Soil Structure Interaction
Prof Kousik Deb
Department of Civil Engineering
Indian Institute of Technology-Kharagpur

Lecture-63
Soil Structure for Pile Foundation (Contd.)

In the last class I derived the governing differential equation for a beam subjected to axial force, as well as lateral load. The equation was first developed for a beam and then implied to pile.

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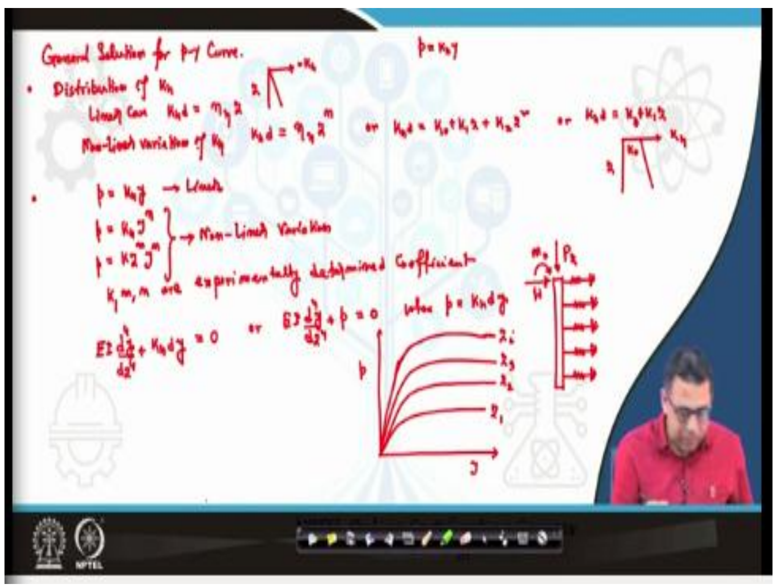

These are the different distributions of k_h discussed in the last class considering both linear and non linear relation between p and y. The expression when the beam is subjected to horizontal load alone without any axial force was also given.

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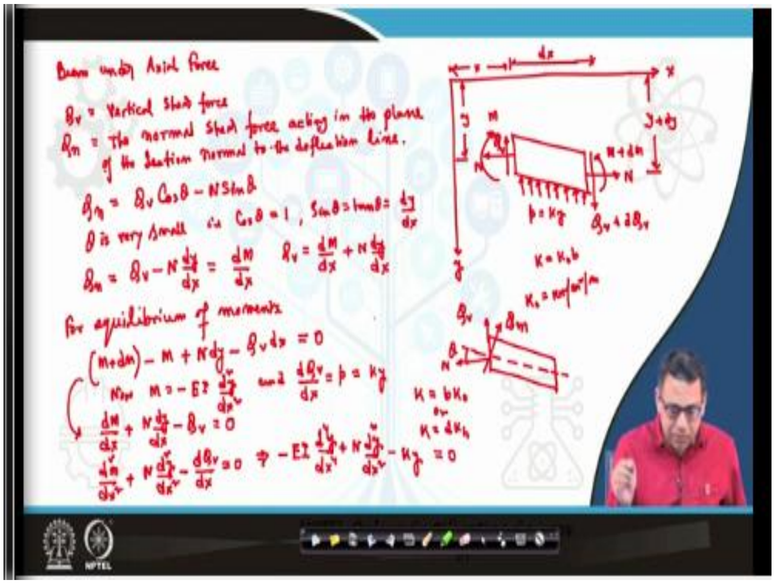

And then the expression for beam under axial force was derived.

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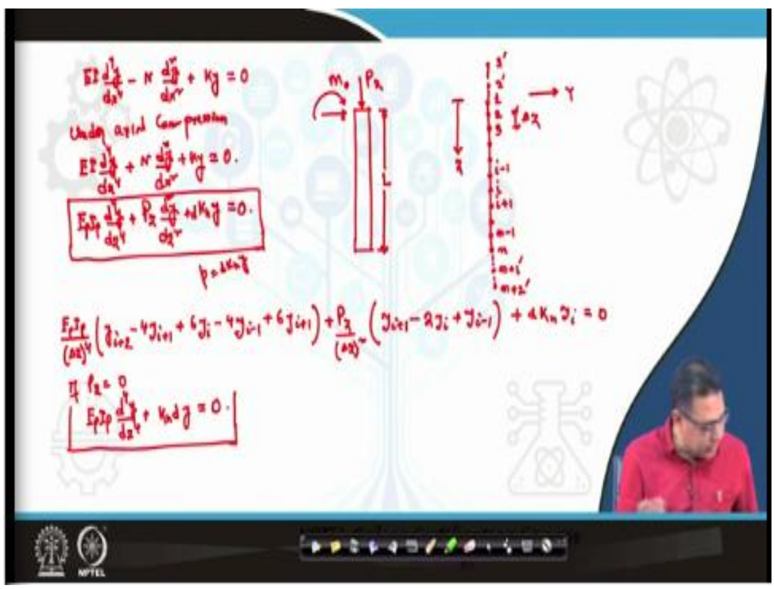

The expression for beam under axial force and the governing differential equation for pile under axial load are given. Now let us solve this equation using finite difference method. The k_h value maybe uniform throughout the soil, or may vary linearly or non-linearly. As long as it is uniform or linearly varying, it is easy to obtain the closed form solution but when the non-linearity comes into picture, it would be difficult to get the closed form solution. This is when the finite difference method proves to be useful.

