Soil Structure Interaction
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Lecture-61
Soil Structure Interaction for Pile Foundation (Contd.)

In the last class I discussed about the determination of pile deflection, slope, bending moment and shear force based on the Hetenyi approach where the pile is modeled as a beam with finite length and I have given the expressions of different coefficients.

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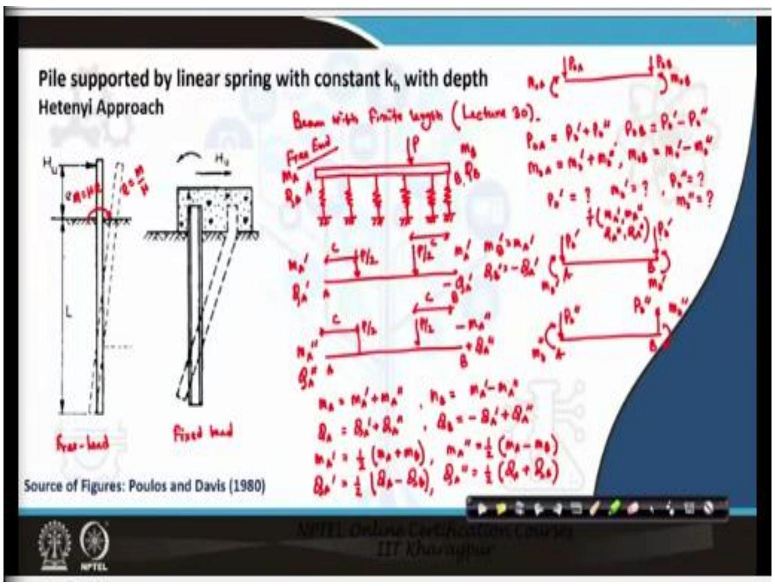

This is the procedure discussed in the last class.

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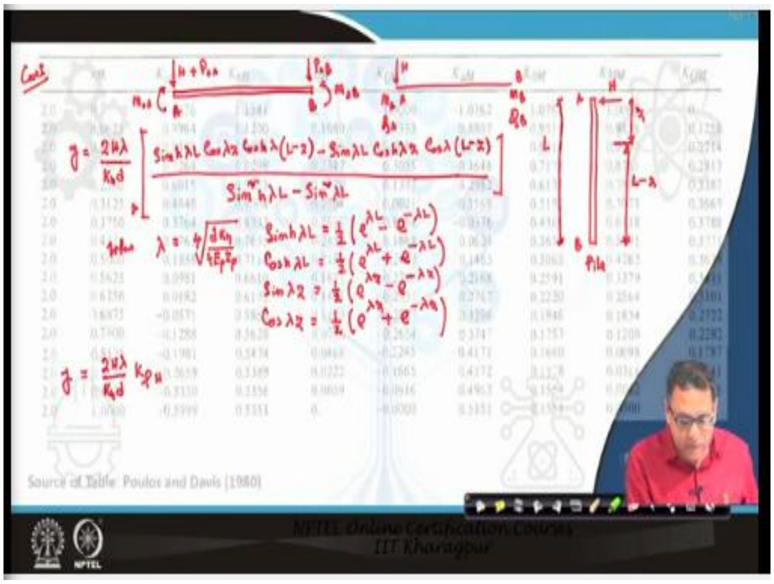

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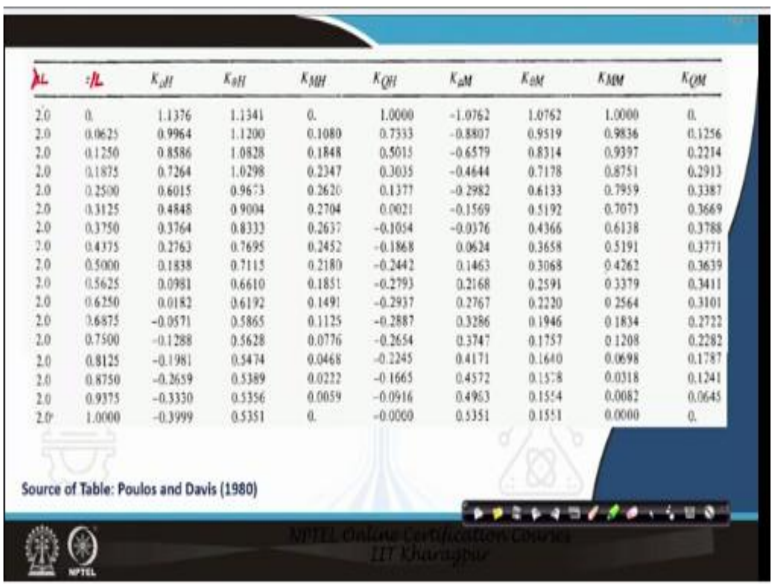

