

Soil Structure Interaction
Prof Kousik Deb
Department of Civil Engineering
Indian Institute of Technology-Kharagpur

Lecture-56
Soil Structure Interaction for Pile Foundation (Contd.)

In this class I will complete the problem of determining the pile group settlement using the elastic analysis approach which I started in the previous class.

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Second Part
Settlement Calculation by Interaction Factor Approach

Solid Pile $R_A = 1$
 $K = \frac{E_p R_A}{L_p} = \frac{27000}{2.6} = 1038.5 \approx 1000$
 $\frac{L_p}{d} = \frac{15}{0.3} = 50$
 $R_{ij} = R_i R_j + R_i \sum_{j \neq i} \left(\frac{P_j d_{ij}}{\sqrt{d_{ij}}} \right)$

Settlement of Pile ①:
 $\rho_A = R_1 \left\{ P_A + P_B \begin{matrix} 0.34 & 0.34 & 0.3 \\ 1-3 & 1-7 & 1-9 \end{matrix} + P_C \begin{matrix} 0.44 & 0.44 & 0.32 \\ 1-2 & 1-4 & 1-6 \end{matrix} \right.$
 $\left. + P_C \begin{matrix} 0.32 & 0.4 \\ 1-8 & 1-5 \end{matrix} \right\}$ $P_A = \text{load on each pile}$
 $1 = \text{group A}$

$\frac{\rho_A}{\rho_1} = 1.98 P_A + 1.52 P_B + 0.4 P_C$ — ①
 $\rho_1 = \text{Settlement of single pile under unit load.}$

Group A: 1, 2, 7, 9 $\rightarrow P_A$
 Group B: 2, 4, 4, 8 $\rightarrow P_B$
 Group C: 5 $\rightarrow P_C$

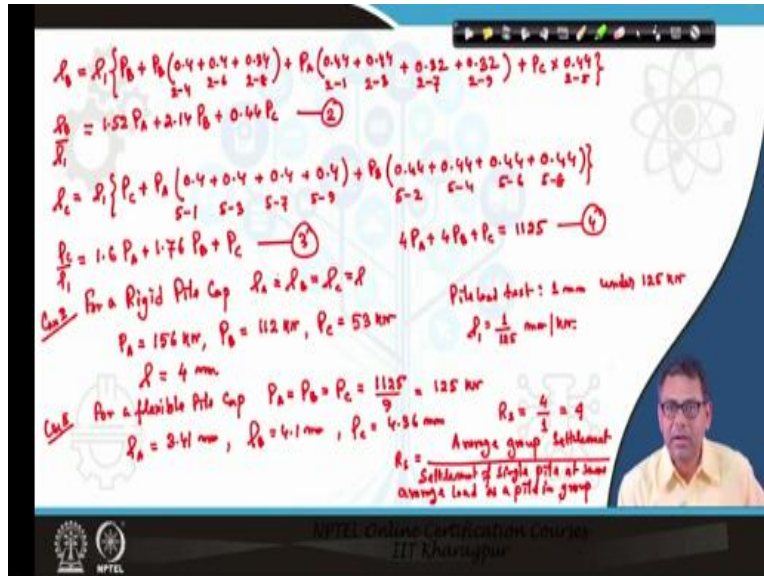
$d_{13} = 0.34$
 $\frac{L_p}{d} = \frac{15}{0.3} = 50$
 $\frac{L_p}{d} = \frac{1}{6.47} = 0.15$
 $d_{13} = 3 = \sqrt{2^2 + 2^2}$

Diagram: A 3x3 grid of piles numbered 1 to 9. Piles 1, 2, 7, 9 are in Group A. Piles 2, 4, 4, 8 are in Group B. Pile 5 is in Group C. Spacing is 1m between adjacent piles.

