Soil Structure Interaction
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Lecture-60
Soil Structure Interaction for Pile Foundation (Contd.)

In this class I will discuss how to determine the settlement of pile under lateral loading based on the subgrade modulus approach.

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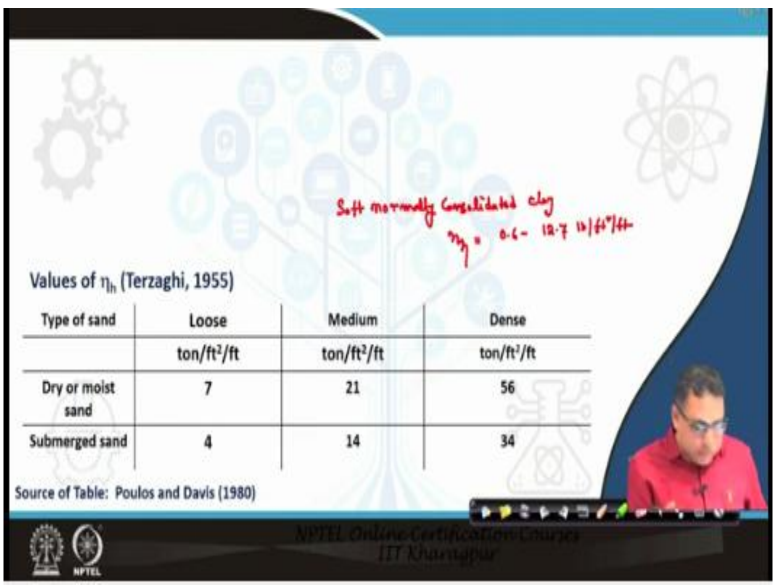

Before starting the topic, there is a small clarification about the values given above (already discussed in the last class). The range of η_h values for soft normally consolidated clay will be between 0.6 to 12.7 lb/ft^2/ft. Somewhere, these values may be in ton/ft^2/ft which should be checked carefully.

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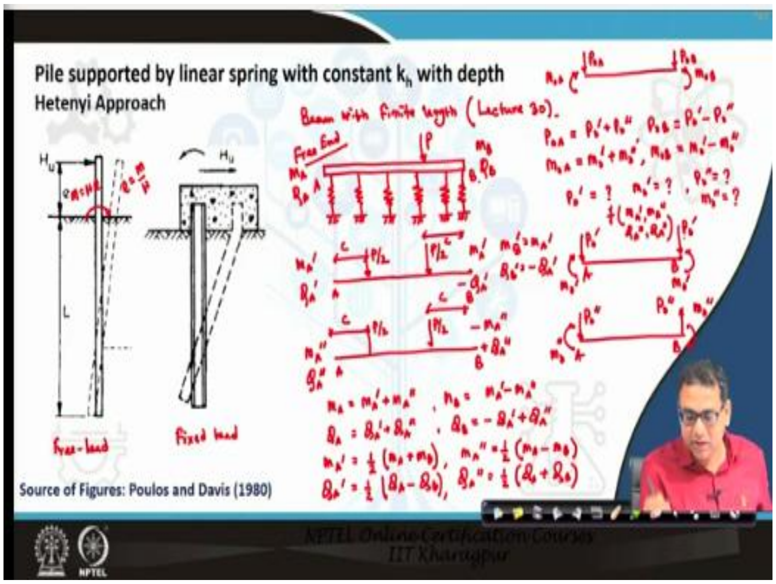

Let us start with the procedure of determining the settlement or slope or other required quantity in a pile under lateral load. The first approach in this regard is the Hetenyi approach where the pile is supported by linear springs with constant k_h. It implies that the k_h value is constant throughout the depth of the pile and so, the pile can be modeled as a beam with finite length.

In case of a beam with finite length the procedure and expressions to determine the deflections, slope, bending moment and shear force were already discussed in lecture 30 (*For detailed explanation of those derivations, please refer lecture 30*). The application of that concept for the determination of the pile settlement under lateral loading will be discussed quickly.

