Soil Structure Interaction
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Lecture – 52
Soil Structure Interaction for Pile Foundation

In this lecture I first will discuss how to use the Pasternak shear model to model an embankment and then I will start the pile foundation part.

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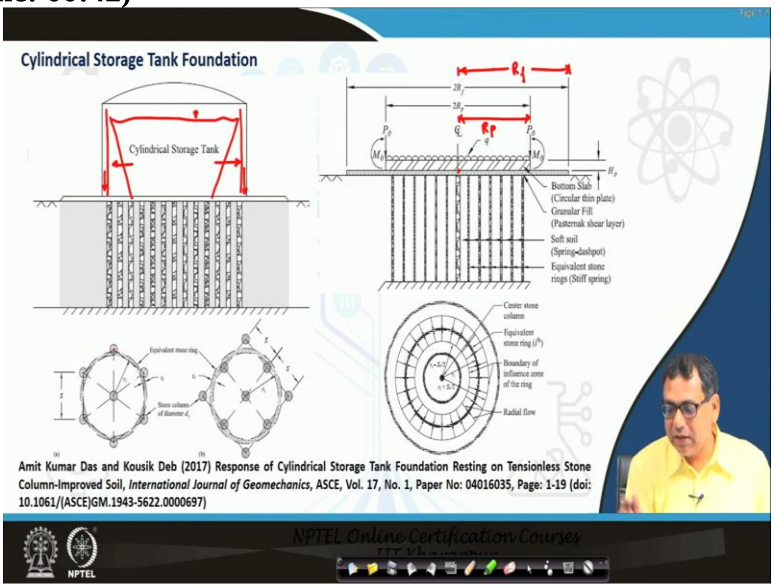

The case of a cylindrical storage tank was already discussed in the last lecture. Now, the procedure to use the boundary conditions here will be discussed. Here, the loading condition is different as the moment, concentrated load and UDL will act altogether if the tank is full. Under empty condition, again a different loading condition comes into picture.

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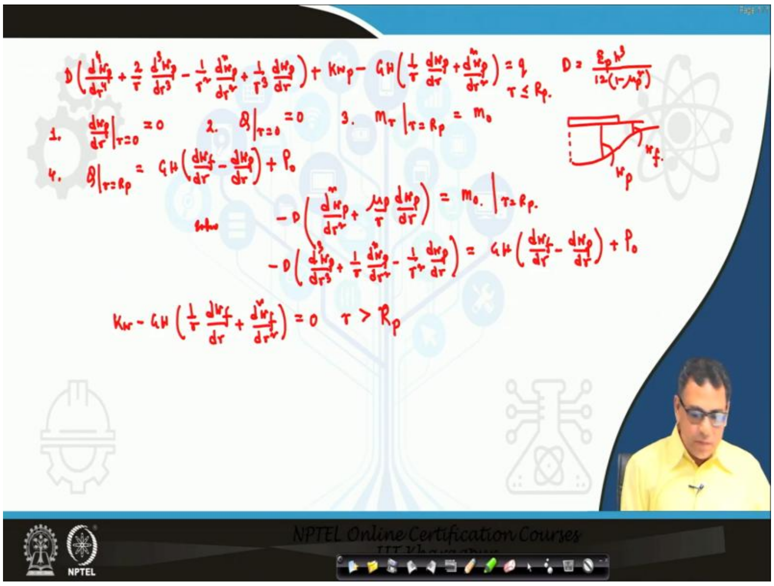

The governing differential equation for the circular tank condition will be:

For $r \leq R_p$: $$D\left(\frac{d^4 w_p}{dr^4} + \frac{2}{r}\frac{d^2 w_p}{dr^2} + \frac{1}{r^3}\frac{dw_p}{dr}\right) + kw_p - GH\left(\frac{1}{r}\frac{dw_p}{dr} + \frac{d^2 w_p}{dr^2}\right) = q$$

$$\text{where, } D = \frac{E_p h^3}{12(1-\mu_p^2)}$$

R_p is the radius of the loaded region and R_f is the radius of the entire model.

The first boundary condition is that the slope at the centre is zero (r = 0) because the load is symmetric:

$$1.)\ \frac{\mathrm{dw_p}}{\mathrm{dr}} = 0\Big|_{r=0}$$

The second boundary condition is that the shear force also will be 0 at the centre (r = 0):

$$2.)\ Q = 0\big|_{r=0}$$

The third boundary condition is that the bending moment at the edge where, $r = R_p$ will be equal to M_o which is nothing but the moment due to the liquid pressure.

$$3.)\ \mathrm{M_r} = M_o\big|_{r=0}$$

$$\text{where, } -D\left(\frac{d^2 w_p}{dr^2} + \frac{\mu_p}{r}\frac{dw_p}{dr}\right) = M_o\big|_{r=R_p}$$

The fourth condition is about the shear force at the edge of the loading ($r = R_p$)

$$4.)\ Q\big|_{r=R_p} = GH\left(\frac{dw_f}{dr} - \frac{dw_p}{dr}\right) + P_o$$

$$-D\left(\frac{d^3w_p}{dr^3}+\frac{1}{r}\frac{d^2w_p}{dr^2}-\frac{1}{r^2}\frac{dw_p}{dr}\right)=GH\left(\frac{dw_f}{dr}-\frac{dw_p}{dr}\right)+P_o$$

The first term in the above expression is due to the shear layer effect and P_o is the shear force due to the point load acting at the edge.