λL	z/L	$K_{\rho H}$	$K_{\theta H}$	K_{MH}	K_{QH}	$K_{\rho M}$	$K_{\theta M}$	K_{MM}	K_{QM}
2.0	0.	1.1376	1.1341	0.	1.0000	-1.0762	1.0762	1.0000	0.
2.0	0.0625	0.9964	1.1200	0.1080	0.7333	-0.8807	0.9519	0.9836	0.1256
2.0	0.1250	0.8586	1.0828	0.1848	0.5015	-0.6579	0.8314	0.9397	0.2214
2.0	0.1875	0.7264	1.0298	0.2347	0.3035	-0.4644	0.7178	0.8751	0.2913
2.0	0.2500	0.6015	0.9673	0.2620	0.1377	-0.2982	0.6133	0.7959	0.3387
2.0	0.3125	0.4848	0.9004	0.2704	0.0021	-0.1569	0.5192	0.7073	0.3669
2.0	0.3750	0.3764	0.8333	0.2637	-0.1054	-0.0376	0.4366	0.6138	0.3788
2.0	0.4375	0.2763	0.7695	0.2452	-0.1868	0.0624	0.3658	0.5191	0.3771
2.0	0.5000	0.1838	0.7115	0.2180	-0.2442	0.1463	0.3068	0.4262	0.3639
2.0	0.5625	0.0981	0.6610	0.1851	-0.2793	0.2168	0.2591	0.3379	0.3411
2.0	0.6250	0.0182	0.6192	0.1491	-0.2937	0.2767	0.2220	0.2564	0.3101
2.0	0.6875	-0.0571	0.5865	0.1125	-0.2887	0.3286	0.1946	0.1834	0.2722
2.0	0.7500	-0.1288	0.5628	0.0776	-0.2654	0.3747	0.1757	0.1208	0.2282
2.0	0.8125	-0.1981	0.5474	0.0468	-0.2245	0.4171	0.1640	0.0698	0.1787
2.0	0.8750	-0.2659	0.5389	0.0222	-0.1665	0.4572	0.1578	0.0318	0.1241
2.0	0.9375	-0.3330	0.5356	0.0059	-0.0916	0.4963	0.1554	0.0082	0.0645
2.0	1.0000	-0.3999	0.5351	0.	-0.0000	0.5351	0.1551	0.0000	0.

Source of Table: Poulos and Davis (1980)

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The values of various coefficients required to determine the various quantities are given in charts (in the slides above and below). These values are given for different z/L ratios and different λL values. The above chart, in particular gives the values of the coefficients for a λL value of 2.

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λL	z/L	$K_{\rho H}$	$K_{\theta H}$	K_{MH}	K_{QH}	$K_{\rho M}$	$K_{\theta M}$	K_{MM}	K_{QM}
3.0	0.	1.0066	1.0004	0.	1.0000	-1.0004	[illegible]	1.0000	0.
3.0	0.0625	0.8210	0.9695	0.1543	0.6575	-0.6589	0.8183	0.9690	0.1545
3.0	0.1250	0.6459	0.8919	0.2508	0.3829	-0.3854	0.6453	0.8913	0.2514
3.0	0.1875	0.4842	0.7870	0.3018	0.1709	-0.1743	0.4857	0.7862	0.3029
3.0	0.2500	0.3515	0.6698	0.3184	0.0141	-0.0184	0.3493	0.6684	0.3202
3.0	0.3125	0.2371	0.5514	0.3101	-0.0956	0.0905	0.2352	0.5491	0.3127
3.0	0.3750	0.1444	0.4394	0.2850	-0.1664	0.1607	0.1429	0.4360	0.2887
3.0	0.4375	0.0716	0.3389	0.2496	-0.2063	0.2002	0.0710	0.3339	0.2544
3.0	0.5000	0.0164	0.2528	0.2091	-0.2223	0.2162	0.0168	0.2458	0.2150
3.0	0.5625	-0.0242	0.1823	0.1673	-0.2205	0.2147	-0.0222	0.1728	0.1744
3.0	0.6250	-0.0529	0.1271	0.1272	-0.2057	0.2011	-0.0489	0.1148	0.1353
3.0	0.6875	-0.0727	0.0864	0.0908	-0.1819	0.1793	-0.0661	0.0709	0.0995
3.0	0.7500	-0.0861	0.0584	0.0594	-0.1519	0.1524	-0.0763	0.0396	0.0684
3.0	0.8125	-0.0953	0.0411	0.0340	-0.1178	0.1227	-0.0816	0.0189	0.0426
3.0	0.8750	-0.1021	0.0321	0.0154	-0.0807	0.0916	-0.0839	0.0069	0.0225
3.0	0.9375	-0.1077	0.0287	0.0039	-0.0414	0.0599	-0.0846	0.0014	0.0083
3.0	1.0000	-0.1130	0.0282	0.	-0.0000	0.0282	-0.0847	0.0000	0.