There are 2 cases that are to be discussed in case of laterally loaded piles: the free headed pile and the fixed head pile. The figure to the right in the slide above shows a free head pile and the figure to the left shows a fixed head pile. So basically, free headed pile is a single pile or when the pile is not under a cap whereas the fixed head pile is the one with cap. So a fixed head pile will be rigid in lateral direction at its head because it is attached to a rigid pile cap. But a free head pile is in a free condition or its head is free to displace in the direction of the load. Note that a pile with a rigid pile cap can only act as a fixed head pile.

Now, for a free headed pile, if a lateral load H_u, is applied on the pile with eccentricity (e), moment will act at the base or at the ground line of the pile and this moment will be h × E. So, the value of eccentricity will be e = M/H. Here the pile is being considered as a beam of finite length resting on springs. The case discussed in lecture 30 was where the beam of finite length with free end is subjected to a load.

Under a point load P on the beam, a moment and a shear force were assumed to develop at both the ends A and B (considering it as an infinite beam). The moment developed at A would be M_A and the shear force developed at A would be Q_A. Similarly, the moment developed at B would be M_B and the shear force developed at B would be Q_B. But as the beam is finite for real, the ends should not have any moment or shear force and hence, some end conditioning forces should be applied to nullify the developed moments and shear forces (M_A, Q_A, M_B and Q_B). The aim is to make the net moment and shear force 0 at both the ends.

Then two cases should be considered by splitting the load: anti symmetric case and symmetric case. Both the cases are shown in the figures in the slide above. For a symmetric case, because of the assumed loads, moment M_A' and shear force Q_A' will develop at point A. Similarly moment M_A' and shear force $-Q_A'$ will develop at point B (i.e., $M_B' = M_A'$ and $Q_B' = -Q_A'$). For this condition, the end conditioning forces, P_o' and M_o' should be applied.

Similarly for the anti symmetric case, because of the assumed loads, moment M_A'' and shear force Q_A'' will develop at point A. Moment $-M_A''$ and shear force Q_A'' will develop at point B (i.e., $M_B'' = -M_A''$ and $Q_B'' = Q_A''$). For this condition, the end conditioning forces, P_o'' and M_o'' should be applied.

Finally,

$$M_A = M_A' + M_A''; \qquad M_B = M_A' - M_A''$$

$$Q_A = Q_A' + Q_A''; \qquad Q_B = -Q_A' + Q_A''$$

By solving the above equations, we get:

$$M_A' = \frac{1}{2}(M_A + M_B); \quad M_A'' = \frac{1}{2}(M_A - M_B)$$

$$Q_A' = \frac{1}{2}(Q_A - Q_B); \qquad Q_A'' = \frac{1}{2}(Q_A + Q_B)$$

These are the developed forces at A and B (both ends) due to the consideration that the beam is infinite. The end conditioning forces, P_{OA}, P_{OB}, M_{OA} and M_{OB} should exactly counteract these forces. The expressions for these forces can be given by:

$$P_{OA} = P_O{'} + P_O{''}; \qquad P_{OB} = P_O{'} - P_O{''}$$

$$M_{OA} = M_O{'} + M_O{''}; \qquad M_{OB} = M_O{'} - M_O{''}$$

The expressions for $P_o{'}$, $P_o{''}$, $M_o{'}$ and $M_o{''}$ were given in lecture 30.

