be determined, but for the fixed head pile, the rotation is 0 at the ground level.

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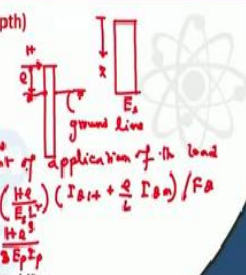

Displacement and Rotation of Single Pile (Constant E_s with depth)
Free-Head Pile

$\rho = \frac{H}{E_s L} \left(I_{\rho H} + \frac{e}{L} I_{\rho M} \right) / F_{\rho}$
 Rotation $\theta = \frac{H}{E_s L} \left(I_{\theta H} + \frac{e}{L} I_{\theta M} \right) / F_{\theta}$

For free head pile, the displacement at the point of application of the load is

$$\delta = \frac{H}{E_s L} \left(I_{\rho H} + \frac{e}{L} I_{\rho M} \right) / F_{\rho} + \left(\frac{H e}{E_s L} \right) \left(I_{\theta H} + \frac{e}{L} I_{\theta M} \right) / F_{\theta}$$

e is the eccentricity of load = M/H
 M is the applied moment at ground line
 H is the horizontal force acting at an eccentricity e above the ground line
 $I_{\rho H}$ is the elastic influence factor for displacement
 F_{ρ} is the yield displacement factor (ratio of pile displacement in elastic soil to pile displacement in yielding soil)
 F_{θ} yield rotation factor

The next concept is about the deflection and rotation of a single pile with constant E_s along the depth. In the concept of subgrade modulus approach, 2 cases were considered: a constant k_h where the subgrade modulus does not change with depth and a variable k_h where the subgrade modulus varies with depth. Similarly two cases will be considered here where the elastic modulus is: constant along the depth of the pile and varying along the pile. As this is the elastic analysis, the two factors E and μ will be focused upon.

The case considered here is of the constant E_s value along the depth which is similar to the uniform k_h value in the subgrade modulus approach. Here the settlement is been indicated with ρ . The displacement will be calculated at the ground level even if the pile head is above the ground level and the load is applied at the head. The determination of deflection wherever H is applied will be shown later on. For now, the deflection of a free-headed pile under a lateral load of H is:

$$\rho = \frac{H}{E_s L} \frac{\left(I_{\rho H} + \frac{e}{L} I_{\rho M} \right)}{F_{\rho}}$$

where, I is the influence factor in which ρ denotes settlement and H denotes horizontal load, $I_{\rho M}$ is the influence factor for settlement under moment, e is the eccentricity of the load (= M/H) or the distance between the point of action of the load and the ground level and F_{ρ} is the yield displacement factor. This factor may be defined as the ratio of pile displacements when soil is in elastic state to that of yielding state. So, it is basically a comparison between the soil in elastic zone and the soil with ultimate resistance. For a purely elastic soil, the value of F_{ρ} will be 1.

Similarly the rotation of a free-headed pile under a lateral load of H is:

$$\theta = \frac{H}{E_s L^2} \left(\frac{I_{\theta H} + \frac{e}{L} I_{\theta M}}{F_\theta} \right)$$

The factors are almost similar to that of those involved in the expression for displacement. $I_{\theta H}$ is the influence factor for rotation under H, $I_{\theta M}$ is the influence factor for rotation under M and F_θ is the ratio of the pile rotation in elastic soil to the pile rotation in yielding soil.

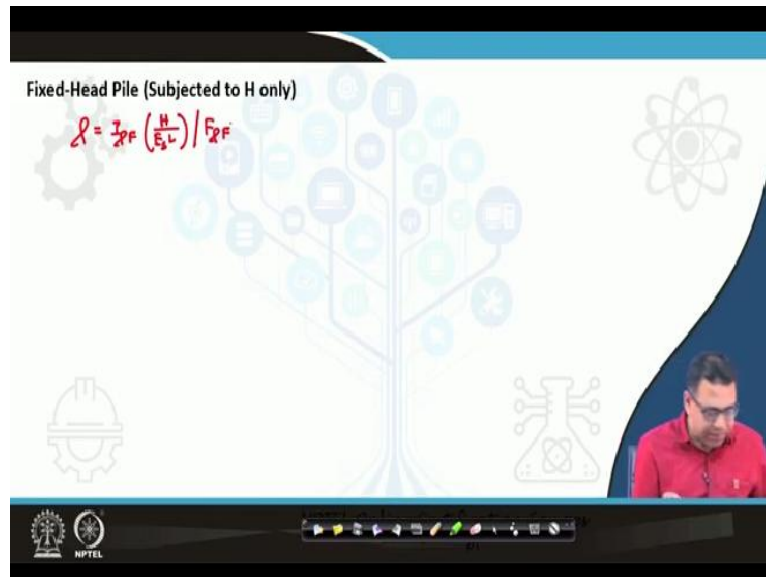
If the displacement should be determined at the point of application of H in a free-headed pile:

