Soil Structure Interaction
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Lecture-62
Soil Structure Interaction for Pile Foundation (Contd.

In this class I will first discuss about the solution for the generalized expression of a beam to determine the deflection and the other quantities for a laterally loaded pile.

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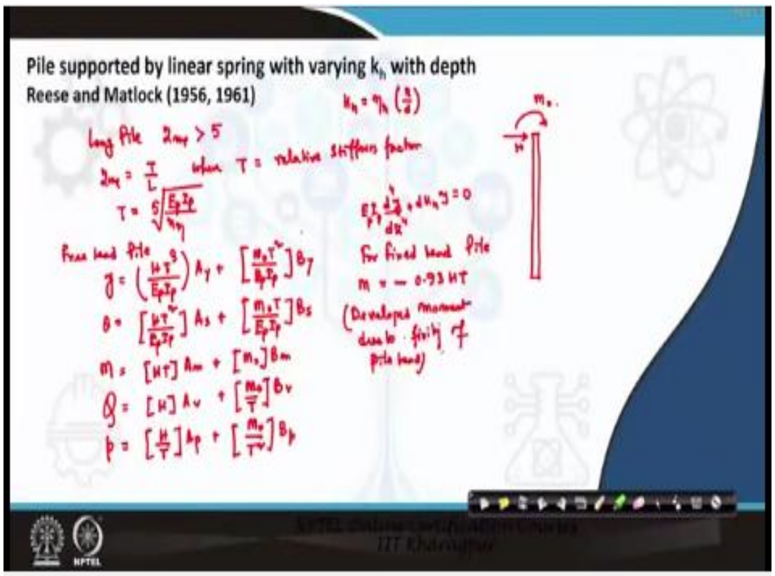

An example using the Reese and Matlock approach was solved already and the determination of deflection for both free head pile and fix head pile was shown. Also, calculating the deflection of a fix head pile and free head pile using Hetenyi approach was discussed. Hetenyi approach is for constant k_h and the Reese and Matlock approach is for linearly varying k_h with depth.

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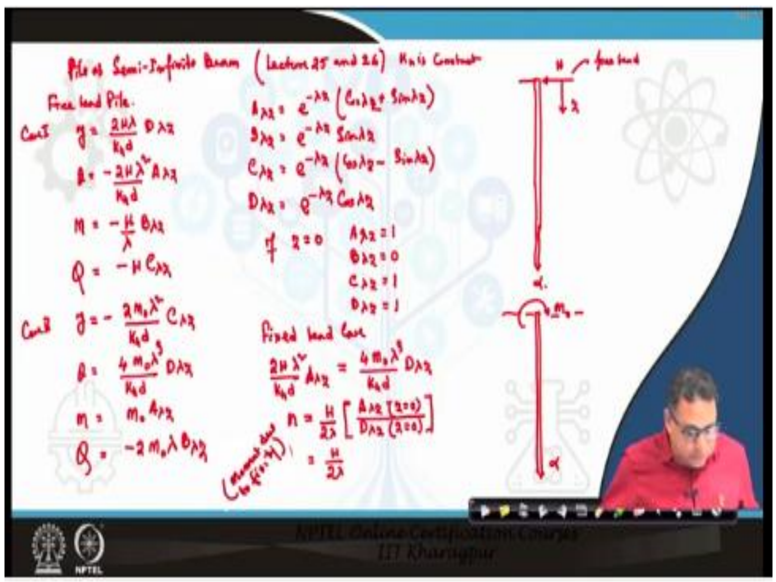

If the pile is to be considered as a semi infinite beam, it is obvious that it should be a very long pile or an infinitely long pile. The concept of semi infinite beam discussed in lectures 25 and 26 can be referred to, for clarity about the analysis of semi-infinite beams.

If a free head pile is long enough to consider it as a semi-infinite beam, it can be considered as a semi-infinite beam with free ends as the boundary condition. Consider the case-1 loading, a horizontal point load of H acting on the pile. In the pile, the head which is at the ground surface (here) is the free end and the base of the pile will be the infinite end. So, for the case-1 loading the end conditioning forces and the procedure discussed for the pile as a beam of finite length can be applied.

