Soil Structure Interaction
Prof. Kousik Deb
Department of Civil Engineering
Indian Institute of Technology Kharagpur

Lecture – 51
Use of Finite Difference Method for Soil Structure Interaction Problems (Contd.)

In this class, I will show you how to represent the equations of soil-structure interaction problems in non-dimensional form, when you solve them by finite difference method. Generally, if you solve this type of problems using numerical techniques, then we should express them in non-dimensional form.

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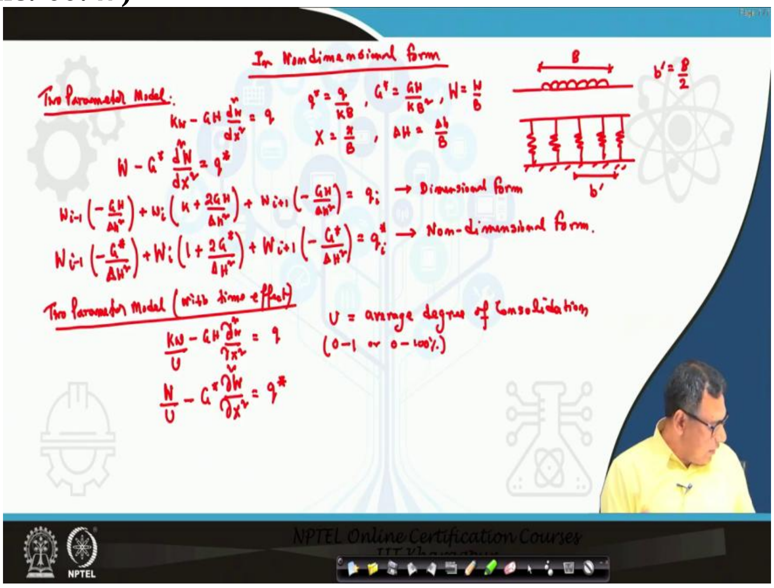

Few of the expressions already discussed will be given in the non-dimensional form and then it will be easy to express all the other equations or the problems that are discussed. Let us get started with the two parameter model. The normal equation when a UDL, q is acting on a two parameter model is:

$$kw - GH\frac{d^2w}{dx^2} = q$$

The UDL is extended over B and half of this value, B/2 is considered as b′.

In the non-dimensional form:

$$q^* = \frac{q}{kB}; \qquad G^* = \frac{GH}{kB^2}$$

$$W = \frac{w}{B}; \qquad X = \frac{x}{B}; \qquad \Delta H = \frac{\Delta h}{B}$$

So, the non-dimensional form of the basic differential equation is:

$$W - G^* \frac{d^2W}{dx^2} = q^*$$

The finite difference form of the basic differential equation is:

$$w_{i-1}\left(-\frac{GH}{\Delta h^2}\right) + w_i\left(k + \frac{2GH}{\Delta h^2}\right) + w_{i+1}\left(-\frac{GH}{\Delta h^2}\right) = q_i \rightarrow \text{Dimensional form}$$

$$W_{i-1}\left(-\frac{G^*}{\Delta H^2}\right) + W_i\left(1 + \frac{2G^*}{\Delta H^2}\right) + W_{i+1}\left(-\frac{G^*}{\Delta H^2}\right) = q^*{}_i \rightarrow \text{Non-Dimensional form}$$

Now, the equation to incorporate the time dependent response in this model:

$$\frac{kw}{U} - GH\frac{\partial^2 w}{\partial x^2} = q$$

$$\frac{W}{U} - G^* \frac{d^2W}{dx^2} = q^*$$

U is the average degree of consolidation that varies from 0 to 1 or 0 to 100 %.

This equation can also be expressed in the finite difference form and then solution can be obtained by forming a matrix.