$$\rho = \frac{H}{E_s L} \left(\frac{I_{\rho H} + \frac{e}{L} I_{\rho M}}{F_\rho} \right) + \frac{He}{E_s L^2} \left(\frac{I_{\theta H} + \frac{e}{L} I_{\theta M}}{F_\theta} \right) + \frac{He^3}{3E_p I_p}$$

The terms are all defined already except $E_p I_p$ which is the flexural rigidity of the pile.

These are the expressions for displacement and rotation for a free-headed pile.

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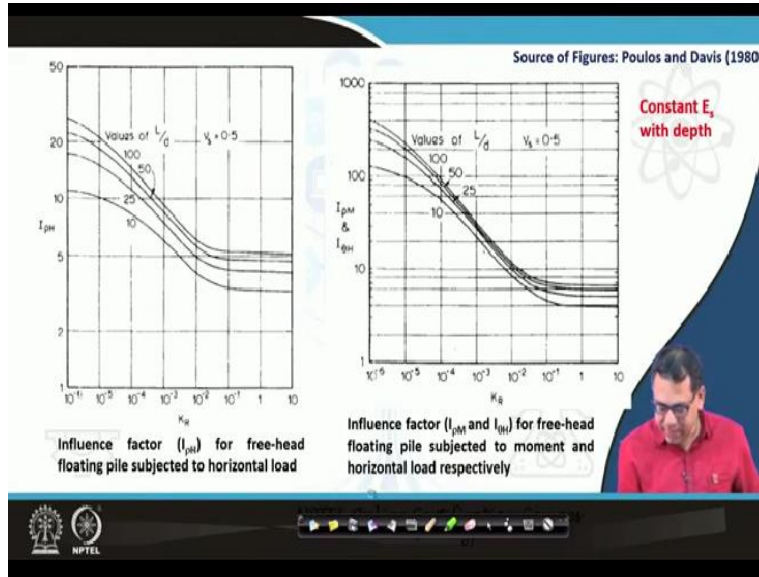


Similarly when a fixed head pile is subjected to H, the expression for displacement will be:

$$\rho = I_{\rho F} \left(\frac{H}{E_s L} \right) / F_{\rho F}$$

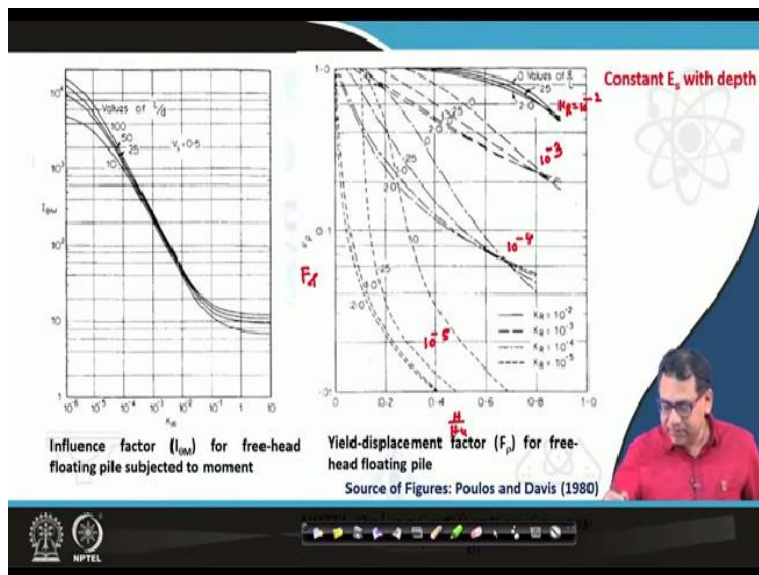
$I_{\rho F}$ is the influence factor settlement in a fixed head pile (here, F denotes fixed), $F_{\rho F}$ is the displacement factor (ratio of pile settlement in soil within elastic zone and the pile settlement in soil at ultimate resistance). Remember that for a pure elastic soil, $F_{\rho F}$ will be 1.