This was the problem definition where the settlement should be determined for a 3×3 pile group with spacing of 1 m. An expression was already determined for the settlement of piles in group A:

$$\Rightarrow \frac{\rho_A}{\rho_1} = 1.98 P_A + 1.52 P_B + 0.4 P_C \rightarrow (1)$$

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Now the settlement of the piles in group B should be determined. So the interaction factors should be calculated for the piles just like the previous case of group A. The distance between the 2 & 4 piles is $\sqrt{1^2 + 1^2}$ because the line joining the centers of both the piles is like a hypotenuse of a right angled isosceles triangle of side 1 m. Similarly the distance between the piles 2 & 6 is also $\sqrt{1^2 + 1^2}$. All the rest are pretty direct to determine except 2-7 and/or 2-9. The distance between the piles 2 & 7 will be $\sqrt{2^2 + 1^2}$. This distance also uses the hypotenuse concept but here, the other two sides of the triangle are 2 m and 1 m (one side is 2 to 1 which is 1 m; other side is 1 to 7 which is 2 m). All the other spacing values can be easily found out. So, the expression for settlement of piles in group B is:

$$\rho_B = \rho_1 \left\{ P_B + P_B \left(\frac{0.4}{2-4} + \frac{0.4}{2-6} + \frac{0.34}{2-8} \right) + P_B \left(\frac{0.44}{2-1} + \frac{0.44}{2-3} + \frac{0.32}{2-7} + \frac{0.32}{2-9} \right) + P_C \times \frac{0.44}{2-5} \right\}$$

$$\Rightarrow \frac{\rho_B}{\rho_1} = 1.52 P_A + 2.14 P_B + 0.44 P_C \rightarrow (2)$$

Similarly, the expression for the settlement of pile in group C is:

$$\rho_C = \rho_1 \left\{ P_C + P_A \left(\frac{0.4}{5-1} + \frac{0.4}{5-3} + \frac{0.4}{5-7} + \frac{0.4}{5-9} \right) + P_B \left(\frac{0.44}{5-2} + \frac{0.44}{5-4} + \frac{0.44}{5-6} + \frac{0.44}{5-8} \right) \right\}$$

$$\Rightarrow \frac{\rho_C}{\rho_1} = 1.6 P_A + 1.76 P_B + P_C \rightarrow (3)$$

Now if there is a rigid pile cap over the piles, it settles uniformly. If the pile cap settles uniformly, it implies that all the piles have the same settlement.

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Example: (a) Design a pile group consisting of RCC solid piles for a column of size 650mm x 650 mm carrying a load of 1125 kN (Total). The exploration data reveal that the sub-soil consists of deposit of clay extending to a greater depth. The other data of the deposit are: Compression index = 0.10, Initial void ratio = 0.9, Saturated unit weight = 20 kN/m³, Unconfined compressive strength = 70kN/m². Proportion the pile group for the permissible settlement of 25 mm. Design the pile group by considering both bearing and settlement criteria. The water table is considered at the ground level. Use a factor of safety 2.5 against bearing and assume adhesion factor of 0.7. Correction factor for the effect of 3-D consolidation or pore water pressure is 0.7.

(b) Determine the settlement of pile group by Interaction Factor Approach. The Elastic Modulus of Pile (E_p) and Soil (E_s) are 27000 MPa and 26 MPa, respectively. $\alpha = 0.5$

Handwritten notes on the slide:

- a) Bearing Capacity
- b) Single Pile failure
- Calculations:

$$Q_{us} = C_u N_c A_b + \alpha E_u A_s$$

$$= 35 \times 9 \times \frac{\pi (0.3)^2}{4} + 0.7 \times 35 \times \pi (0.3) \times 15$$

$$= 354.6 \text{ kN}$$

$$Q_{ug} = 9 \times 354.6 = 3191.4 \text{ kN}$$
- Diagram parameters:
 - $d = 300 \text{ mm}$
 - $\alpha = 0.5$
 - $L = 15 \text{ m}$
 - $s = 2 \text{ m}$
 - $q_u = 70 \text{ kN/m}^2$
 - $C_u = q_u/2 = 35 \text{ kN/m}^2$

The total load coming on to the pile group will be divided in the following way as there are 4 piles in group A, 4 piles in group B and 1 pile in group C. There are 4 unknowns in total: P_A, P_B, P_C and the settlement of the pile cap, ρ.

$$4P_A + 4P_B + P_C = 1125 \rightarrow (4)$$

By solving these equations, the ρ value can be determined. But the problem is to get the ρ, the value of ρ₁ should be known. Suppose if a pile load test is done the ρ₁ value can be obtained from the data of that test. Suppose for a particular load x is the settlement, then the value of ρ₁ will be: x divided by that load. So ρ₁ is the settlement of the pile under unit load.