Source of Table: Poulos and Davis (1980)

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The above chart is for a λL value of 3.

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λL	z/L	$K_{\rho H}$	$K_{\theta H}$	K_{MH}	K_{QH}	$K_{\rho M}$	$K_{\theta M}$	K_{MM}	K_{QM}
4.0	0.	1.0008	1.0015	0.	1.0000	-1.0015	1.0021	1.0000	0.
4.0	0.0625	0.7550	0.9488	0.1926	0.5616	-0.5624	0.7567	0.9472	0.1929
4.0	0.1250	0.5323	0.8247	0.2907	0.2411	-0.2409	0.5344	0.8229	0.2910
4.0	0.1875	0.3452	0.6693	0.3218	0.0234	-0.0220	0.3478	0.6673	0.3219
4.0	0.2500	0.1979	0.5101	0.3092	-0.1108	0.1136	0.2010	0.5082	0.3090
4.0	0.3125	0.0890	0.3641	0.2717	-0.1810	0.1855	0.0926	0.3626	0.2705
4.0	0.3750	0.0140	0.2403	0.2226	-0.2055	0.2118	0.0178	0.2397	0.2203
4.0	0.4375	-0.0332	0.1419	0.1715	-0.1996	0.2079	-0.0255	0.1430	0.1671
4.0	0.5000	-0.0590	0.0682	0.1243	-0.1758	0.1858	-0.0558	0.0720	0.1176
4.0	0.5625	-0.0692	0.0163	0.0843	-0.1432	0.1545	-0.0674	0.0242	0.0749
4.0	0.6250	-0.0687	-0.0176	0.0529	-0.1084	0.1200	-0.0696	-0.0043	0.0406
4.0	0.6875	-0.0615	-0.0379	0.0299	-0.0756	0.0858	-0.0665	-0.0178	0.0149
4.0	0.7500	-0.0505	-0.0488	0.0147	-0.0475	0.0538	-0.0616	-0.0206	-0.0025
4.0	0.8125	-0.0376	-0.0536	0.0057	-0.0255	0.0242	-0.0568	-0.0166	-0.0122
4.0	0.8750	-0.0239	-0.0552	0.0014	-0.0101	-0.0033	-0.0535	-0.0096	-0.0148
4.0	0.9375	-0.0101	-0.0555	0.0001	-0.0016	-0.0296	-0.0520	-0.0029	-0.0106
4.0	1.0000	0.0038	-0.0555	-0.	0.0000	-0.0555	-0.0517	-0.0000	-0.

Source of Table: Poulos and Davis (1980)

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Similarly the above chart is for a λL value of 4.