As discussed in the previous classes, the first step in the finite difference method is to consider that the structure of interest (here, pile) is divided into number of segments. The points which divide the pile into segments are called nodes and it is considered that here, there are n number of nodes.

As the order of the differential equation is 4, we need 4 boundary conditions (just like the beam problem) to obtain a solution for this differential equation. For this, we need four imaginary nodes that may be named: 1′, 2′, (n+1)′ and (n+2)′. The finite difference for of this differential equation is:

$$\frac{E_p I_p}{(\Delta z)^4}\left(y_{i+2}-4y_{i+1}+6y_i-4y_{i-1}+6y_{i+1}\right)+\frac{P_z}{(\Delta z)^2}\left(y_{i+1}-2y_i+y_{i-1}\right)+dk_h y_i=0$$

If $P_z = 0$, the original differential equation will be:

$$E_p I_p \frac{d^4 y}{dx^4}+k_h dy=0$$

This equation is for the case when there is no axial force and this can also be written in the finite difference form.

Note that in these 2 equations, the p-y relationships were considered to be linear which means that k_h is uniform along the depth or that k_h is constant. But using the finite difference method, solutions can be obtained for any type of distribution. The various types of distributions or variations were already discussed earlier.

The variation of k_h is nothing but the shape of p-y curve. When talking about the p-y curve, understand that in some cases it may have different curves at different depths. This is shown in the first slide of this lecture. In that curve, there are a number of graphs which represent different depths (say, z = 1, 2, 3, etc.). The initial portion of the curve is more or less the linear variation and after that, it tends to reach the ultimate value. So, the important observation that is to be made here is that the relation between p and y also changes with depth in some cases.

This is true for the sandy soil, where it was mentioned that the variation of k_h is not uniform, but varies with depth. So the relation between stress and displacement that is valid at one depth for a soil may not be valid at another depth for the same soil. The p-y relationship may change and the k_h variation may change. These two things should be taken care of, when this type of problem is being solved. This is why it is considered that k_h is a function of depth while solving these governing differential equations.

So, a nonlinear relationship between p and y should be considered along with the variation of k_h with depth. The finite difference method makes it possible to incorporate these things in the equation and solve.

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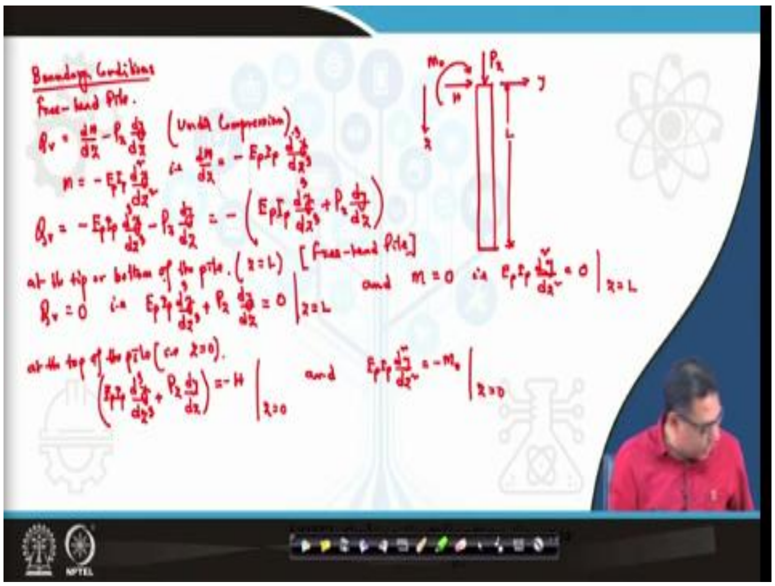

Now let us see the boundary conditions under the loading conditions considered. The loading conditions considered here are: an axial force of P_z, a horizontal load of H and a moment M_o. Under these loading conditions, for a free head pile the boundary conditions will be discussed now. Firstly, the relation between moment and shear force was formulated as:

$$Q_v = \frac{dM}{dx} + N\frac{dy}{dx}$$

This was the relation when the axial force considered was tensile in nature, but here the axial force is P_z (which should be substituted in N) and compressive in nature. So the sign for the axial force term will change. Also, during the derivation for the beam the axial direction was represented by x but here it is the depth which can be represented by z. So the modified equation will be:

$$Q_v = \frac{dM}{dz} - P_z \frac{dy}{dz} \text{ (under compression)}$$

It is known that: $M = -E_p I_p \frac{d^2 y}{dx^2} \Rightarrow \frac{dM}{dx} = -E_p I_p \frac{d^3 y}{dx^3}$