For example, from a pile load test it is known that the pile undergoes 1 mm settlement under 125 kN load. Then the ρ₁ value will be $\frac{1}{125}$ mm/kN . If the equations are solved by using this value of

ρ₁, the obtained values will be: P_A = 156 kN, P_B = 112 kN, P_C = 53 kN and the settlement of the pile cap (in this case, uniform settlement for all piles) will be ρ = ρ_A = ρ_B = ρ_C = ρ_D = 4 mm. Here, the ρ₁ value is considered arbitrarily.

Consider that the pile cap is flexible in case 2. For a flexible pile cap the settlement will be different at different places of the cap, but the loading will be same throughout its area. So in that case $P_A = P_B = P_C = \frac{1125}{9}$ i.e., all the piles will take the same amount of load. So, this value is 125 kN which is the load carried by a single pile.

For a flexible pile cap, if P_A , P_B and P_C are known along with ρ_1 , from the 1, 2, 3 equations we get: $\rho_A = 3.41$ mm, $\rho_B = 4.1$ mm and $\rho_C = 4.36$ mm. If these values are observed properly, it can be seen that the value of ρ_B is almost equal to the average of ρ_A and ρ_C . Also that ρ_B and ρ are almost equal. The conclusion is that in a pile group, for a flexible pile cap consideration, the settlement of the piles which are neither corner piles nor the centre piles will be almost equal to the settlement of a rigid pile cap.

It was mentioned that the ρ_1 value is obtained from the pile load test. But, even if the test data is not available, ρ_1 can be calculated by elastic analysis also.

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For Single Pile Settlement.

$$I = I_0 R_h R_k R_\mu$$

(Floating pile)
Pile is very rigid

$I_0 = I_0$
 $\mu_s = 0.5$
 $d = 0.3$
 $h = 50, K = 1000, \frac{d}{B} = 1, I_0 = 0.043$
 $I = 125$

To determine the settlement of a floating single pile, the factor I should be calculated first:

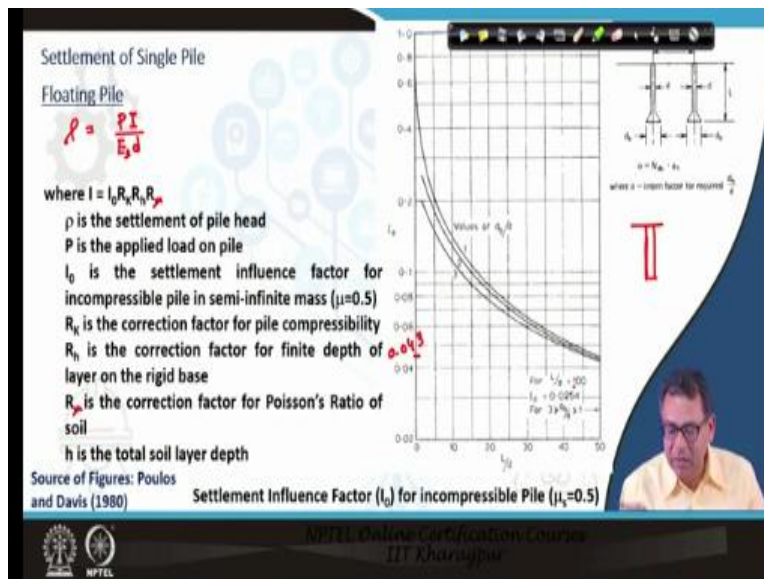
$$I = I_0 R_h R_k R_\mu$$

Let us use this formula for the present problem. In the current problem, the pile is very rigid and so the correction for pile compression, R_k is not required. The soil has a Poisson's ratio of 0.5

and hence R_μ is not required. It was also mentioned in the problem that the soil layer depth is infinite and hence the correction for finite depth, R_h is also not required. So, the above expression reduces to: $I = I_o$.