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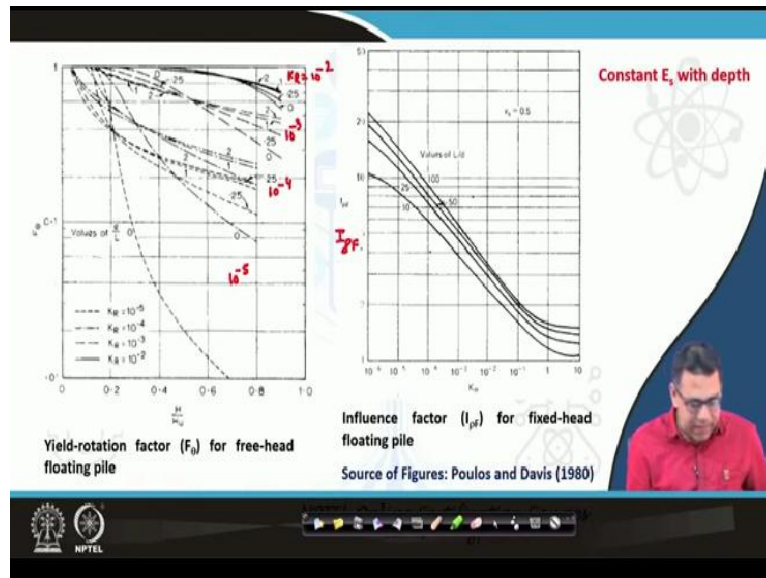
The above slide shows the charts to determine the influence factor values settlement under H, settlement under M and rotation under M in case of free-headed piles. The influence factor value will be read against the respective K_R value on the x axis for an appropriate L/d ratio. Note that these charts are developed for free headed floating piles resting in a soil of constant E_s and with a Poisson's ratio of 0.5.

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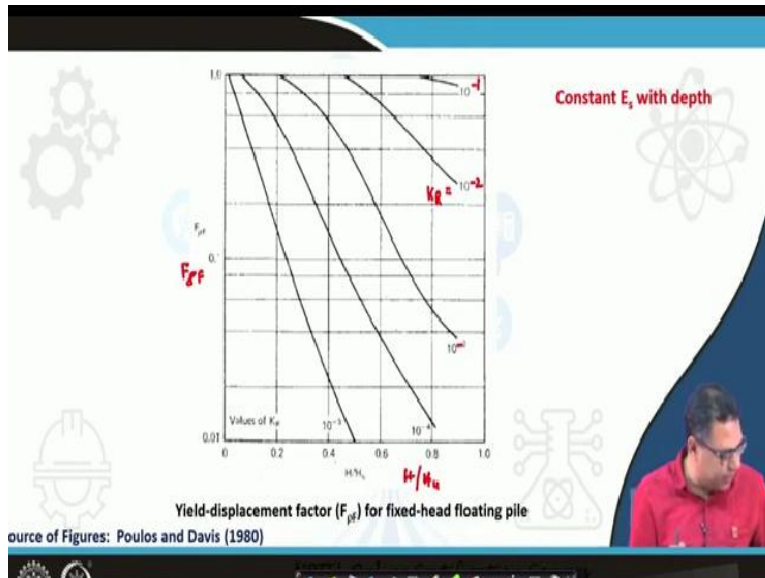
Above shown are the charts for rotation influence factor under moment ($I_{\theta M}$) and for the yield displacement factor, F_p for free-headed floating pile. The factor $I_{\theta M}$ depends upon the L/d ratio and K_R value of soil just like the previous influence factors. But the factor F_p depends upon the e/L ratio and H/H_u ratio on the x axis. H is the horizontal load applied on the pile and H_u is the ultimate horizontal load carrying capacity of the pile.

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Similarly the chart on the left in the above slide is for F_{θ} , the yield-rotation factor for a free-headed floating pile. The chart on the right shows the influence factor for settlement in case of a fixed-head pile (I_{pF}). It is already mentioned that no moment can act on the fixed-head piles and as there is only settlement in this case, the influence factor is also formulated for the settlement.