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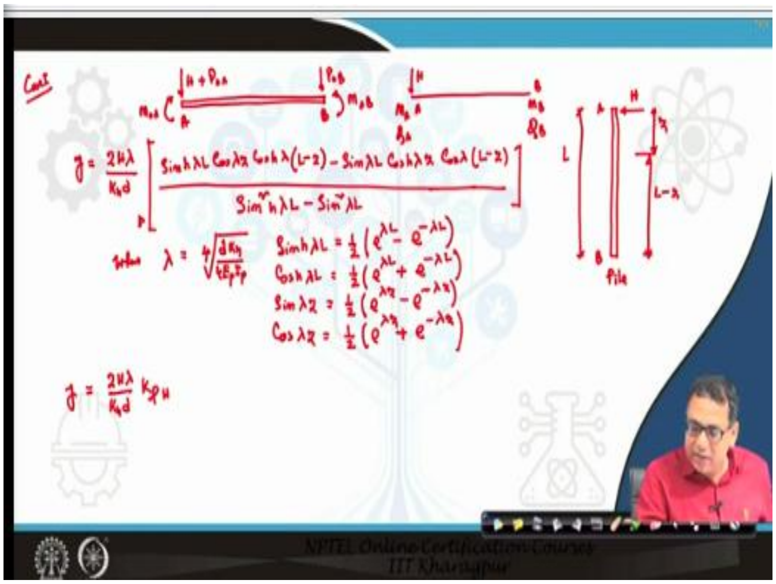

To determine the end conditioning forces and then to find out the deflection along with the other required quantities, first consider the case where the horizontal load, H is applied on the pile as shown in the figure above. In addition to the applied load, end conditioning forces need to be applied. As H is applied at one of the ends, the point of action for H and the end conditioning forces would be the same in this case.

The four reactions in the beam due to the external load and the end conditioning forces are shown in the slide above. Similar to a beam, for the present case of pile, the top free end is considered as point A and the bottom end is considered as point B. After this, the expressions for deflection and other quantities under these five loads (4 end conditioning forces + 1 external load) should be formulated.

For that, the reactions at both the ends due to the external load, M_A, M_B, Q_A and Q_B should be calculated first. Once these forces are determined, the values of M_A', M_A'', Q_A' and Q_A'' can be found out. From these values, P_o', P_o'', M_o' and M_o'' can be calculated (P_o', P_o'', M_o' and M_o'' are a function of M_A', M_A'', Q_A', Q_A'' and length of the pile). Finally using these values, P_{OA}, P_{OB}, M_{OA} and M_{OB} can be calculated. This is the step by step procedure to obtain the expressions of the end conditioning forces. The general expressions for the four quantities along the beam can be determined now.

For the loading condition considered here (case-1) and for the free end condition, the expression for deflection will be:

$$y = \frac{2H\lambda}{k_h d}\left[\frac{\sinh \lambda L \times \cos \lambda x \times \cosh \lambda (L-x) - \sin \lambda L \times \cosh \lambda x \times \cos \lambda (L-x)}{\sin^2 h\lambda L - \sin^2 \lambda L}\right]$$

If the value within the parentheses is replaced with a coefficient, $k_{\rho H}$:

$$y = \frac{2H\lambda}{k_h d} k_{\rho H}$$

$$\text{where, } \lambda = \sqrt[4]{\frac{dk_h}{4E_p I_p}}$$

$$\sinh \lambda L = \frac{1}{2}\left(e^{\lambda L} - e^{-\lambda L}\right); \qquad \cosh \lambda L = \frac{1}{2}\left(e^{\lambda L} + e^{-\lambda L}\right)$$

$$\sin \lambda z = \frac{1}{2}\left(e^{\lambda z} - e^{-\lambda z}\right); \qquad \cos \lambda z = \frac{1}{2}\left(e^{\lambda z} + e^{-\lambda z}\right)$$