The differential equation for the zone beyond the loaded region is:

For $r > R_p$:
$$kw_f - GH\left(\frac{1}{r}\frac{dw_f}{dr}+\frac{d^2w_f}{dr^2}\right)=0$$

All these expressions and are for the case when the tank is full. If the tank is empty, M_o will be 0 because there will be no moment and q value also will be in the reduced form. Also, the q value would be very less as the tank is empty and the only weight is of the foundation. If the weight of the foundation is neglected, then q will be also equal to 0. In such case, only 2 concentrated loads at the two edges will act.

After this, the procedure to solve is the same as that of the beam or plate problem.

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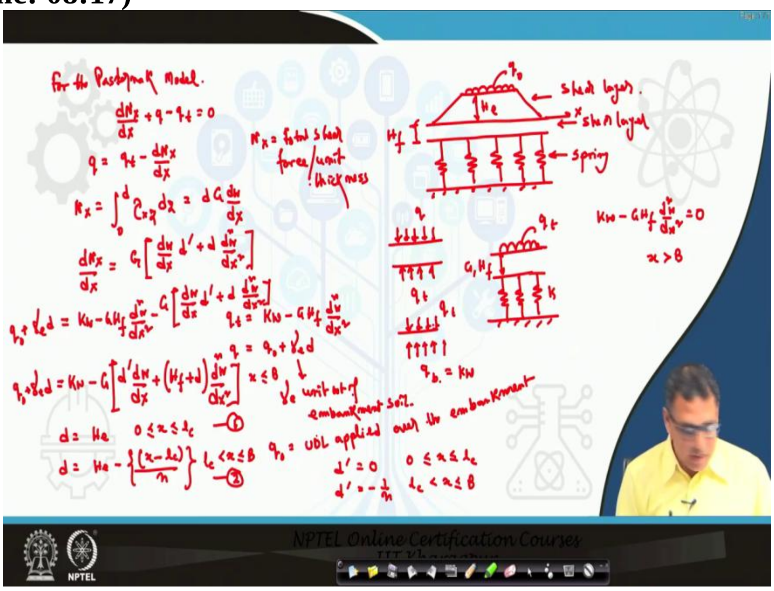

The next concept is about how to model an embankment. For the Pasternak model, the equation is:

$$\frac{dN_x}{dx}+q-q_t=0$$

The condition is shown in the figure where the embankment is resting on a shear layer which is underlain by a layer of springs. The embankment will also be modelled as a shear layer. But the thickness of the shear layer above the springs is constant, but it is not the case when the embankment is considered as the shear layer. The above differential expression can be written as:

$$q = q_t - \frac{dN_x}{dx}$$

N_x is the total shear force per unit thickness

$$N_x = \int_0^d \tau_{xz} dz = dG \frac{dw}{dx}$$

$$\frac{dN_x}{dx} = G\left[\frac{dw}{dx} d' + d \frac{d^2 w}{dx^2}\right]$$

where, d is the depth of the embankment which changes along the embankment length and d′ is the derivative with respect to length = $\frac{\mathrm{d}}{\mathrm{dx}}(\mathrm{d})$.

On the top of the second Pasternak layer (granular layer), a UDL of q is acting and at the bottom of this layer, a reaction of q_t is acting upwards. This q_t acts as a load on the soft soil and a reaction of q_b acts upwards at this face due to the spring reaction. As q_b is the reaction of the springs with spring constant, k: $q_b = kw$.

Now, the condition is that a UDL acts on the shear layer of thickness H_f and shear modulus, G. This shear layer is resting on the soft soil with spring constant, k.

$$q_t = kw - GH_f \frac{d^2 w}{dx^2}$$

$$q = q_o + \gamma_e d$$

where, q_o is the intensity of the UDL, γ_e is the unit weight of the embankment and so q will be the sum of external load and the soil weight. Now, the basic governing differential equation can be written as:

$$q_o + \gamma_e d = kw - G\left[d' \frac{dw}{dx} + \left(H_f + d\right)\frac{d^2 w}{dx^2}\right] \qquad \text{for } x \le B$$