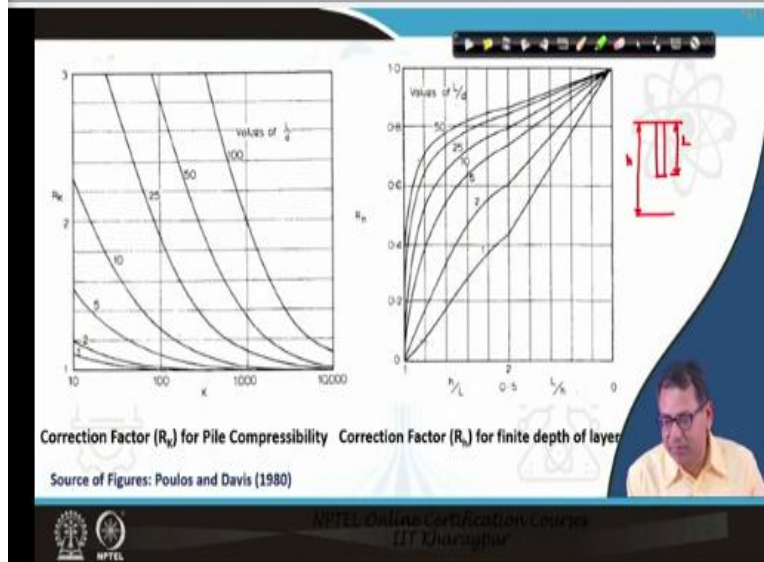
The value of I_o can be read from the chart which depends upon L/d , μ , k and d_b/d . In this case, $\mu_s = 0.5$, $L/d = 50$, $k = 1000$ and $d_b/d = 1$.

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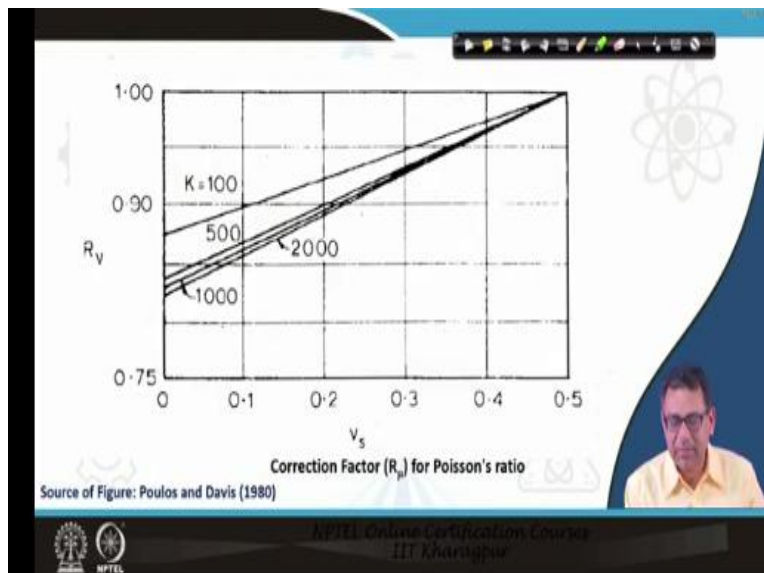


For an incompressible floating pile, with all the above mentioned values, I_o will be 0.043 from the above chart. Now if there are different values of μ or different conditions of the pile or soil, these corrections should be applied to I_o .

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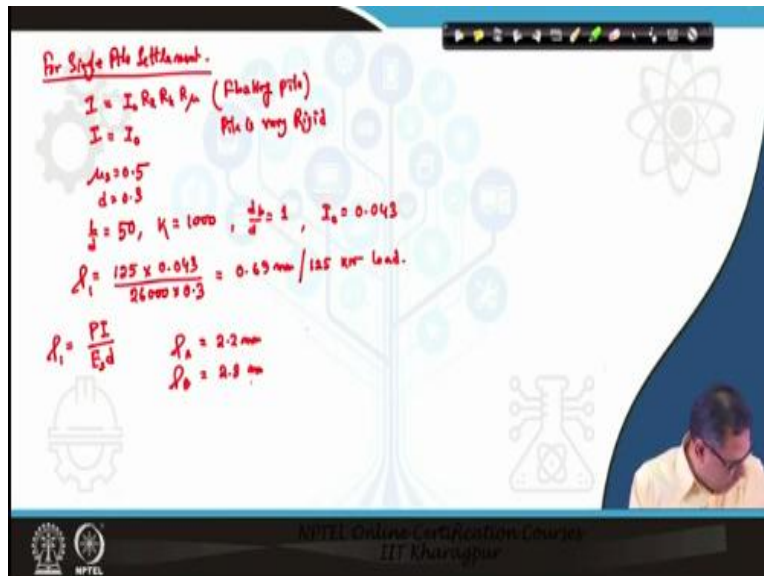


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But here the corrections are not required and so the I_0 value is 0.043.