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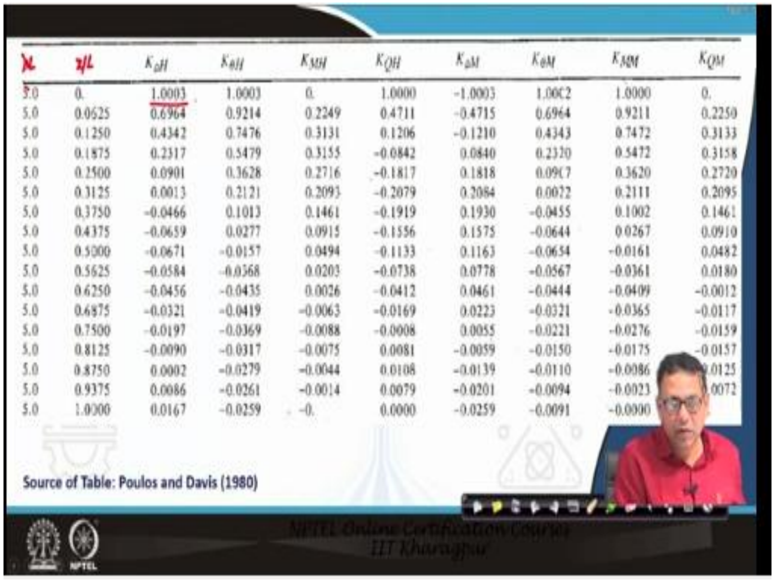

λL	z/L	$K_{\rho H}$	$K_{\theta H}$	K_{MH}	K_{QH}	$K_{\rho M}$	$K_{\theta M}$	K_{MM}	K_{QM}
5.0	0.	1.0003	1.0003	0.	1.0000	-1.0003	1.00C2	1.0000	0.
5.0	0.0625	0.6964	0.9214	0.2249	0.4711	-0.4715	0.6964	0.9211	0.2250
5.0	0.1250	0.4342	0.7476	0.3131	0.1206	-0.1210	0.4343	0.7472	0.3133
5.0	0.1875	0.2317	0.5479	0.3155	-0.0842	0.0840	0.2320	0.5472	0.3158
5.0	0.2500	0.0901	0.3628	0.2716	-0.1817	0.1818	0.09C7	0.3620	0.2720
5.0	0.3125	0.0013	0.2121	0.2093	-0.2079	0.2084	0.0022	0.2111	0.2095
5.0	0.3750	-0.0466	0.1013	0.1461	-0.1919	0.1930	-0.0455	0.1002	0.1461
5.0	0.4375	-0.0659	0.0277	0.0915	-0.1556	0.1575	-0.0644	0.0267	0.0910
5.0	0.5000	-0.0671	-0.0157	0.0494	-0.1133	0.1163	-0.0654	-0.0161	0.0482
5.0	0.5625	-0.0584	-0.0368	0.0203	-0.0738	0.0778	-0.0567	-0.0361	0.0180
5.0	0.6250	-0.0456	-0.0435	0.0026	-0.0412	0.0461	-0.0444	-0.0409	-0.0012
5.0	0.6875	-0.0321	-0.0419	-0.0063	-0.0169	0.0223	-0.0321	-0.0365	-0.0117
5.0	0.7500	-0.0197	-0.0369	-0.0088	-0.0008	0.0055	-0.0221	-0.0276	-0.0159
5.0	0.8125	-0.0090	-0.0317	-0.0075	0.0081	-0.0059	-0.0150	-0.0175	-0.0157
5.0	0.8750	0.0002	-0.0279	-0.0044	0.0108	-0.0139	-0.0110	-0.0086	[illegible]
5.0	0.9375	0.0086	-0.0261	-0.0014	0.0079	-0.0201	-0.0094	-0.0023	[illegible]
5.0	1.0000	0.0167	-0.0259	-0.	0.0000	-0.0259	-0.0091	-0.0000	[illegible]

Source of Table: Poulos and Davis (1980)

Finally above is the chart for the λL value 5.

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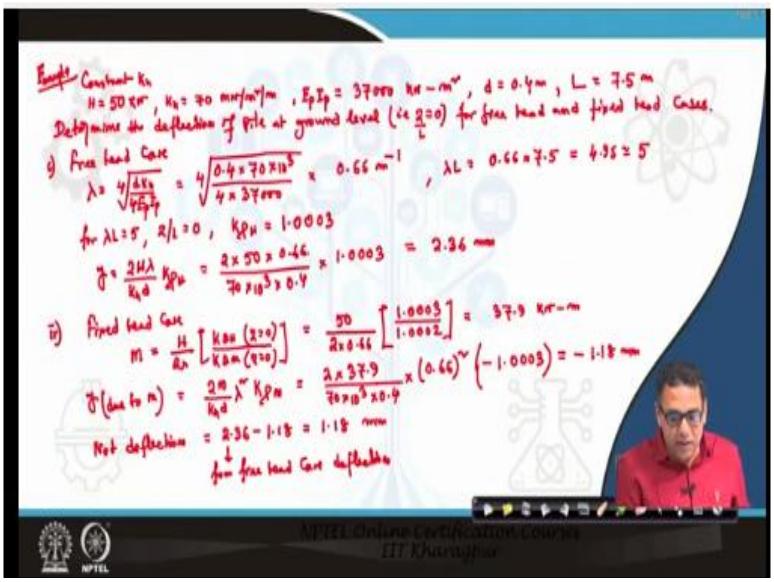