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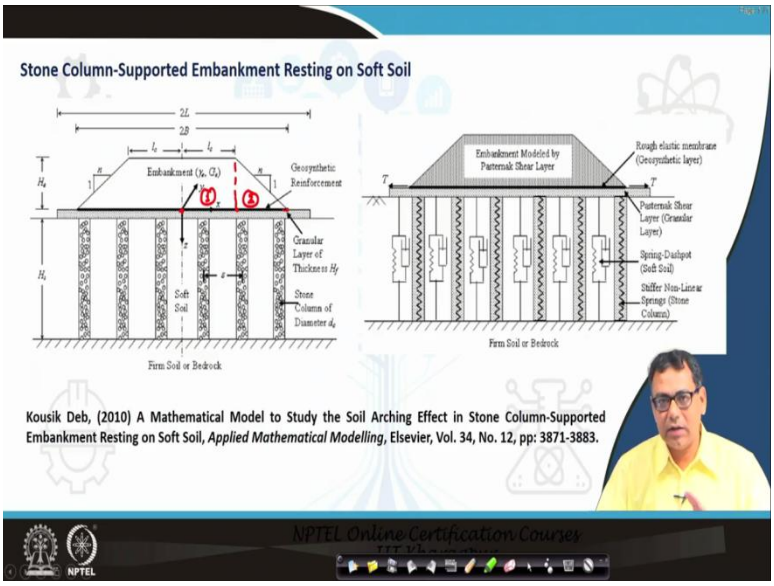

The length from the centre of the embankment to the point where the slope starts is taken as l_c. In that case, we can write:

$$d = H_e \quad \text{If}: 0 \le x \le l_c \rightarrow (1)$$

$$d = H_e - \left[\frac{(x - l_c)}{n}\right] \quad \text{If}: l_c \le x \le B \rightarrow (2)$$

where, n is the side slope of the embankment (1V : nH) and B is the half width of the embankment.

The value of d′ will be zero as long as x is within l_c as the height of embankment is constant in that portion. For the side slope part of the embankment, d′ is given below.

$$d' = 0 \quad \text{If}: 0 \le x \le l_c$$

$$d = -\frac{1}{n} \quad \text{If}: l_c \le x \le B$$

The differential expression for the region beyond the embankment is given below:

$$kw - GH_f \frac{d^2 w}{dx^2} = 0 \qquad \text{for } x > B$$

This way, the embankment problem can be solved using the Pasternak model. The difference between the original Pasternak model and this is that the thickness of the shear layer is not constant here, but is a function of x.

If the time dependent effect should be incorporated, the kw term in the differential equation should be divided with U and can be solved for that.

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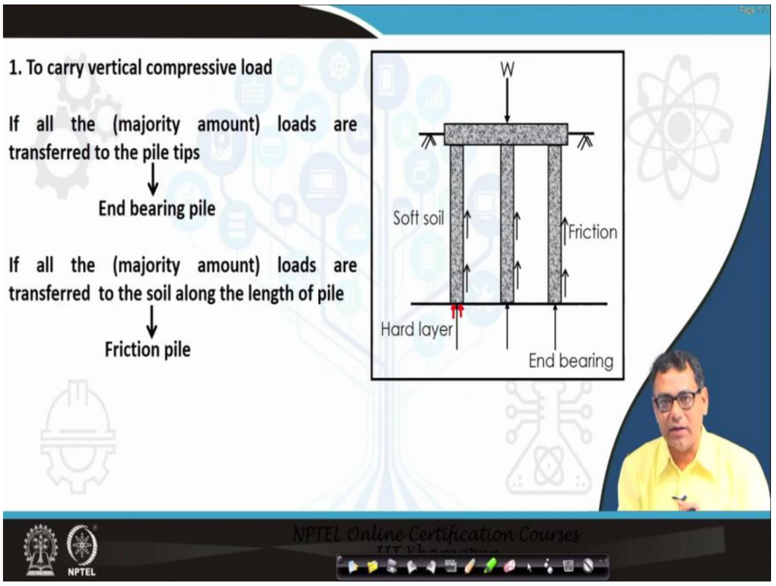

Now, let us start the concept of soil-structure interaction in the pile foundation. The piles can be classified into three groups: piles under compressive load, piles under lateral load and piles subjected to uplift. If a pile is subjected to a compressive load, the load transfer happens from the pile to the soil through friction between the pile shaft and the soil throughout the pile length. If the pile rests on a hard stratum, the load transfer may occur due to the end bearing also. If the majority of the resistance is due to the friction then it is called a friction pile and if majority of the resistance is due to the end bearing, then it is called an end bearing pile.