Substituting the above value of M in the equation for Q_v, we get:

$$Q_v = -E_p I_p \frac{d^3 y}{dz^3} - P_z \frac{dy}{dz} = -\left(E_p I_p \frac{d^3 y}{dz^3} + P_z \frac{dy}{dz} \right)$$

Now, the pile is considered long enough (considered as a semi-infinite beam) that at the tip or bottom of the pile, the shear force and moment will be 0. The bottom of pile is when the depth is equal to the pile length, z = L. So:

$$Q_v = 0 \qquad \Rightarrow E_p I_p \frac{d^3 y}{dz^3} + P_z \frac{dy}{dz}\bigg|_{z=L} = 0$$

Now the terms d^3y/dz^3 and dy/dz should be expressed in the finite difference form. From that, 4 unknowns relating to the imaginary nodes, 1′, 2′, (n+1)′ and (n+2)′ can be obtained. Then the imaginary unknowns can be converted or related to the real unknowns. After this using the four boundary conditions (about to be determined) the unknowns can be solved. Once all the unknowns are real, they can be solved with the n number of equations to find out the deflection and other quantities at any point.

There is another boundary condition; the moment at the base of the pile is 0:

$$M = 0 \Rightarrow -E_p I_p \frac{d^2 y}{dx^2}\bigg|_{z=L} = 0$$

The above is the boundary condition at the base of the pile at a depth of L. At z = 0 or at the pile head, a moment M_o is applied and as it is a free head pile, the moment value at z = 0 will remain M_o. So, the boundary condition for bending moment at z = 0 will be:

$$E_p I_p \frac{d^2 y}{dx^2}\bigg|_{z=0} = -M_\circ$$

Similarly, the boundary condition related to shear force at z = 0 will be:

$$\left(E_p I_p \frac{d^3 y}{dz^3} + P_z \frac{dy}{dz} \right)\bigg|_{z=0} = -H$$

The signs in the above two boundary conditions depend upon the directions of the applied forces. Here, both M_o and H are considered to be positive, but the sign may vary if their direction changes.

The four unknowns that are in excess because of the four imaginary nodes are now balanced with the four boundary conditions. Now, as the number of unknowns and number of equations are equal, the solution can be obtained.

Remember that if the effect P_z is neglected, then in the place of P_z, 0 should be substituted only when the responses will be free from the effect of P_z. If the response for axial force is to be studied, the appropriate value of P_z should be substituted in the equations.

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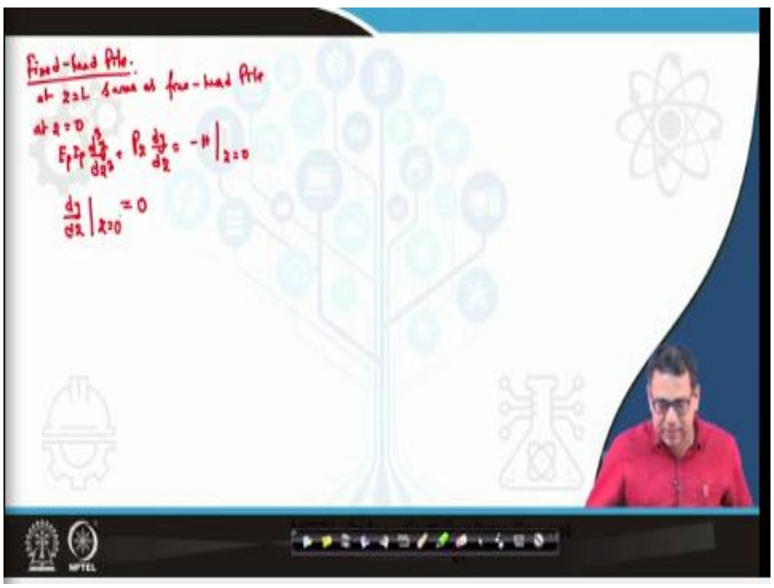

Till now, the case of free head pile has been dealt with. Now, let us start the discussion about fixed head pile. The boundary conditions at z = L will be the same as that of free head pile because the only difference between fixed head and free head piles is the fixity at pile head. Also, at z = 0, the shear force boundary condition is the same:

$$\left(E_p I_p \frac{d^3 y}{dz^3} + P_z \frac{dy}{dz} \right)\Bigg|_{z=0} = -H$$

The only difference in both the cases is the other boundary condition. Here, due to the fixity of the pile head, the slope will also be zero at the pile head. By considering this, the generalized solution for lateral loaded pile using subgrade modulus concept can be determined.