So, the expression for deflection, y is:

$$y = \frac{2H\lambda}{k_h d} D_{\lambda x}$$

$$\theta = -\frac{2H\lambda^2}{k_h d} A_{\lambda x}$$

$$M = -\frac{H}{\lambda} B_{\lambda x}$$

$$Q = -HC_{\lambda x}$$

If the pile is subjected to a concentrated moment (case-2 loading):

$$y = -\frac{2M_\circ \lambda^2}{k_h d} C_{\lambda x}$$

$$\theta = \frac{4M_\circ \lambda^3}{k_h d} D_{\lambda x}$$

$$M = M_\circ A_{\lambda x}$$

$$Q = -2M_\circ \lambda B_{\lambda x}$$

The fixed head piles can also be solved similarly. By equating the deflection in free end to the slope restricted due to the fixity, the additional moment developed at the fixed end can be determined.

$$\frac{2H\lambda^2}{k_h d} A_{\lambda z} = \frac{4M_\circ \lambda^3}{k_h d} D_{\lambda x}$$

$$\Rightarrow M = \frac{H}{2\lambda}\left[\frac{A_{\lambda z}}{D_{\lambda z}}\right]_{z=0}$$

$$\therefore M = \frac{H}{2\lambda} \quad \text{(As, at z = 0. } A_{\lambda z} = D_{\lambda z} = 1\text{)}$$

This is the moment that has to be applied at the ground level.

Now the question is when can these piles be considered as semi infinite beams.

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λL	z/L	$K_{\rho H}$ ≈ $D_{\lambda z}$	$K_{\theta H}$	K_{MH}	K_{QH}	$K_{\rho M}$	$K_{\theta M}$	K_{MM}	K_{QM}
2.0	0.	1.1376	1.1341	0.	1.0000	-1.0762	1.0762	1.0000	0.
2.0	0.0625	0.9964	1.1200	0.1080	0.7333	-0.8807	0.9519	0.9836	0.1256
2.0	0.1250	0.8586	1.0828	0.1848	0.5015	-0.6579	0.8314	0.9397	0.2214
2.0	0.1875	0.7264	1.0298	0.2347	0.3035	-0.4644	0.7178	0.8751	0.2913
2.0	0.2500	0.6015	0.9673	0.2620	0.1377	-0.2982	0.6133	0.7959	0.3387
2.0	0.3125	0.4848	0.9004	0.2704	0.0021	-0.1569	0.5192	0.7073	0.3669
2.0	0.3750	0.3764	0.8333	0.2637	-0.1054	-0.0376	0.4366	0.6138	0.3788
2.0	0.4375	0.2763	0.7695	0.2452	-0.1868	0.0624	0.3658	0.5191	0.3771
2.0	0.5000	0.1838	0.7115	0.2180	-0.2442	0.1463	0.3068	0.4262	0.3639
2.0	0.5625	0.0981	0.6610	0.1851	-0.2793	0.2168	0.2591	0.3379	0.3411
2.0	0.6250	0.0182	0.6192	0.1491	-0.2937	0.2767	0.2220	0.2564	0.3101
2.0	0.6875	-0.0571	0.5865	0.1125	-0.2887	0.3286	0.1946	0.1834	0.2722
2.0	0.7500	-0.1288	0.5628	0.0776	-0.2654	0.3747	0.1757	0.1208	0.2282
2.0	0.8125	-0.1981	0.5474	0.0468	-0.2245	0.4171	0.1640	0.0698	0.1787
2.0	0.8750	-0.2659	0.5389	0.0222	-0.1665	0.4572	0.1578	0.0318	0.1241
2.0	0.9375	-0.3330	0.5356	0.0059	-0.0916	0.4963	0.1554	0.0082	0.0645
2.0	1.0000	-0.3999	0.5351	0.	-0.0000	0.5351	0.1551	0.0000	0.

Source of Table: Poulos and Davis (1980)

The expressions for piles with finite length and for semi-infinite piles are similar apart from a minor difference of the co efficient. Above chart is already given for the piles with finite length and may replicate for the semi-infinite pile too. The $k_{\rho H}$ term is almost equivalent to $D_{\lambda z}$ in the semi-infinite case. But here, the value of $D_{\lambda z}$ for z/L = 0 is 1.1376 which is not the same. So, for a λL value of 2, these charts cannot be used for semi-infinite piles. So, these charts can be used only if the pile is of finite length.