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Once the I value is determined, use it in the expression for single pile settlement:

$$\rho_1 = \frac{PI}{E_s \times d}$$

$$\Rightarrow \rho_1 = \frac{125 \times 0.043}{26000 \times 0.3} = 0.69 \text{ mm} / 125 \text{ kN}$$

So, the value of ρ_1 is 0.69 mm under a load of 125 kN. By substituting this ρ_1 in the equations developed in the problem for the flexible pile cap case, we get: $\rho_A = 2.2$ mm, $\rho_B = 2.8$ mm and $\rho_C = 3$ mm.

Similarly the settlement in case of a rigid pile cap can also be found out and that is, $\rho = 2.76$ mm. R_s is a factor defined by:

$$R_s = \frac{\text{group settlement}}{\text{single pile settlement}} = \frac{2.76}{0.69} = 4$$

If the R_s value is to be determined for the rigid pile cap case where the ρ_1 is taken as 1 mm per 125 kN load:

$$R_s = \frac{\text{group settlement}}{\text{single pile settlement}} = \frac{4}{1} = 4$$

The definition of R_s is the average group settlement divided by the settlement of the single pile at same average load as a pile in group.

Here, as all the piles have the same settlement of 4 mm, the average settlement will also be 4 mm. Each pile is taking an average load of 125 kN and is undergoing a settlement of 1 mm. So, the settlement of a single pile at the same average load as a pile in a group is 1 mm. If the average load of a pile in a group is not the same as the single pile load, then the R_s value will be different. This is because the R_s value should be converted here such that the average load carried by a pile in group will be the same as the single pile load.

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L/d	s/d	k	4				9				16				25			
			10	100	1000	∞	10	100	1000	∞	10	100	1000	∞	10	100	1000	∞
2	2	1.83	2.22	2.24	2.02	2.78	3.80	4.42	4.40	3.75	2.60	2.40	2.23	4.57	7.20	8.48	8.68	
3	2	1.40	1.73	1.88	1.80	1.83	2.40	2.82	2.85	2.26	1.55	1.74	1.87	2.68	3.98	4.70	4.75	
5	2	1.21	1.29	1.40	1.30	1.42	1.76	1.97	1.99	1.63	1.19	1.40	1.48	1.87	2.63	2.93	2.93	
10	2	1.09	1.14	1.21	1.10	1.21	1.44	1.54	1.54	1.20	0.88	1.04	1.08	1.30	1.80	1.98	1.98	
2	3	1.67	1.74	2.09	2.10	1.98	2.61	3.48	3.74	2.46	1.54	1.46	1.34	2.95	4.40	4.50	4.51	
3	3	1.23	1.40	1.74	1.70	1.49	1.90	2.27	2.72	1.74	1.20	1.42	1.47	1.90	2.58	2.88	2.88	
5	3	1.03	1.10	1.26	1.10	1.10	1.29	1.52	1.52	1.21	0.84	1.00	1.00	1.18	1.60	1.70	1.70	
10	3	0.73	0.81	1.10	1.04	0.84	1.04	1.10	1.10	0.84	0.60	0.72	0.72	0.88	1.18	1.28	1.28	
2	5	1.28	1.30	1.78	2.04	1.74	2.04	2.70	3.29	2.08	1.29	1.10	1.00	1.60	2.40	2.60	2.60	
3	5	0.88	0.98	1.28	1.28	1.04	1.28	1.40	1.40	1.10	0.78	0.88	0.88	1.08	1.40	1.50	1.50	
5	5	0.78	0.88	1.04	1.04	0.80	1.04	1.10	1.10	0.80	0.60	0.70	0.70	0.80	1.00	1.10	1.10	
10	5	0.67	0.76	0.94	0.94	0.70	0.94	1.00	1.00	0.70	0.50	0.60	0.60	0.70	0.90	1.00	1.00	

Settlement ratio (R_s) of friction pile group with rigid cap in deep uniform soil mass

$\frac{L}{d} = 50, \frac{s}{d} = \frac{1}{0.3} = 3.33, k = 1000, R_s = 4, 3 \times 3 = 9$

Source of Table: Poulos and Davis (1980)

For a rigid pile cap on floating piles, the R_s value depending upon the L/d ratio, s/d ratio, k value and number of piles can be read from the above chart. A value of 4 for the number of piles refers to a 2×2 pile group and 9 piles refer to a 3×3 pile group.

Let us check the R_s value obtained from the calculations for the given problem agrees with the value given in the chart. For the case given in the problem, L/d ratio was 50, s/d was $1/0.3 = 3.33$, k value was 1000. For these values and a number of piles of 9, the R_s value is 4.52 for an s/d ratio of 2 and is 3.51 for an s/d ratio of 5. By interpolation, the value for an s/d ratio of 3.33 would be 3.96 which is almost equal to 4.