Let us see an example problem in this concept where the soil has a constant k_h value throughout its depth. The horizontal point load acting on the pile is H = 50 kN, k_h value is 70 MN/m^2/m and the flexural rigidity of the pile, E_pI_p is 37,000 kN-m^2. Diameter of the pile, d = 0.4 m, length of the pile, L = 7.5 m. The aim is to determine the deflection of the pile at ground surface for both free head case and fixed head case.

Consider the free head case as the case-1:

$$\lambda = \sqrt[4]{\frac{dk_h}{4E_pI_p}} = \sqrt[4]{\frac{0.4\times 70\times 10^3}{4\times 3700}} = 0.66m^{-1}$$

$$\Rightarrow \lambda L = 0.66\times 7.5 = 4.96 \approx 5$$

As the deflection is to be determined at the ground surface, the z value is 0 and hence z/L = 0.

$$\text{So, for } \lambda L = 5 \textit{ and } z/L = 0 \quad \Rightarrow k_{\rho H} = 1.0003$$

Substituting these values in the expression for the deflection:

$$y = \frac{2H\lambda}{k_h d}k_{\rho H} = \frac{2\times 50\times 0.66}{7\times 10^3\times 0.4}\times 1.0003 = 2.36mm$$

This is the value of the pile deflection at the pile head which is at ground surface.

Now, for the fixed head pile, firstly a moment M should be applied to make the slope at the pile head zero and then the deflection this moment causes should be calculated.

$$M = \frac{H}{2\lambda}\left[\frac{k_{\theta H}}{k_{\theta M}}\right]\Bigg|_{z=0} = \frac{50}{2\times 0.66}\left[\frac{1.0003}{1.0002}\right] = 37.9kN - m$$

$$y_{\text{(due to M)}} = \frac{2M\lambda^2}{k_h d}k_{\rho M} = \frac{2\times 37.9\times 0.66^2}{70\times 10^3\times 0.4}\times[-1.0003] = -1.18mm$$

The above value of deflection exists only in case of a free head pile. If this is a free head pile, the deflection would be 2.36 mm but another deflection came into picture due to the fixity. So, the net deflection at the pile head for a fixed head pile would be:

$$\text{Net deflection, } \mathrm{y_{net}} = 2.36 - 1.18 = 1.18mm$$

As the deflection due to fixity is in the opposite direction (negative sign) to the pile deflection in free head case, in the net deflection calculation, it is subtracted.

In the free head case, if the load is only H, deflection arises because of H alone. But in the fixed head case, both H and the moment developed due to fixity should be considered to calculate the deflection.

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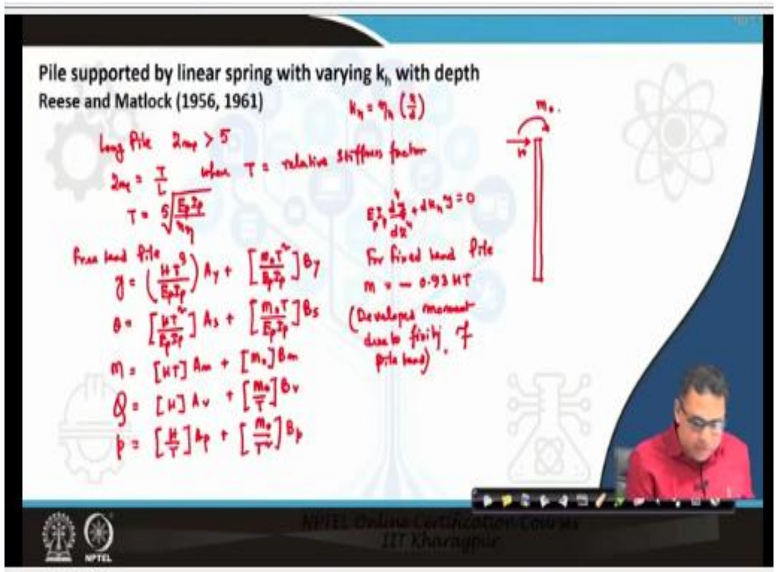