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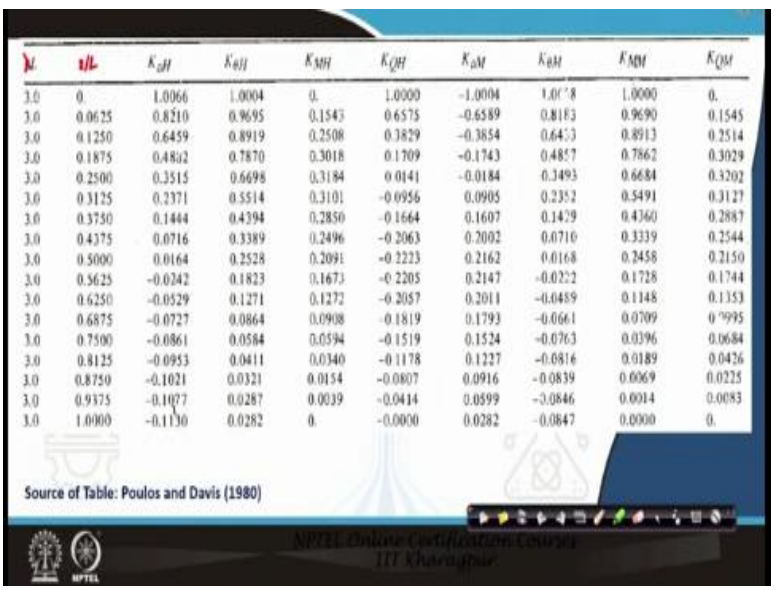

λL	z/L	$K_{\rho H}$	$K_{\theta H}$	K_{MH}	K_{QH}	$K_{\rho M}$	$K_{\theta H}$	K_{MM}	K_{QM}
3.0	0.	1.0066	1.0004	0.	1.0000	-1.0004	1.0078	1.0000	0.
3.0	0.0625	0.8210	0.9695	0.1543	0.6575	-0.6589	0.8183	0.9690	0.1545
3.0	0.1250	0.6459	0.8919	0.2508	0.3829	-0.3854	0.6433	0.8913	0.2514
3.0	0.1875	0.4842	0.7870	0.3018	0.1709	-0.1743	0.4857	0.7862	0.3029
3.0	0.2500	0.3515	0.6698	0.3184	0.0141	-0.0184	0.3493	0.6684	0.3202
3.0	0.3125	0.2371	0.5514	0.3101	-0.0956	0.0905	0.2352	0.5491	0.3127
3.0	0.3750	0.1444	0.4394	0.2850	-0.1664	0.1607	0.1429	0.4360	0.2887
3.0	0.4375	0.0716	0.3389	0.2496	-0.2063	0.2002	0.0710	0.3339	0.2544
3.0	0.5000	0.0164	0.2528	0.2091	-0.2223	0.2162	0.0168	0.2458	0.2150
3.0	0.5625	-0.0242	0.1823	0.1673	-0.2205	0.2147	-0.0222	0.1728	0.1744
3.0	0.6250	-0.0529	0.1271	0.1272	-0.2057	0.2011	-0.0489	0.1148	0.1353
3.0	0.6875	-0.0727	0.0864	0.0908	-0.1819	0.1793	-0.0661	0.0709	0.0995
3.0	0.7500	-0.0861	0.0584	0.0594	-0.1519	0.1524	-0.0763	0.0396	0.0684
3.0	0.8125	-0.0953	0.0411	0.0340	-0.1178	0.1227	-0.0816	0.0189	0.0426
3.0	0.8750	-0.1021	0.0321	0.0154	-0.0807	0.0916	-0.0839	0.0069	0.0225
3.0	0.9375	-0.1077	0.0287	0.0039	-0.0414	0.0599	-0.0846	0.0014	0.0083
3.0	1.0000	-0.1130	0.0282	0.	-0.0000	0.0282	-0.0847	0.0000	0.

Source of Table: Poulos and Davis (1980)

For a λL value of 3, the coefficient value is approaching 1 (1.0066). As λL value increases the mean length of the pile increases.