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Finally this chart is for the yield-displacement factor for a fixed-head pile, F_{pF} . The last two charts discussed are for the fixed-head piles (I_{pF} & F_{pF}) and all the remaining charts are applicable for free-headed piles.

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Displacement and Rotation of Single Pile (Linearly varying E_s with depth) E_s

$E_s = N_{\eta} z$ $N_{\eta} = \text{rate of increase of } E_s \text{ with depth}$

$N_{\eta} = \eta h$ $K_{\eta} = \eta h \left(\frac{z}{d} \right)$ $N_{\eta} = k_h \left(\frac{z}{d} \right)^m$

Pile flexibility factor $K_N = \frac{E_p I_p}{N_{\eta} L^3}$

Free-head pile: $\delta = \frac{H}{N_{\eta} L^2} \left(I_{pH} + \frac{e}{L} I_{p\theta} \right) F'_{pF}$

Fixed-head pile: $\delta = \frac{H}{N_{\eta} L^2} I'_{pF} / F'_{pF}$

$\theta = \frac{H}{N_{\eta} L^3} \left(I'_{\theta H} + \frac{e}{L} I'_{\theta\theta} \right) / F'_{\theta}$

Let us now look into the concept of displacement and rotation for single pile in a soil with linearly varying E_s with depth. Here, the expression for E_s can be written as:

$$E_s = N_{\eta} \times z$$

where, N_{η} is the rate of increase of E_s with depth

$$N_{\eta} = \eta h; \quad \text{and} \quad k_h = \eta h \left(\frac{z}{d} \right)$$

The pile flexibility factor is given by:

$$k_N = \frac{E_p I_p}{N_\eta L^5}$$

For the free-headed pile, the expression for settlement is:

$$\rho = \frac{H}{N_\eta L} \frac{\left(I'_{\rho H} + \frac{e}{L} I'_{\rho M} \right)}{F'_\rho}$$

The expression is same as that of the constant E_s , but the only difference is the dash terms, which indicate that the E_s is linearly varying with depth. The influence factors also denote the same as the constant E_s case.

Similarly, the expression for the rotation will be:

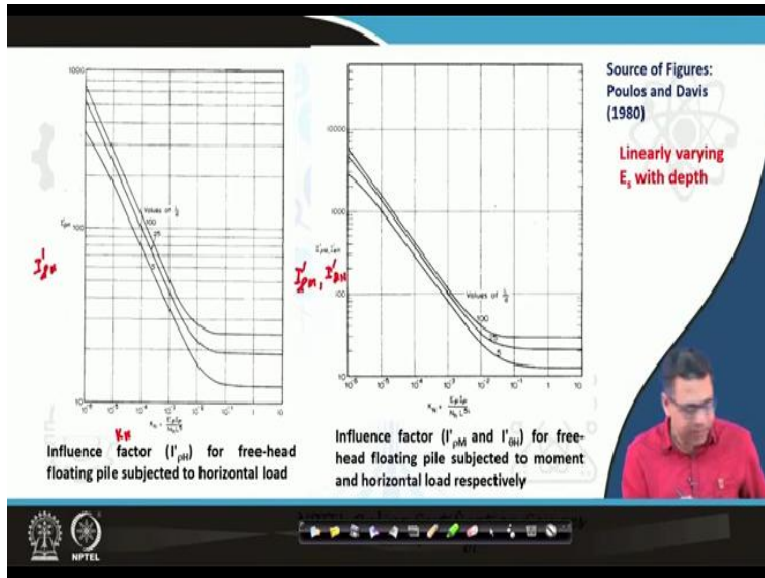
$$\theta = \frac{H}{N_\eta L^3} \frac{\left(I'_{\theta H} + \frac{e}{L} I'_{\theta M} \right)}{F'_\theta}$$