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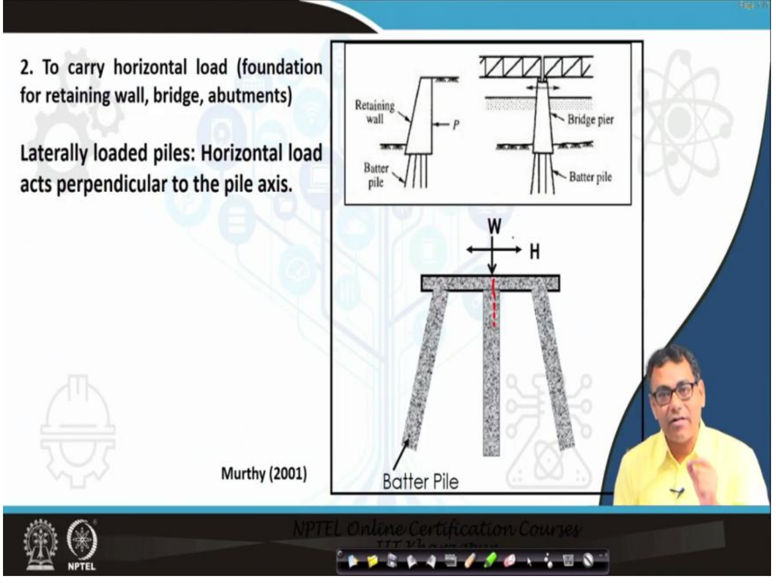

Next is about the pile subjected to horizontal or lateral load. This type of piles can be encountered with below retaining walls, bridges, apartments, or a pile in any off-shore structure where the load from a water wave acts on the pile. If the load axis is perpendicular to the pile axis, it can be called as the lateral loaded pie.

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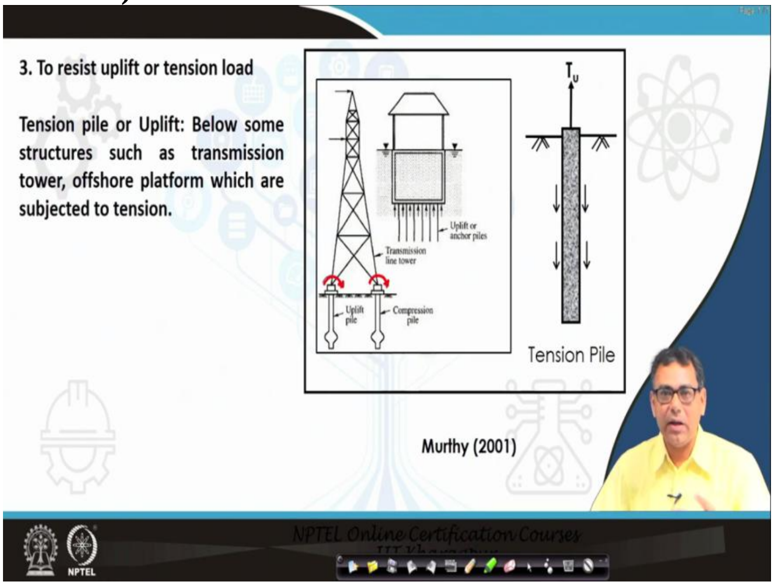

The third type of pile is the pile under uplift. Suppose there is a pile in the foundation of a tower. Due to the wind load, there is a chance that the tower will be subjected to a moment or rotation and in such case, piles in one side of the foundation will be subjected to tension. So, whenever a pile is under towers or tall buildings, they have to be checked under tension.

These are the three different types of loading possible in piles. The soil-structure interaction part within the pile will only be considered and basically, the settlement calculation will be dealt with. Also, the conventional design of the pile foundation under compressive load, how to consider the interaction of the piles in a group to calculate the settlement will be explained.

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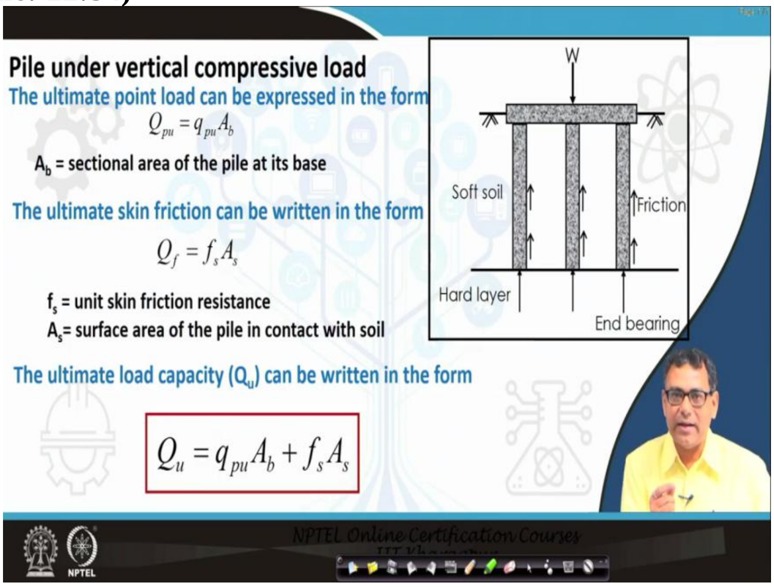

For a pile under vertical compressive load the ultimate load may come from bearing and from friction. The frictional resistance is f_s and bearing resistance is q_p. By multiplying the surrounding area of the pile, the total frictional resistance can be calculated.

$$Q_f = f_s \times A_s$$

A_s value can be determined by multiplying the perimeter of the pile and the pile length. If the area of the pile base is multiplied with q_p, then the Q_{pu} value is obtained. So, the ultimate load carrying capacity of the pile is:

$$Q_u = q_{pu} A_b + f_s A_s$$

The first term is the end bearing resistance and the second term is the frictional resistance.