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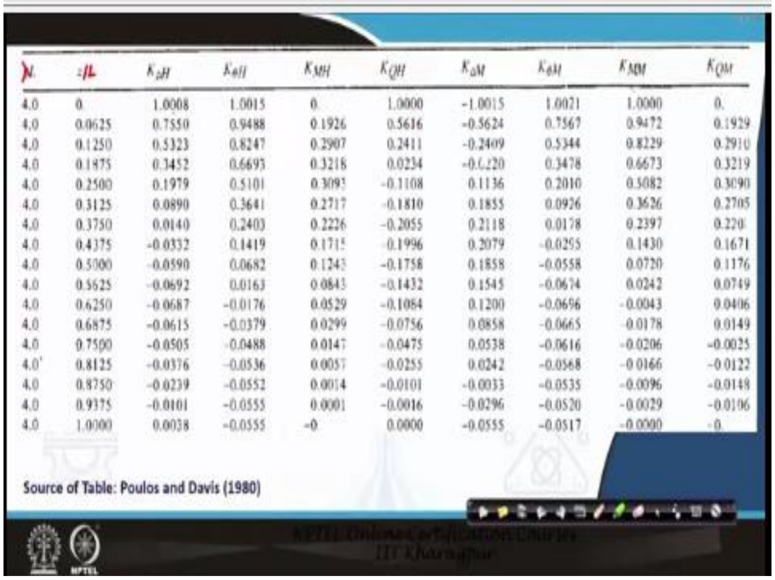

λL	z/L	$K_{\rho H}$	$K_{\theta H}$	K_{MH}	K_{QH}	$K_{\rho M}$	$K_{\theta M}$	K_{MM}	K_{QM}
4.0	0	1.0008	1.0015	0.	1.0000	-1.0015	1.0021	1.0000	0.
4.0	0.0625	0.7550	0.9488	0.1926	0.5616	-0.5624	0.7567	0.9472	0.1929
4.0	0.1250	0.5323	0.8247	0.2907	0.2411	-0.2409	0.5344	0.8229	0.2910
4.0	0.1875	0.3452	0.6693	0.3218	0.0234	-0.0220	0.3478	0.6673	0.3219
4.0	0.2500	0.1979	0.5101	0.3093	-0.1108	0.1136	0.2010	0.5082	0.3090
4.0	0.3125	0.0890	0.3641	0.2717	-0.1810	0.1855	0.0926	0.3626	0.2705
4.0	0.3750	0.0140	0.2403	0.2226	-0.2055	0.2118	0.0178	0.2397	0.220
4.0	0.4375	-0.0332	0.1419	0.1715	-0.1996	0.2079	-0.0255	0.1430	0.1671
4.0	0.5000	-0.0590	0.0682	0.1243	-0.1758	0.1858	-0.0558	0.0720	0.1176
4.0	0.5625	-0.0692	0.0163	0.0843	-0.1432	0.1545	-0.0674	0.0242	0.0749
4.0	0.6250	-0.0687	-0.0176	0.0529	-0.1084	0.1200	-0.0696	-0.0043	0.0406
4.0	0.6875	-0.0615	-0.0379	0.0299	-0.0756	0.0858	-0.0665	-0.0178	0.0149
4.0	0.7500	-0.0505	-0.0488	0.0147	-0.0475	0.0538	-0.0616	-0.0206	-0.0025
4.0	0.8125	-0.0376	-0.0536	0.0057	-0.0255	0.0242	-0.0568	-0.0166	-0.0122
4.0	0.8750	-0.0239	-0.0552	0.0014	-0.0101	-0.0033	-0.0535	-0.0096	-0.0148
4.0	0.9375	-0.0101	-0.0555	0.0001	-0.0016	-0.0296	-0.0520	-0.0029	-0.0106
4.0	1.0000	0.0038	-0.0555	-0	0.0000	-0.0555	-0.0517	-0.0000	-0

Source of Table: Poulos and Davis (1980)

Similarly for a λL value of 4, the coefficient value is 1.0003. So, if the λL is greater than 2, the coefficient values are almost matching and hence, usually if λL is greater than 2.5 then it is called as a long pile. The concept of semi infinite beam can also be used only in such case. But, usually the pile with finite length will be used for a λL value up to 5. If the λL is greater than 5, then the pile must be considered as a semi-infinite pile only because there are no coefficients available for such case.

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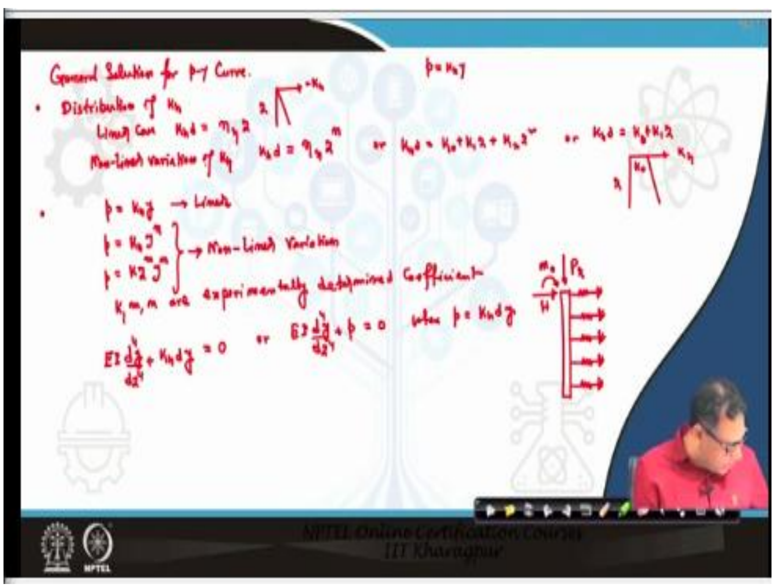