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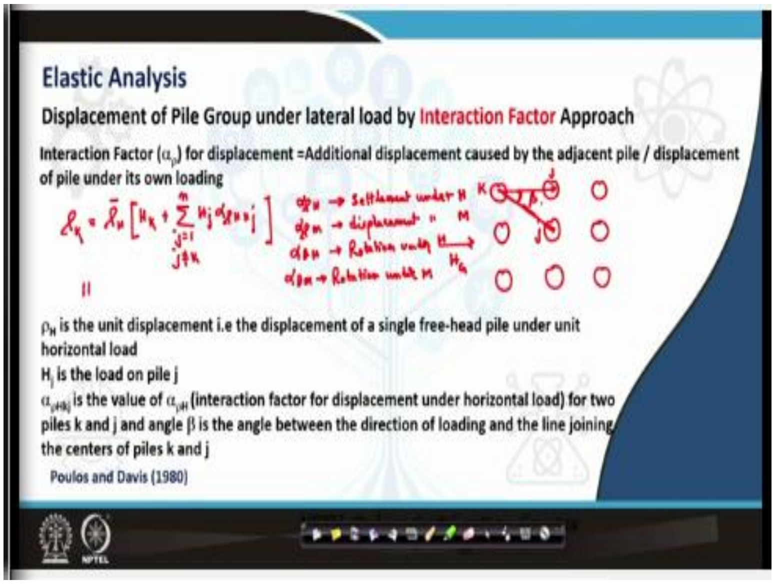

The next concept under the laterally loaded piles is the elastic analysis or the interaction factor approach. Till now, the laterally loaded piles are analyzed using the subgrade modulus approach and the quantities determined were for the single pile. But piles are often used as groups and so the pile group response under lateral load should also be studied.

The pile group response, the procedure to determine the expressions and the method to use them were discussed for the pile groups under compressive load using elastic analysis. Now, in a similar way the elastic analysis approach will be dealt for the laterally loaded piles. The interaction factor approach basically deals with the interaction among piles which effects the settlement. In a pile group of n piles, settlement of the kth pile can be given as:

$$\rho_k = \bar{\rho}_H \left[H_k + \sum_{\substack{j=1 \\ j \neq k}}^{n} H_j \alpha_{\rho Hkj} \right]$$

where, H_G is the horizontal load applied on the pile group, H is the horizontal load applied on a single pile, $\bar{\rho}_H$ is the unit settlement or displacement of a single free head pile under unit horizontal load, H_k is the horizontal load taken by the kth pile (part of H_G that acts on the kth pile),

$\alpha_{\rho Hkj}$ is the interaction factor for settlement between the k and j piles (here, ρ refers to settlement, H to horizontal load, kj to the piles between which the factor is being calculated), H_j is the horizontal load taken by the j^{th} pile.

Another term which is not used in the above expression directly, but will be required during the calculation is β. β is the angle between the load direction and the line joining k and j piles. The $\bar{\rho}_H$ is multiplied directly with the H_k, (the load on the pile for which settlement is determined) because the interaction factor for the own pile will be 1 (i.e., $\alpha_{\rho Hkk} = 1$).

Using this information, the settlement of a particular pile in a group can be calculated considering the effect of all other piles in that group. In the above expression, only the settlement was taken care of, but interaction between the piles will affect other quantities also. There are 3 more interaction factors to be considered.

$\alpha_{\rho H}$ is for settlement under H

$\alpha_{\rho M}$ is for displacement under M

$\alpha_{\theta H}$ is for rotation under H

$\alpha_{\theta M}$ is for rotation under M

There is a chance that moment may also come into picture in the lateral direction on a pile group. The best example is when the pile cap is at a height above the ground level and a horizontal load is applied at the pile cap. So, there are two types of interaction factor here: one for settlement / displacement and one for moment. If the rotation should be calculated, the interaction factor rotation should be considered. If any pile group is subjected to both horizontal load and moment, both type of interaction factors should be considered.

Before calculating the load each pile is taking in a group, it should be remembered that the sum of an individual load each pile is subjected to, is the group horizontal load, H_G. There are some assumptions involved in this analysis as: for a free headed pile group, horizontal load and moment may be applied, but for a fixed head pile group only horizontal load can be applied. Also, both for the fixed head and fee headed pile groups, all the piles are subjected to equal amount of displacement.

In the next class I will show how to determine these interaction factors under different conditions (free headed or fixed head). Then, we will see how to determine the settlement and the rotation of a single pile. To calculate the deflection and rotation of a pile group, we need to know the settlement and rotation of a single pile. Thank you.