The above two are the expressions for a free-headed pile in soil of constant E_s value. Now for the fixed head pile, similar expression will be valid:

$$\rho = \frac{H}{N_\eta L^2} \frac{I'_{\rho F}}{F'_{\rho F}}$$

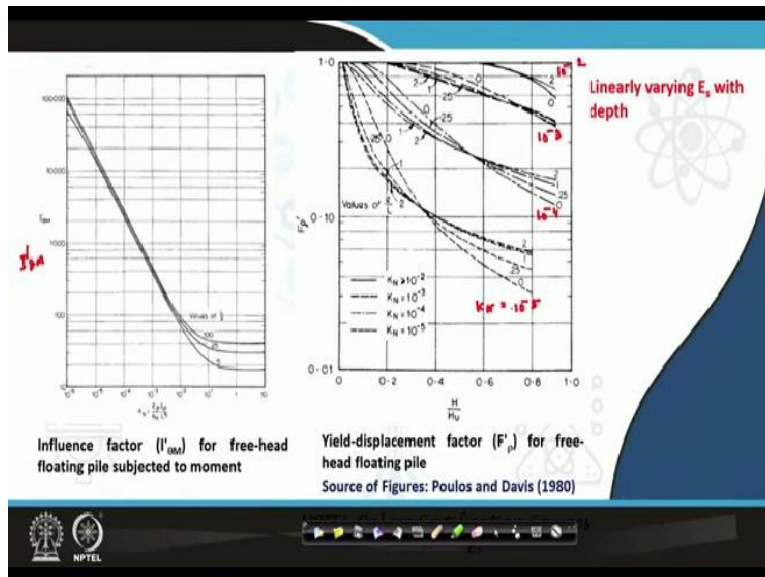
In case of fixed head piles, only settlement is valid and the letter 'F' in the suffix of the factors indicates that the pile is fixed but is not any loading condition.

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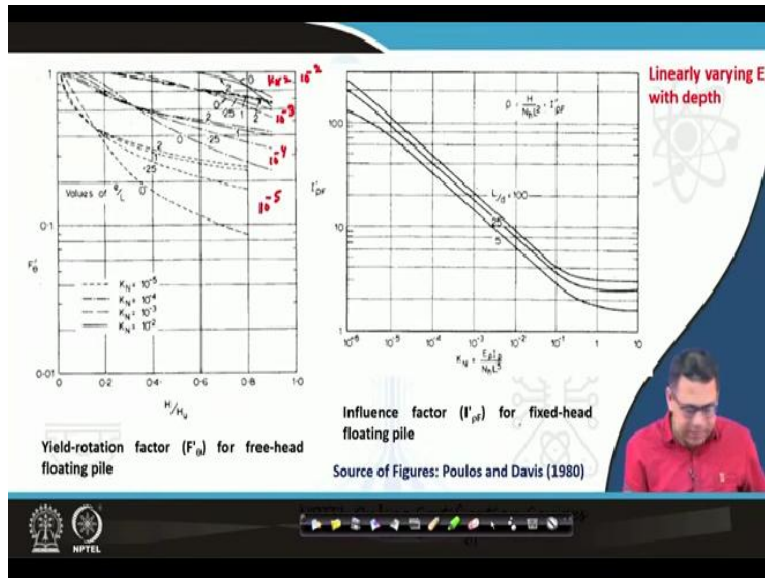
Similar to that of the influence factors in case of a constant E_s soil, the factors for linearly varying E_s can also be determined from the charts. The charts for $I'_{\rho H}$, $I'_{\rho M}$ and $I'_{\theta H}$ are given on the above slide. Both the factors depend upon the K_N value which was discussed already.

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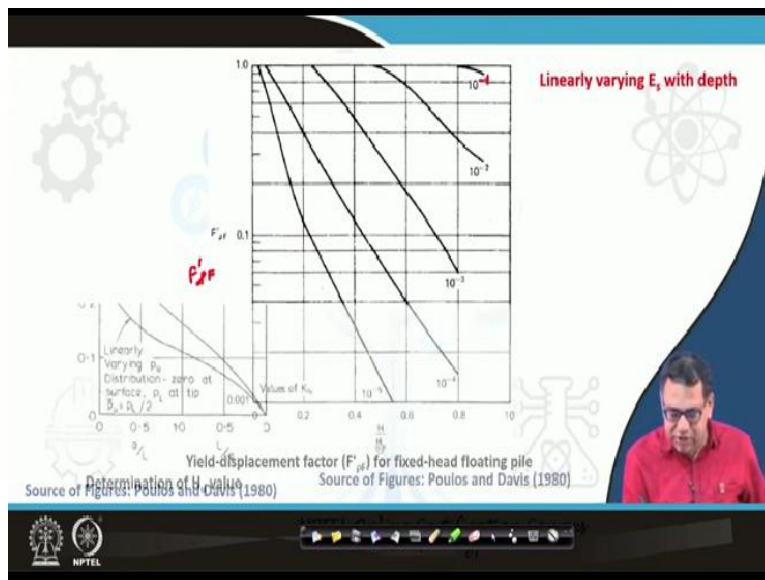


These are the charts for $I'_{\theta M}$ and F'_{ρ} for free headed pile under linearly varying E_s . The influence factor is dependent on the K_N value just like the other two already discussed. But the yield-displacement factor, F'_{ρ} varies with the H/H_u ratio (also similar to constant E_s case).