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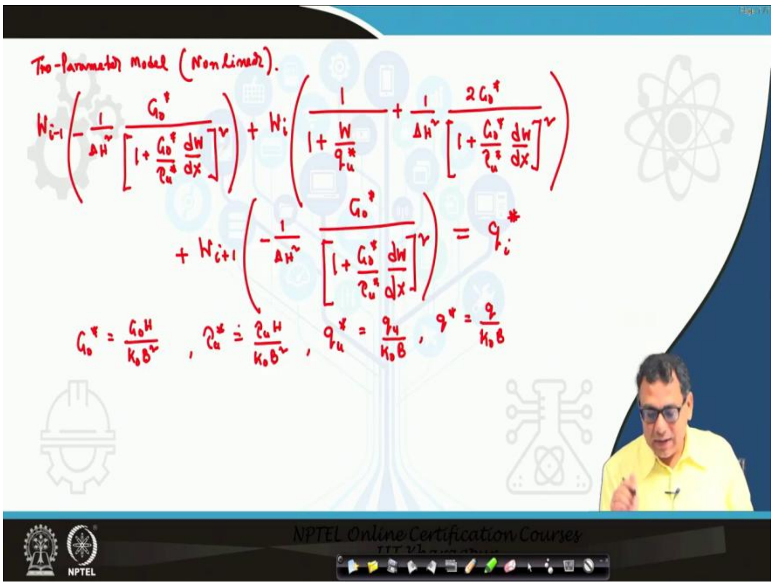

Another case that will be dealt with the non-dimensional form is the non-linear behavior of a two parameter model:

$$W_{i-1}\left[-\frac{1}{\Delta H^2}\frac{G_o^*}{\left(1+\frac{G_o^*}{\tau_u^*}\frac{dw}{dx}\right)^2}\right]+W_i\left[\frac{1}{1+\frac{W}{q_u^*}}+\frac{1}{\Delta H^2}\frac{2G_o^*}{\left(1+\frac{G_o^*}{\tau_u^*}\frac{dw}{dx}\right)^2}\right]+W_{i+1}\left[-\frac{1}{\Delta H^2}\frac{G_o^*}{\left(1+\frac{G_o^*}{\tau_u^*}\frac{dw}{dx}\right)^2}\right]=q_i^*$$

$$\text{where, } q_u^*=\frac{q_u}{k_o B}; \qquad G_o^*=\frac{G_o H}{k_o B^2}; \qquad \tau_u^*=\frac{\tau_u H}{k_o B^2}; \qquad q^*=\frac{q}{kB}$$

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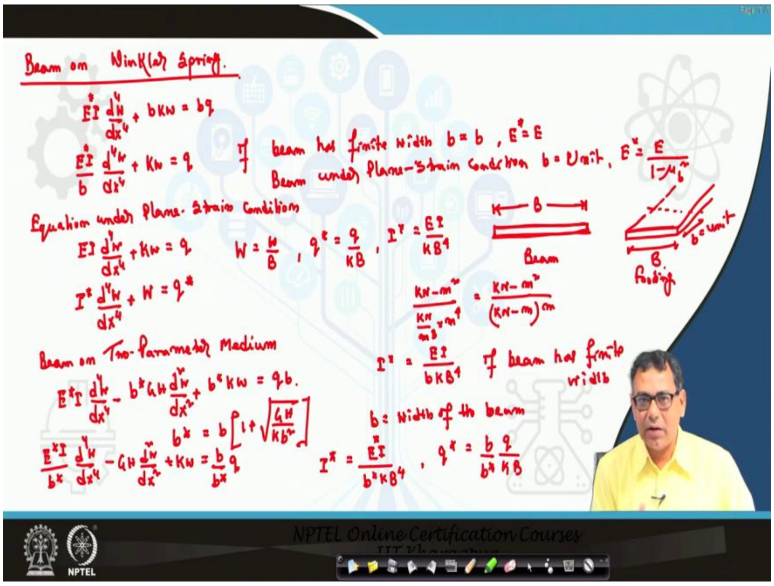

The expressions discussed till now are for the load directly acting on the soil medium. Now let us see the equation for a beam resting on Winkler springs.