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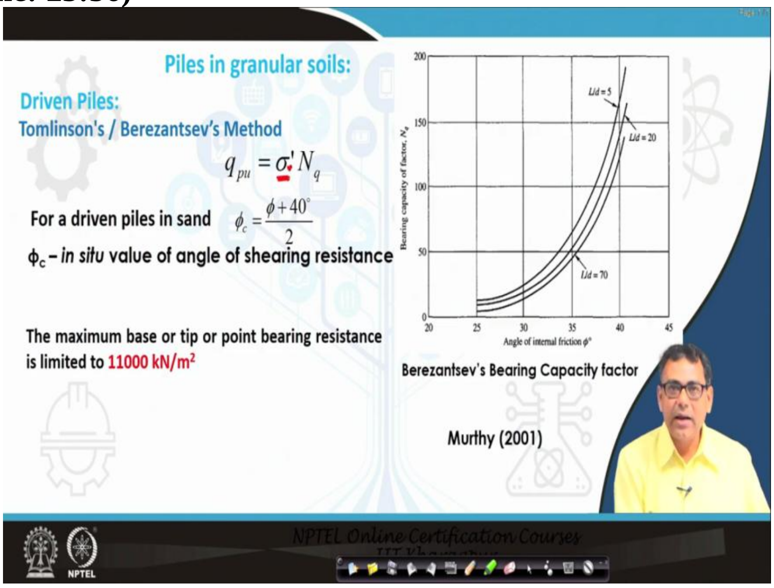

The end bearing resistance of a pile can be calculated using the correlation given above:

$$q_{pu} = \sigma'_v N_q$$

So, if the angle of the friction angle of the soil is known, the chart to read the N_q value is given and the effective vertical stress can be calculated. If the case is a driven pile in sand and if $\varphi > 40^o$, then the φ value should be corrected using the expression:

$$\phi_c = \frac{\phi + 40^\circ}{2}$$

Remember that it is recommended to restrict the bearing resistance to 11,000 kN/m^2. So, if the bearing resistance is calculated to be more than this value, consider it to be equal to 11,000 kN/m^2.

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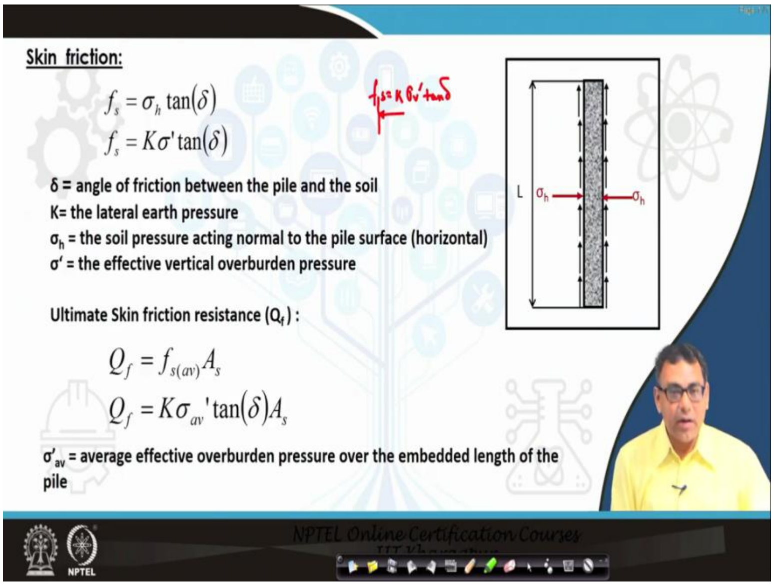