The next case is when a pile is supported by linear springs with varying k_h or the k_h is varying linearly with depth. The approach suggested by Reese and Matlock will be followed for this concept. The expression for k_h in this case was already discussed:

$$k_h = \eta_h\left(\tfrac{z}{d}\right)$$

Expressions for various quantities were also given for long piles defined by $Z_{max} > 5$.

where, $Z_{\max} = \dfrac{L}{T}$ (T is the relative stiffness factor); $\Rightarrow T = \sqrt[5]{\dfrac{E_p I_p}{\eta_h}}$

η_h is the unit subgrade modulus reaction.

For the free head pile the expressions given are:

$$y = \left[\frac{HT^3}{E_p I_p}\right]A_y + \left[\frac{M_\circ T^2}{E_p I_p}\right]B_y$$

$$\theta = \left[\frac{HT^2}{E_p I_p}\right]A_s + \left[\frac{M_\circ T^2}{E_p I_p}\right]B_s$$

$$M = [HT]A_M + [M_\circ]B_M$$

$$Q = [H]A_v + \left[\frac{M_\circ}{T}\right]B_v$$

$$p = \left[\frac{H}{T}\right]A_p + \left[\frac{M_\circ}{T^2}\right]B_p$$

These coefficients are obtained from the solution of the basic differential equation given below for a beam on elastic foundation by FDM:

$$E_p I_p \frac{d^4 y}{dz^4} + dk_h y = 0$$

The procedure to use FDM to find the solution in this case will also be discussed. First consider all the factors, formulate in the finite difference form and then after solving the FDM equations the coefficients A and B can be determined. The process of using a finite difference form in a general way to solve a general problem for pile deflection based on the subgrade modulus approach will be discussed.

Remember that these expressions are valid when k_h varies linearly with depth. For a fixed head pile, it is mentioned that the moment developed due to the fixity will be: M = –0.93 HT. In the previous case also there was an expression given for the moment developed due to fixity, but here the expression has a minus value, which means that the moment will be developed in the opposite direction, to that of H.

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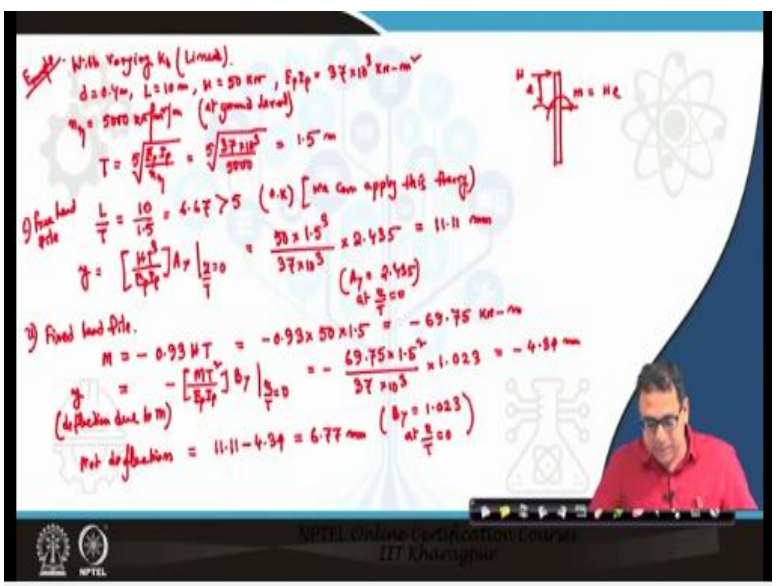

Now an example problem will be solved for a case where the k_h varies linearly.