Now, a generalized solution of pile and the procedure to solve it using the finite difference method will be discussed. That generalized solution is proposed by Reese and Matlock. This is nothing but the generalized solution of p-y curve. The relation between the deflection and the stress is given by: $p = k_h \times y$.

So the distribution of k_h for the first case where k_h is linear, will be: $k_h \times d = \eta_h \times z$

Similarly, if the k_h is non linear, the distribution can be written in the following forms:

$$k_h \times d = \eta_h \times z^n$$

OR

$$k_h\, d = k_0 + k_1\, z + k_2\, z2 + \ldots.$$

OR

$$k_h\, d = k_0 + k_1\, z$$

The first two representations are truly non-linear variations. The last one is a linear variation, but with an initial value of k_0.

From the initial discussions, from the p-y curve: $p = k_h \times y$ which is the linear variation.

If the non-linearity has to be introduced in this, we may consider that:

$$p = k_h \times y^n \text{ or } p = k_h \times z^m \times y^n$$

The coefficients k_h, m and n are experimentally determined. So any type of variation of k_h or any type of relationship between the p and y can be considered (linear or nonlinear).

Now consider the general beam expression applied to the case of piles:

$$EI\frac{d^4y}{dz^4} + k_h dy = 0 \text{ (or) } EI\frac{d^4y}{dz^4} + p = 0 \ \{\text{As } p = k_h\, dy\}$$

This is the general beam expression which is being used for the pile and springs where horizontal load, H or moment, M may be applied. But in addition to H, pile is also subjected to an axial force. That means that the pile is subjected force along with the horizontal load in almost all the cases. So, in addition to H, the pile experiences a P_z and sometimes moment. But in the normal beam expression, the consideration is only for H or M. So considering this case the expression for beam, rather pile under axial load should be developed.

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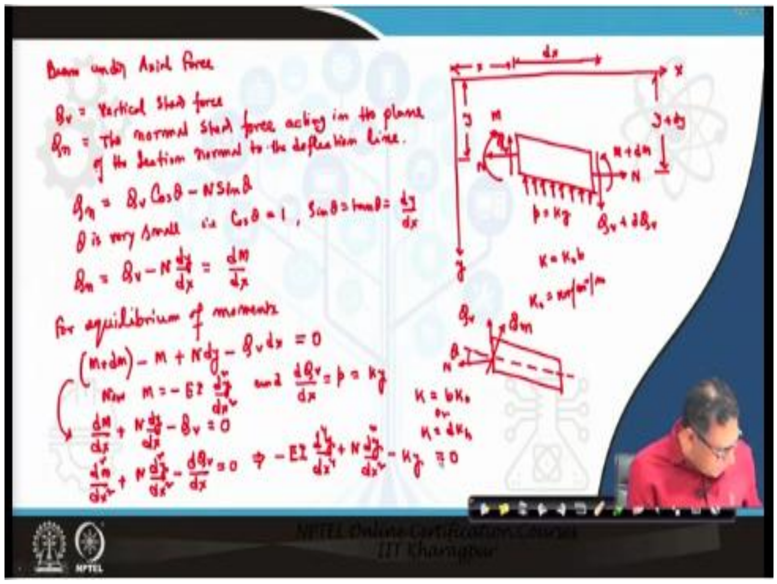

Now the expression for beam under axial load or axial force will be developed. First the expression will be developed for the beam which can be used for the pile. Consider a small segment of beam with length dx as shown in the first figure above. As the beam rests on soil and the soil can be idealized by springs, the reaction it experiences at the bottom will be: $p = k \times y$. Remember that here, $k = k_o \times b$ and that the units of k_o are $kN/m^2/m$.