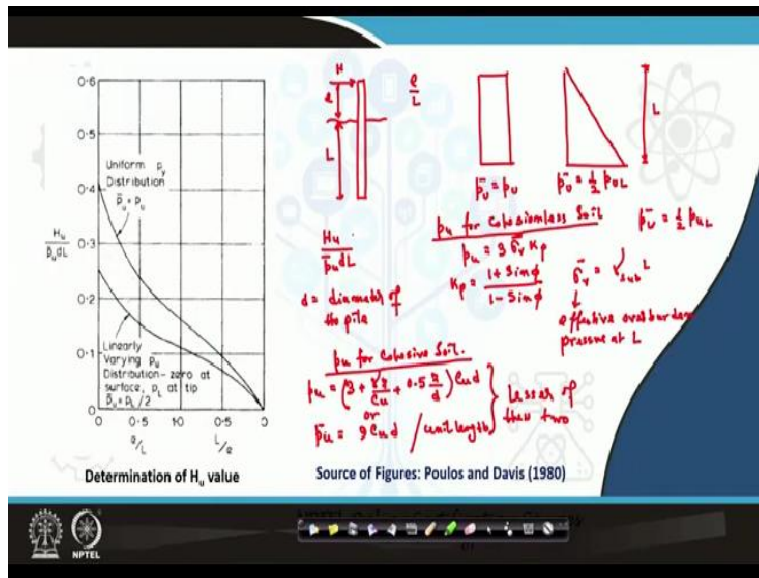
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Similarly the yield-rotation factor, F'_θ can be read from the chart on the left in above slide. For the fixed headed pile, the interaction factor for rotation, $I'_{\theta F}$ is given in the right side chart above.
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Finally, the yield-displacement factor, $F'_{\rho F}$ for a fixed head pile is given here.
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H_u value can be obtained from the above chart. The x axis consists of the e/L and L/e values and the y axis, $\frac{H_u}{\bar{p}_u dL}$. The curve on the top is for uniform distribution of p_u and the other curve is for a linearly varying p_u distribution. This means that the first (top) curve can be used for a soil where E_s is constant along the depth and the other curve is for the case when E_s varies with depth. To determine the yield-rotation or yield-displacement factors, the H/H_u ratio is to be determined and the H_u value can be determined from the above chart.

So to use the chart and determine the H_u value, d and L are already known but the \bar{p}_u should also be known. There are two cases mentioned in the chart for p_u . One is where there is uniform distribution which implies $\bar{p}_u = p_u$. The other case is the linearly varying p_u value where

$\bar{p}_u = P_{ul} / 2$. The p_u value for cohesion less soil can be approximately given as:

$$p_u = 3\bar{\sigma}_v k_p$$

where, $\bar{\sigma}_v$ is the vertical effective over burden pressure at a depth of L , k_p is the coefficient of

passive earth pressure given by: $k_p = \frac{1 + \sin\phi}{1 - \sin\phi}$.

Similarly, for cohesive soils the p_u should be considered the lesser of:

$$p_u = \left(3 + \frac{\gamma_z}{c_u} + 0.5 \frac{z}{d} \right) c_u d$$

OR

$$p_u = 9c_u d$$

Note that p_u value obtained for the cohesive soil is per unit length of the pile shaft. So, when using this p_u value in the expression, $\frac{H_u}{\bar{p}_u d L}$ substitute the value of p_u in place of $\bar{p}_u d$ because the

d is already multiplied it in the expression. Also, the unit of p_u in the above expressions is kN/m but it should be kN in the H_u expression and hence multiply with the length, L alone. But, in the cohesionless soils this confusion is not there and the \bar{p}_u or p_u should be multiplied with $d \times L$.

In the next class, I will show how to determine the settlement of single pile and pile group using these charts and expression. Thank you.