So these values can be directly taken, but it should be remembered that these values are calculated for $\mu = 0.5$, and an infinite depth of the layer along with a uniform d value. But this R_s value will be different if the soil conditions change.

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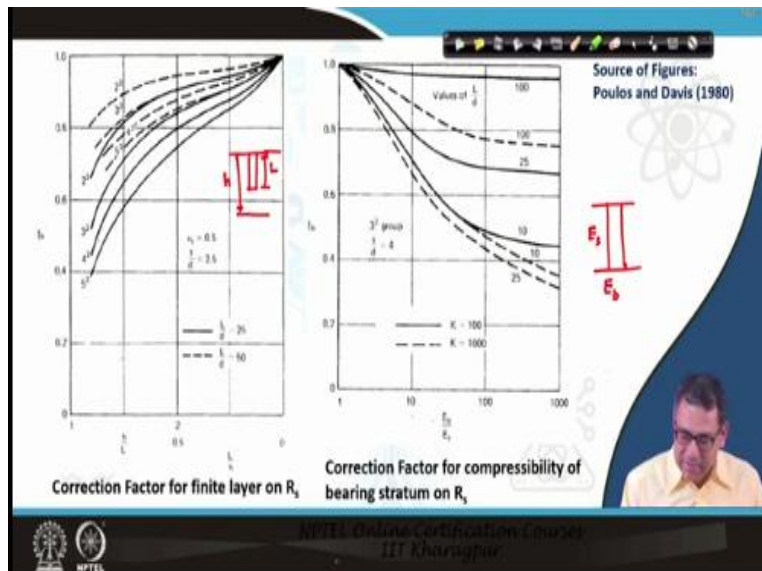
No. of Piles in Group		4				9				16				25			
L/D	L/D	10	100	1000	=	10	100	1000	=	10	100	1000	=	10	100	1000	=
10	2	1.23	1.14	1.05	1.00	2.02	1.81	1.60	1.00	3.20	2.49	1.80	1.00	3.70	2.83	2.00	1.00
	3	1.13	1.08	1.00	1.00	1.73	1.53	1.32	1.00	2.80	2.14	1.62	1.00	3.33	2.53	1.83	1.00
	10	1.22	1.07	1.00	1.00	1.94	1.62	1.30	1.00	3.04	2.52	1.80	1.00	3.62	2.82	2.00	1.00
25	2	1.88	1.62	1.00	1.00	2.84	2.57	1.86	1.00	3.70	3.28	2.33	1.00	4.88	4.13	3.00	1.00
	3	1.28	1.26	1.08	1.00	1.67	1.70	1.16	1.00	1.94	2.00	1.23	1.00	2.15	2.23	1.29	1.00
	10	1.14	1.12	1.04	1.00	1.23	1.26	1.06	1.00	1.39	1.37	1.07	1.00	1.33	1.38	1.08	1.00
100	2	2.49	2.24	1.39	1.00	4.06	3.59	1.96	1.00	5.83	5.27	2.62	1.00	7.62	7.06	3.41	1.00
	3	1.78	1.75	1.32	1.00	2.50	2.34	1.72	1.00	3.28	3.34	2.14	1.00	4.04	4.23	2.63	1.00
	10	1.39	1.43	1.25	1.00	1.78	1.81	1.46	1.00	2.30	2.29	1.71	1.00	3.42	3.71	1.97	1.00
1000	2	3.24	2.20	1.83	1.00	4.40	3.93	2.04	1.00	6.24	5.89	4.41	1.00	8.14	7.93	6.40	1.00
	3	1.82	1.84	1.67	1.00	2.71	2.77	2.32	1.00	3.54	3.74	3.47	1.00	4.33	4.68	4.41	1.00
	10	1.44	1.44	1.46	1.00	1.84	1.89	1.98	1.00	2.21	2.48	2.53	1.00	2.53	2.98	3.10	1.00

Settlement ratio (R_s) of end bearing pile group with rigid cap resting on a rigid stratum

Source of Table: Poulos and Davis (1980)

The above chart is to determine the R_s value for an end bearing pile group resting on a rigid stratum. This condition was discussed as the second case out of the three in the last class.