The formulae to calculate the skin friction is given:

$$f_s = \sigma_h \tan\delta$$

$$\Rightarrow f_s = K\sigma' \tan\delta$$

σ'_v is the effective vertical overburden pressure, δ is the angle of friction between the pile & soil, K is the lateral earth pressure coefficient and σ_h is the soil pressure acting normal to the pile surface. This skin friction value should be multiplied with the area of pile that offers friction to get the ultimate skin frictional resistance:

$$Q_f = f_{s(av)} A_s$$

$$\Rightarrow Q_f = K\sigma'_{av} \tan(\delta) A_s$$

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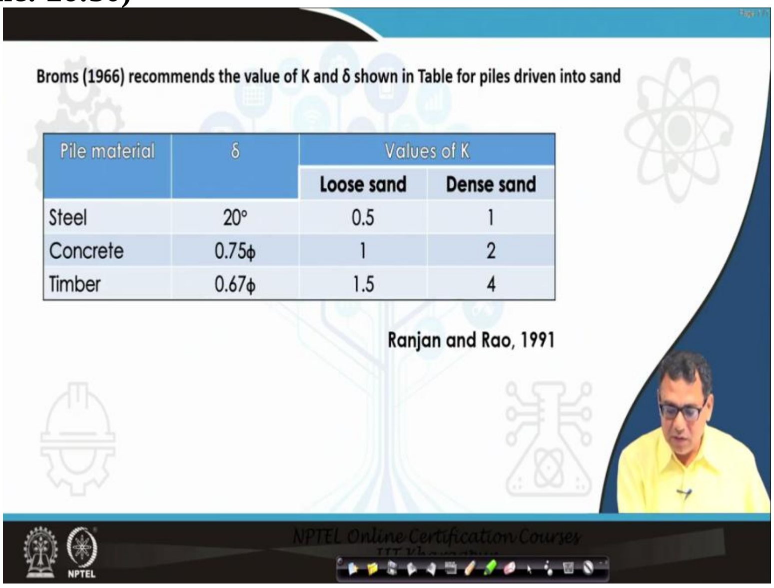

Pile material	δ	Values of K	
		Loose sand	Dense sand
Steel	20°	0.5	1
Concrete	0.75ϕ	1	2
Timber	0.67ϕ	1.5	4

The above slide shows the table to determine the values of δ and k for various materials and combinations.

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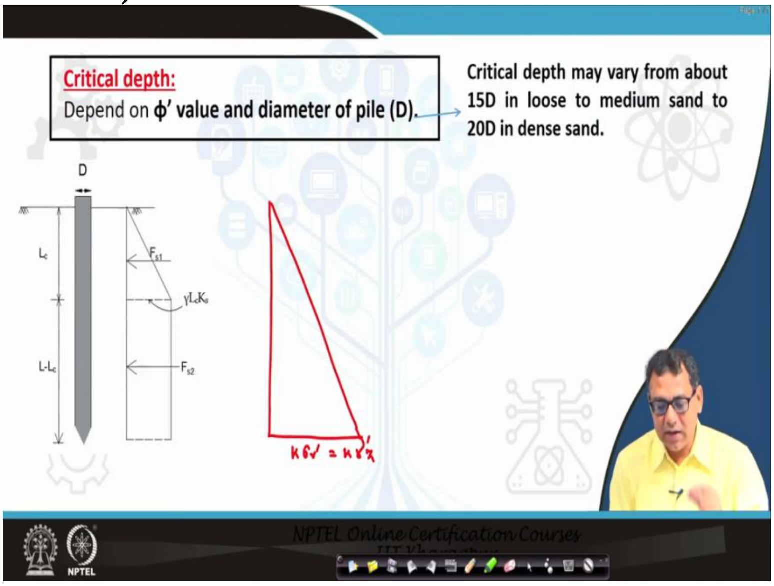

Now let us see the concept of critical depth. Usually, the lateral stress due to soil is assumed to increase linearly with depth. But in case of piles, it is recommended to restrict this value as shown in the left side figure above. So, the stress is assumed to increase up to a certain depth and then to be uniform beyond that. The depth beyond which the lateral earth pressure remains constant is termed as the critical length or critical depth (L_c).

The value of critical depth is 15D for loose to medium sand and for dense sand it is 20 D (D is the diameter of the pile). So, in a dense sand, the lateral stress on pile increases up to a depth of 20D and beyond that depth, it will become constant.

To calculate the tip resistance, the σ'_v value is needed which should be calculated using the reduced value or the uniform value.

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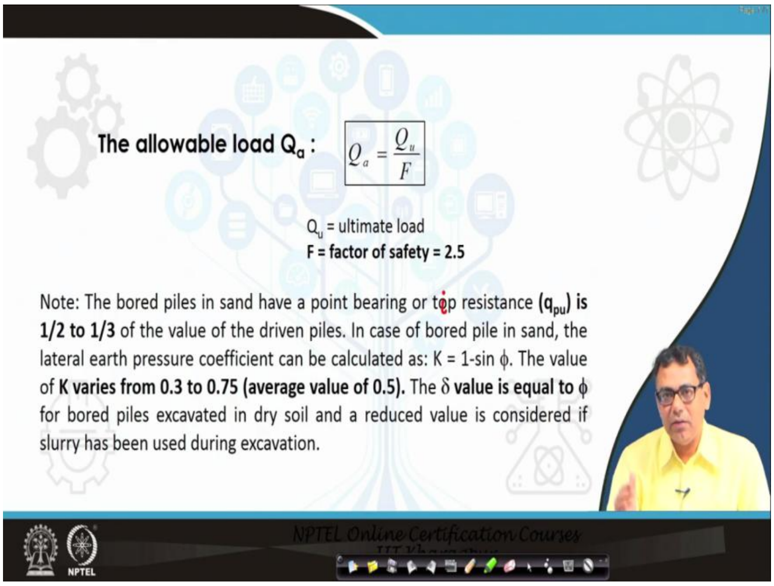

Ultimately, both the frictional resistance and end bearing resistance should be added to determine the ultimate load carrying capacity of the pile (Q_u). This ultimate load capacity is then divided by a factor of safety (usually 2.5), to get the allowable load, Q_a.