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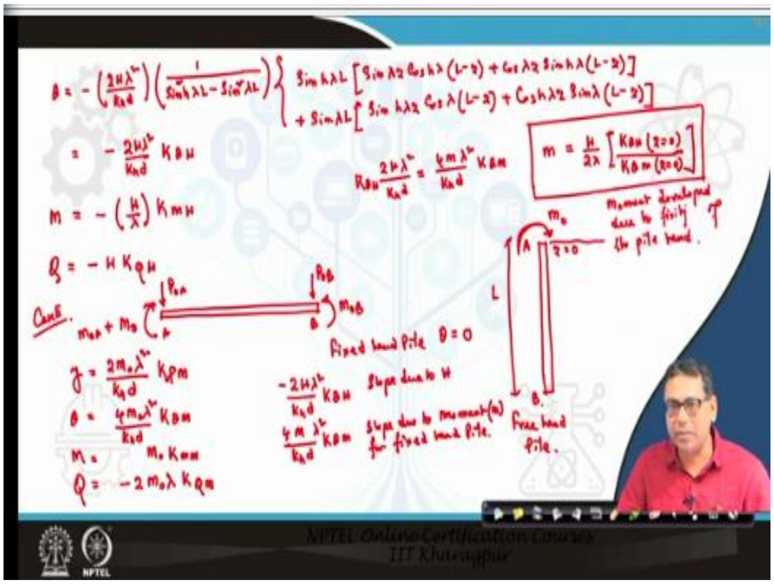

Similarly, the expression for slope is:

$$\theta = -\left(\frac{2H\lambda^2}{k_h d}\right)\left(\frac{1}{\sinh^2 \lambda L - \sin^2 \lambda L}\right)\{\sinh \lambda L[\sin \lambda x \cosh \lambda(L-z) + \cos \lambda x \sinh \lambda(L-z)]$$

$$+ \sin \lambda L[\sinh \lambda z \cos \lambda(L-z) + \cosh \lambda z \sin \lambda(L-z)]\}$$

$$\theta = -\left(\frac{2H\lambda^2}{k_h d}\right)k_{\theta H}$$

Similarly the expression for moment is:

$$M = -\left(\frac{H}{\lambda}\right)k_{MH}$$

Similarly the expression for shear force is:

$$Q = -Hk_{QH}$$

These are all the expressions for the required quantities when a horizontal point load acts at the top of a free end pile which is considered as case-1 here. In the case-1, the point load was acting at the pile head which is above the ground level. Moving on to the case-2, where a moment is considered to be acting at the top of a free head pile. Here, the pile head is considered to be at the ground level. The expressions for case-2 will be:

$$y = \frac{2M_{\circ}\lambda^2}{k_h d}k_{\rho M}$$

$$\theta = \frac{4M_{\circ}\lambda^2}{k_h d}k_{\theta M}$$

$$M = M_{\circ} k_{MM}$$

$$Q = -2M_{\circ} \lambda k_{QM}$$

These are the expressions in case of a free end pile. These expressions can be derived in the similar way in case of a fix head pile also. For a fix head pile, the condition is that slope will be 0 at the pile head. In a free head pile, certain slope developed at the pile head due to the applied load or moment. When a horizontal point load is on a fixed head pile, the fixity restrains any sort of slope in the pile head and that restrained slope will show up as moment. Technically any moment at pile head should create some slope.

The slope due to point load is: $\theta = -\left(\frac{2H\lambda^2}{k_h d}\right) k_{\theta H}$

The slope due to moment is: $\theta = \frac{4M_{\circ}\lambda^2}{k_h d} k_{\theta M}$

These two has to be equal and if they are equated, the moment developed due to fixity can be determined.

$$\left(\frac{2H\lambda^2}{k_h d}\right) k_{\theta H} = \frac{4M_{\circ}\lambda^2}{k_h d} k_{\theta M}$$

$$\Rightarrow M = \frac{H}{2\lambda}\left[\frac{k_{\theta H}}{k_{\theta M}}\right]\Bigg|_{z=0}$$

So this amount of moment should be applied in a free end pile to make it a fixed end pile. As the H is acting towards the left, the moment that acts on the pile will be in clockwise direction, as considered.

In the next class I will show you what happens if k_h varies linearly with depth. The expressions of the deflection, slope, bending moment and shear force of the pile will also be discussed along with an example problem. Thank you.