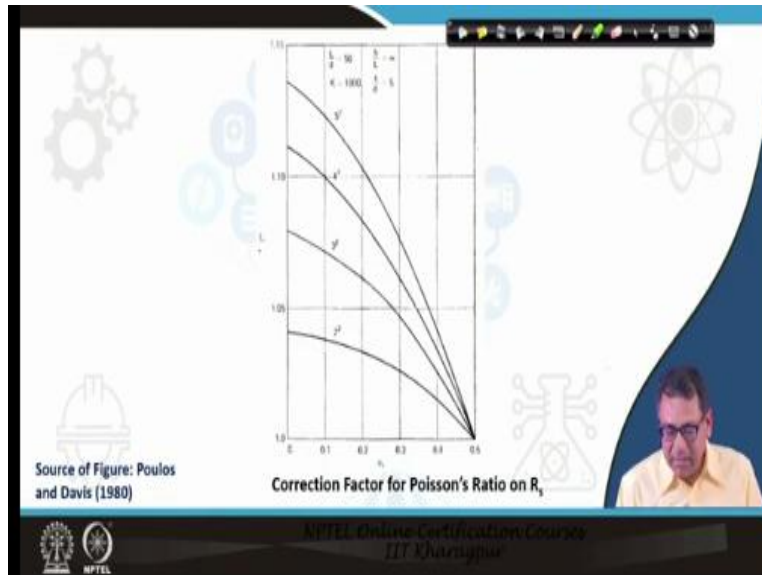
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If the R_s value is directly picked for any problem, then for different conditions it should be checked whether corrections should be applied or not. If a floating pile rests in a soil layer with finite depth, the finite layer correction should be applied which can be read from the left side chart in the above slide.

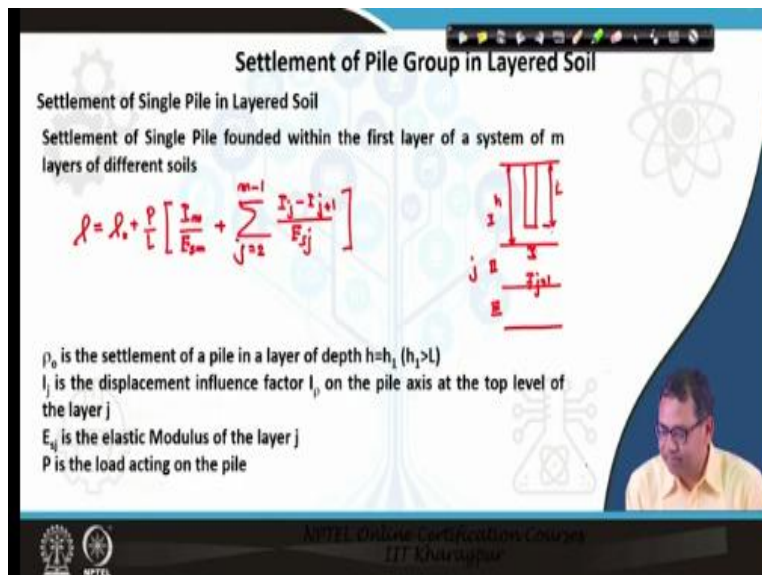
If an end bearing pile rests on a compressible stratum, the correction for compressibility of the bearing strata should be applied. The chart for this is shown on the right in the slide above.

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As it can be seen, all the charts discussed so far in regard to R_s value and its corrections were developed for a Poisson's ratio of 0.5. But if the soil has a Poisson's ratio less than 0.5, the above correction should be used for the R_s value when R_s is read from the charts.

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Till now, the settlement of a pile group either in homogeneous soil or resting on a hard stratum was discussed. But what if there are multiple layers within the pile depth and its influence zone? So the expression for settlement of a single pile in layered soil is given by:

$$\rho = \rho_o + \frac{P}{L} \left[\frac{I_m}{E_{sm}} + \sum_{j=2}^{m-1} \frac{I_j - I_{j-1}}{E_{sj}} \right]$$

Settlement of Pile Group in Layered Soil

Settlement of Single Pile in Layered Soil

Settlement of Single Pile founded within the first layer of a system of m layers of different soils

$$\rho = \rho_o + \frac{P}{L} \left[\frac{I_m}{E_{sm}} + \sum_{j=2}^{m-1} \frac{I_j - I_{j-1}}{E_{sj}} \right]$$

ρ_o is the settlement of a pile in a layer of depth $h=h_1$ ($h_1 > L$)
 I_j is the displacement influence factor I_j on the pile axis at the top level of the layer j
 E_{sj} is the elastic Modulus of the layer j
 P is the load acting on the pile

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The first term is ρ_o which is the settlement of a pile in a layer of depth $h = h_1$, ($h_1 > L$). So, it is nothing but the settlement calculated by single layer theory (as discussed earlier) of a pile of length, L that is resting in a soil layer of depth more than L . In the second part of the expression, the ratio I_m/E_{sm} refers to the settlement of the single first layer and the other part (under summation) refers to the settlement of all the other (lower) layers.