Then consider an axial force or axial tension, N acts on either side of the beam element. A shear force in the vertical direction, Q_v is assumed to act on the left side phase of the element while a shear force of Q_v+dQ_v is assumed to act on the right side. Similarly a moment of M acts on the left side and a moment of M+dM acts on the right side of the element. The distance from the origin to the left side face of the element is x. The distance from the x axis to the axial force, N acting on the left side face is y and the distance from the x axis to the N on the right side face is y+dy.

In the initial case, the pile or the beam was not subjected to axial tension N, but now it has been introduced. The deformed shape of the beam would be like the second figure in the above slide. The deflection line or the axis of the deformed shape is indicated by the dashed line in the second figure and in this deformed element, the Q_v acts in the vertical direction as discussed already, but the Q_b will act with a slight deviation from the vertical as it will be normal to the deflection line. This slight deviation is quantified as an angle, θ. Q_v is the vertical shear force, Q_n is the normal shear force acting in the plane of the section normal to the deflection line. Now the relationship between these two shear forces will be:

$$Q_n = Q_v \cos\theta - N \sin\theta$$

As θ is very small, $\cos\theta = 1$ and $\sin\theta \approx \tan\theta \approx \dfrac{dy}{dx}$

$$\Rightarrow Q_n = Q_v - N\frac{dy}{dx} = \frac{dM}{dx}$$

Considering the equilibrium of moments:

$$(M + dM) - M + Ndy - Q_v dx = 0$$

$$\Rightarrow \frac{dM}{dx} + N\frac{dy}{dx} - Q_v = 0$$

Differentiating the above expression w.r.t x:

$$\Rightarrow \frac{d^2M}{dx^2} + N\frac{d^2y}{dx^2} - \frac{dQ_v}{dx} = 0$$

It is known that: $M = -EI\dfrac{d^2y}{dx^2}$ and $\dfrac{dQ_v}{dx} = p = ky$

$$\Rightarrow -EI\frac{d^4y}{dx^4} + N\frac{d^2y}{dx^2} - ky = 0$$

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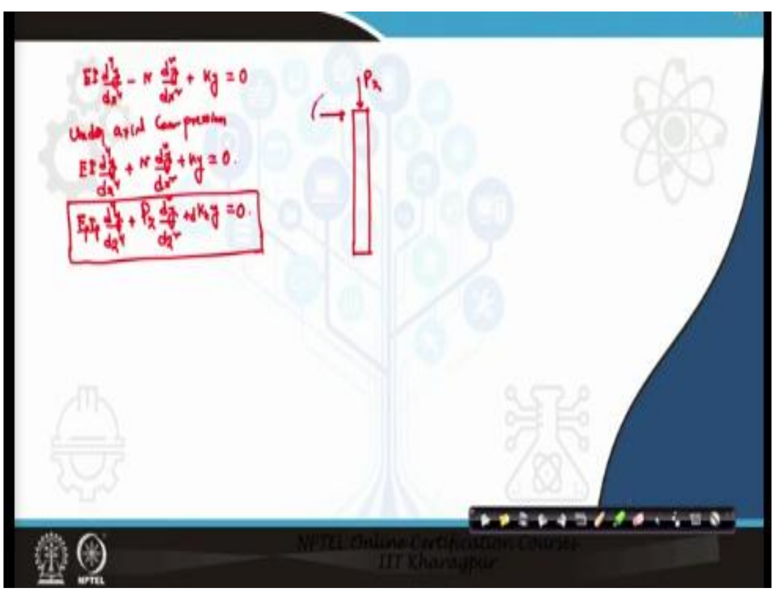

So the final expression will be:

$$EI\frac{d^4y}{dx^4} - N\frac{d^2y}{dx^2} + ky = 0$$

The above is the expression for beam under axial force when that force is tensile in nature. But for piles, the axial force will be compressive in nature in most of the cases and hence the expression in case of a compressive axial force will be:

$$EI\frac{d^4y}{dx^4} + N\frac{d^2y}{dx^2} + ky = 0$$

This is the governing differential equation for a beam under axial compression. If this is implied to the piles under an axial compression force of P_z:

$$E_pI_p\frac{d^4y}{dx^4} + P_z\frac{d^2y}{dx^2} + ky = 0$$

Here the k_h is considered to be constant throughout the pile length.

In the next class I will provide the boundary conditions for this governing differential equation because we have to solve this governing differential equation. So, I will show you how to solve this differential equation by using the boundary conditions and to apply the finite difference method to solve these equations. So, I will discuss that boundary conditions and solution procedure using the finite different scheme in the next class. Thank you.