For bored piles in sand, the point bearing resistance or the tip resistance will be 1/2 to 1/3 of the value of driven pile.

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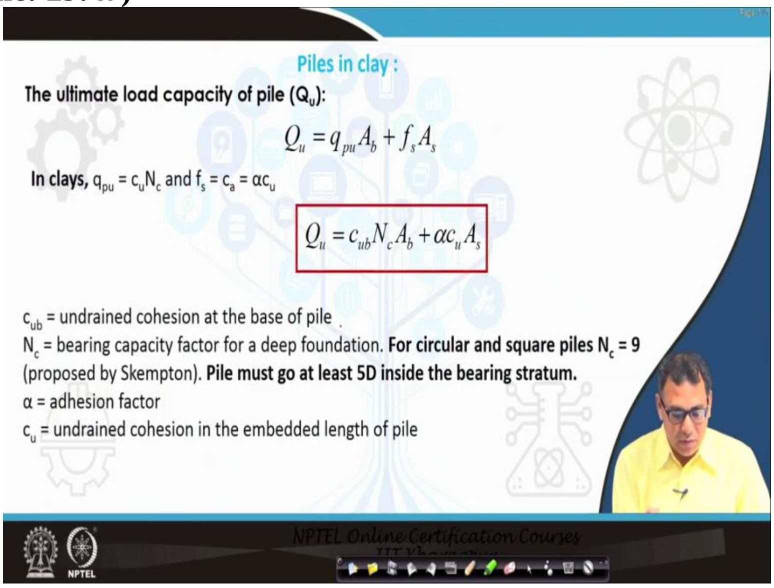

The ultimate load capacity of a pile can be given by:

$$Q_u = q_{pu}A_b + f_sA_s$$

In clays: $q_{pu} = c_u N_c$ and $f_s = c_a = \alpha c_u$

$$Q_u = c_{ub}N_cA_b + \alpha c_uA_s$$

where, α is the adhesion factor, c_{ub} is the undrained cohesion of the soil at the base of the pile, N_c is the bearing capacity factor (usually, 9 for piles) and c_{ub} is the undrained cohesion of the soil throughout the embedded length of the pile.

The adhesion between the clay to the pile material is similar to that of the δ in case of pile in granular soil.

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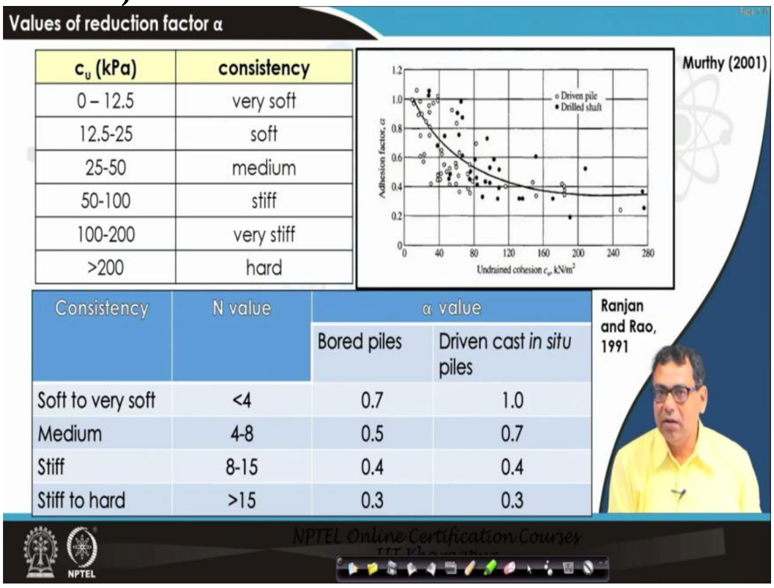

The above table gives the c_u values based on the consistency of the clayey soil. Also, the N values are given depending upon the consistency and the type of installation of the pile (bored or cast in-situ). In case of sand the value for driven pile can be calculated and then for bored pile it would be 1/2 to 1/3 of the driven pile value.