I_j is the displacement influence factor on the pile axis at the top level of the j^{th} layer. If the settlement for layer 2 should be calculated, I_j will be at the top of the second layer and I_{j+1} will be at the top of the 3rd layer.

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Settlement of Pile Group in Layered Soil

Settlement of Single Pile in Layered Soil

Settlement of Single Pile founded within the first layer of a system of m layers of different soils

$$\rho = \rho_0 + \frac{P}{L} \left[\frac{I_m}{E_{sm}} + \sum_{j=2}^{m-1} \frac{I_j - I_{j+1}}{E_{sj}} \right]$$

ρ_0 is the settlement of a pile in a layer of depth $h=h_1$ ($h_1 > L$)
 I_j is the displacement influence factor I_p on the pile axis at the top level of the layer j
 E_{sj} is the elastic Modulus of the layer j
 P is the load acting on the pile

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E_{sj} is the elastic modulus of the j^{th} layer soil ok and similarly, I_m is the m^{th} layer or last layer influence factor. The first layer is not considered in the summation because it is already taken into account with the ρ_0 term.

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Equivalent Diameter (D_p) of single pier for same settlement as pile group
 Source of Figures: Poulos and Davis (1980)

Equivalent length (L_p) of single pier for same settlement as pile group

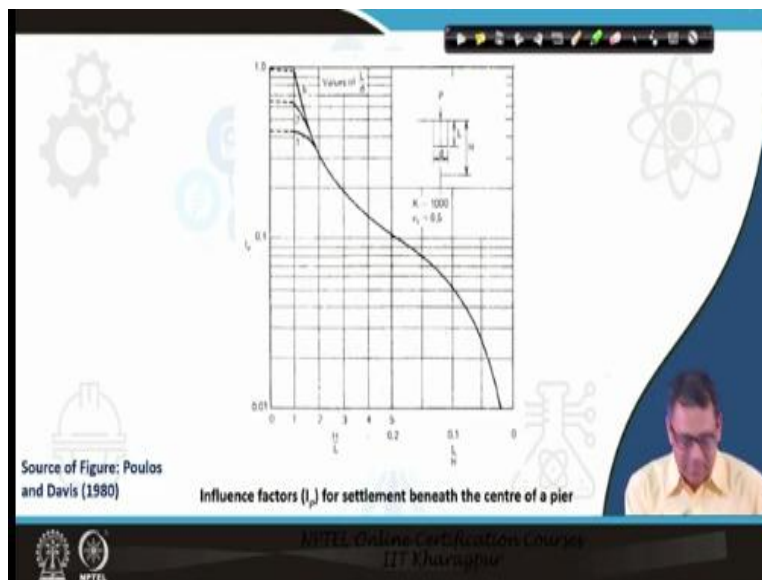
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If there is a pile group resting in a layered soil, then the settlement calculation will be discussed now. If there is a pile group in a square formation as shown in the figure above, the side of it should be determined first. Then, the entire group should be converted into an equivalent single pile with diameter large enough to be equivalent to the group. This equivalent pile may be called as a pier because of its large diameter. The equivalent diameter of the pier is shown in blue in the above figure.

There are two approaches to convert this group to a pier: equivalent length approach and equivalent diameter approach. If the equivalent diameter approach is to be followed, then depending upon the s/d ratio, L/d ratio and k value, the equivalent diameter can be read from the chart to the left in the above slide.

If the equivalent length concept is to be used, then depending upon the L/d and s/d ratios, the equivalent length of the pier can be obtained from the chart on right side. Once the group is converted to an equivalent single pier, the expression for single pile settlement in multi layer can be used to calculate the settlement of the group.

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In the next class, I will solve an example problem for pile in layered soil by determining the ρ value and influence factor value. I will explain in the next class how to use the above chart and how to solve using the equivalent diameter and equivalent length concept. Thank you.