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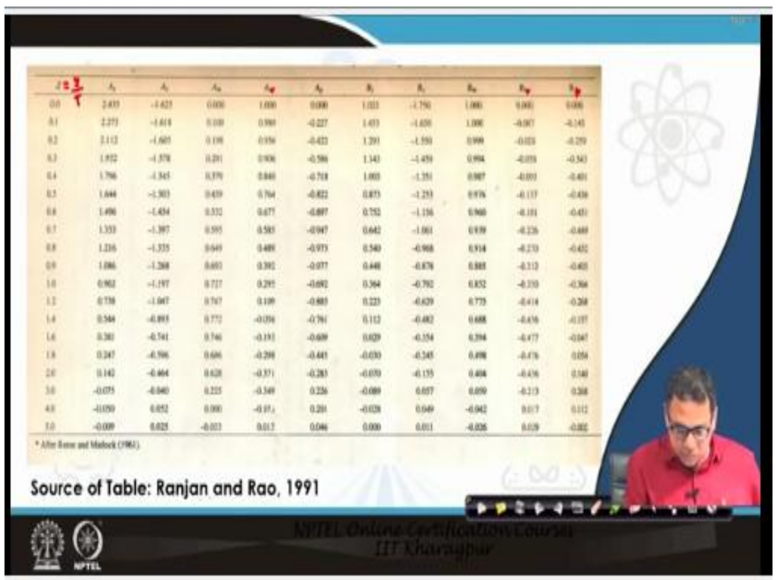

The coefficients required to determine the various quantities for the pile are given in the tabular form above. According to this table, the values of the coefficients are a function of Z (where Z = z/T).

Coming to the example problem, the deflection of the pile at ground surface should be determined, where the diameter of the pile is 0.4 m, length of the pile, L = 10 m and H = 50 kN (no moment is applied). The horizontal point load, H is applied at the ground surface, the flexural rigidity of the pile, $E_pI_p = 37 \times 10^3$ kN-m^2.

Note that here; H is applied at the ground level. If H is applied at a certain height from the ground level, the effect of H as well as the effect of the moment it causes at the ground level should be considered in the calculations. (For example, suppose if H acts at a height of ‘e’ from the ground level, a moment H × e will act on the pile at the ground level). But now, the eccentricity value is 0. The coefficient of modulus of subgrade reaction is 5000 kN/m^2/m. The ranges of these values for sand were already given depending upon the soil condition.

The expression for T is:

$$T = \sqrt[5]{\frac{E_p I_p}{\eta_h}} = \sqrt[5]{\frac{37 \times 10^3}{5000}} = 1.5m$$

Now, the value of Z_{max} or L/T is: $\frac{\mathrm{L}}{\mathrm{T}} = \frac{10}{1.5} = 6.67$. As L/T > 5, this method is applicable. The expression for deflection is:

$$y = \left[\frac{HT^3}{E_p I_p}\right] A_y + \left[\frac{M_\circ T^2}{E_p I_p}\right] B_y$$

(Since there is no moment acting and H is acting at ground surface, M_o is zero)

$$\Rightarrow y = \left[\frac{HT^3}{E_p I_p}\right] A_y \bigg|_{\frac{z}{T}=0} = \frac{50 \times 1.5^3}{37 \times 10^3} \times 2.435 = 11.11mm$$

This is the deflection of a free head pile at the pile head under the given conditions.

Now, for a fixed head pile it was discussed that the moment developed will be –0.93 × HT:

$$M = -0.93 \times HT = -0.93 \times 50 \times 1.5 = -69.75kN - m$$

In addition to the 11.11 mm (due to H), the pile faces another deflection due to the above moment. So, the deflection due to the moment, M will be:

$$y = \left[\frac{HT^3}{E_p I_p}\right] A_y + \left[\frac{M_\circ T^2}{E_p I_p}\right] B_y \Rightarrow y = \left[\frac{M_\circ T^2}{E_p I_p}\right] B_y$$

(Since, the deflection due to the moment alone is being calculated now)

$$\Rightarrow y = \left[\frac{-69.75 \times 1.5^2}{37 \times 10^3}\right] \times 1.023 = -4.34mm$$

So, the net deflection will be = 11.11 – 4.34 = 6.77 mm.

Till now, two different approaches were discussed considering constant k_h and linearly varying k_h along that depth of the pile.

We have discussed the Hetenyi approach where we considered the pile as a beam with finite length. But I will show you how to solve for the pile by considering it as a semi-infinite beam in the next class. After that, I will discuss about a general solution and how to use the finite difference method to solve that generalized solution. Thank you.