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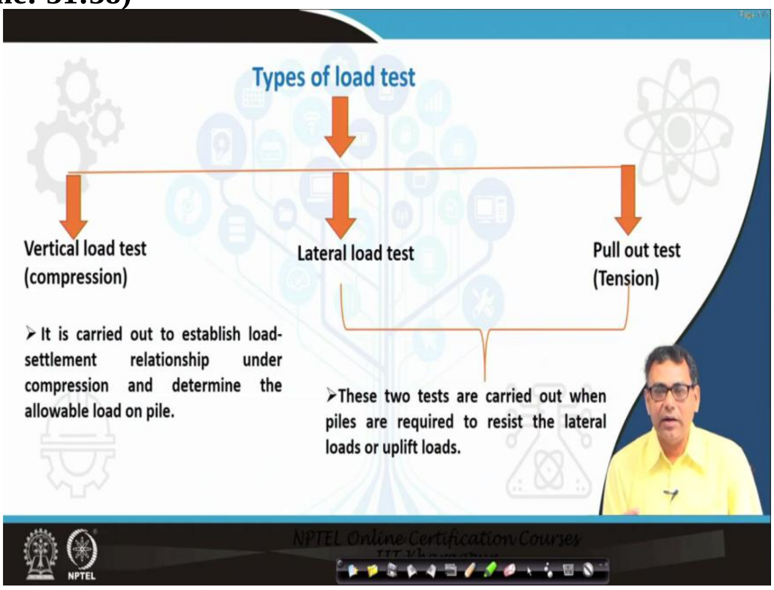

The above slide shows the different types of pile load tests through which the load carrying capacity of a pile can be determined. The pile load test is similar to the plate load test for shallow foundation but the pile load test helps to calculate the load carrying capacity of piles. There are even dynamic formulae to calculate the bearing capacity of driven piles.

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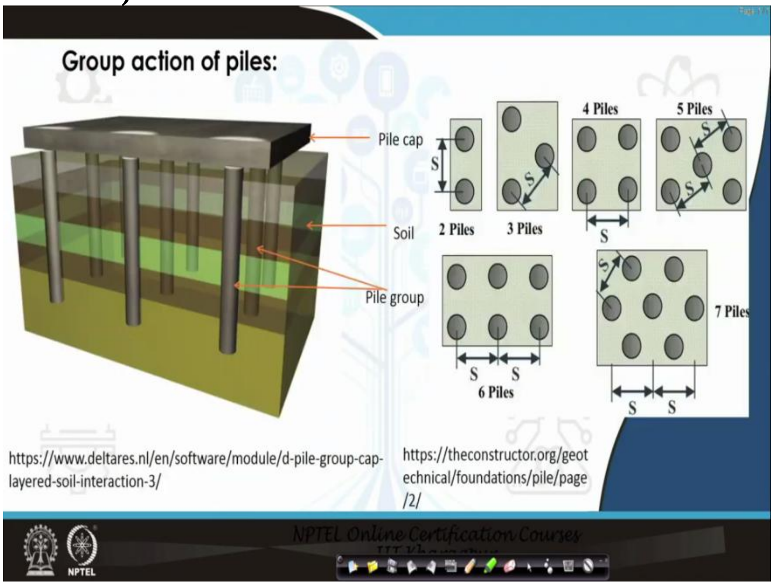

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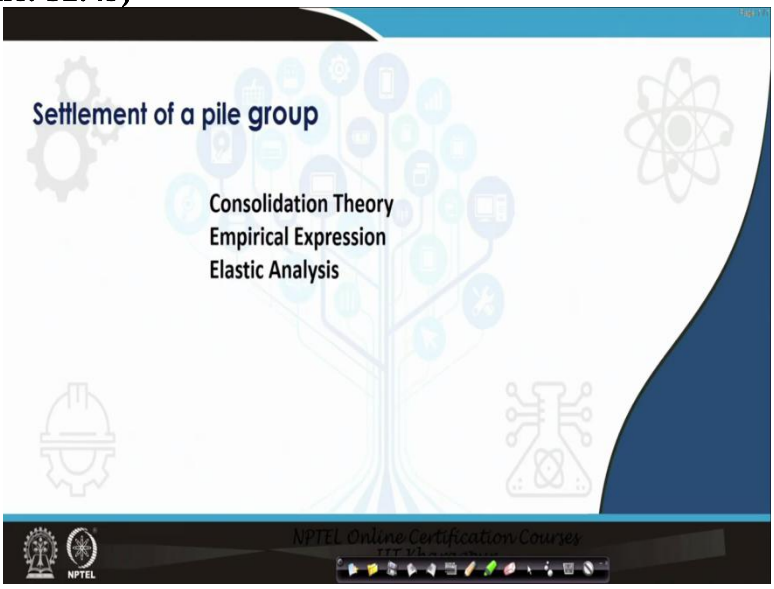

In the next class, I will discuss the pile load test. After that, I will discuss the group pile interaction. Then I will discuss the settlement calculation considering the interaction among the piles in a group. Thank you.