$$E^*I\frac{d^4w}{dx^4}+bkw=bq$$

$$\Rightarrow \frac{E^*I}{b}\frac{d^4w}{dx^4}+kw=q$$

If the beam has finite width: b = b and E^* = E

If the beam is under plane strain condition: b = unit and $E^* = E/(1-\mu_b^2)$

So, the equation under plane strain condition is:

$$E^*I\frac{d^4w}{dx^4}+kw=q$$

$$q^*=\frac{q}{kB}; \qquad W=\frac{w}{B}; \qquad I^*=\frac{EI}{kB^4}$$

$$I^* \frac{d^4 w}{dx^4} + W = q^*$$

The length of the footing is actually considered as the width of the beam. So, if it is under plane strain condition, it refers to a strip footing which has a very high value for length. Out of this, unit length of the footing is considered as the unit width of the footing for evaluation. If the formula for I^* is observed, it is not non-dimensional. The reason for this is there is another b in the expression which is already replaced with unit in that expression. The original expression for I^* is:

$$I^* = \frac{EI}{bkB^4}$$

If the beam has finite width, the above expression will be used as it is, but if it is in plane strain b can be replaced with 1.

Similarly for the two parameter model:

$$E^* I \frac{d^4 w}{dx^4} - b^* GH \frac{d^2 w}{dx^2} + b^* kw = bq$$

$$b^* = b\left[1 + \sqrt{\frac{GH}{kb^2}}\right]$$

$$\frac{E^* I}{b^*} \frac{d^4 w}{dx^4} - GH \frac{d^2 w}{dx^2} + kw = \frac{b}{b^*} q$$

$$I^* = \frac{EI}{b^* kB^4} \ ; \quad q^* = \frac{b}{b^*} \frac{q}{kB}$$

These are some of the different non-dimensional forms that have been discussed. The other equations can also be expressed in this non-dimensional form.

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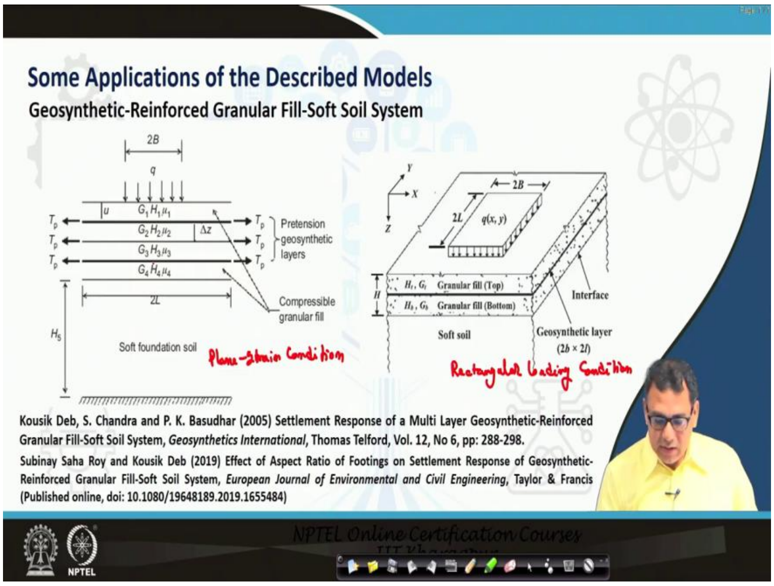

The next concept is about some applications of the described models. Below each application, references of few papers are given that can be looked into, for detailed explanation. There are a number of applications of these described models and people have done a tremendous amount of work in this area applying into the real field. *A few examples of my research are given, but there are other researchers also they have done not lots of work on these areas.*

Consider a granular fill placed over a soft soil and within the granular fill geosynthetic reinforcement is provided. In this model, the soft soil is modeled as a combination of spring and dashpot to incorporate the time effect, and then the granular fill is modeled as a shear layer of the Pasternak model. Geosynthetic reinforcement is modeled as rough elastic membrane. The right side figure is an example for the plane-strain condition and the left side figure shows a rectangular loading condition.

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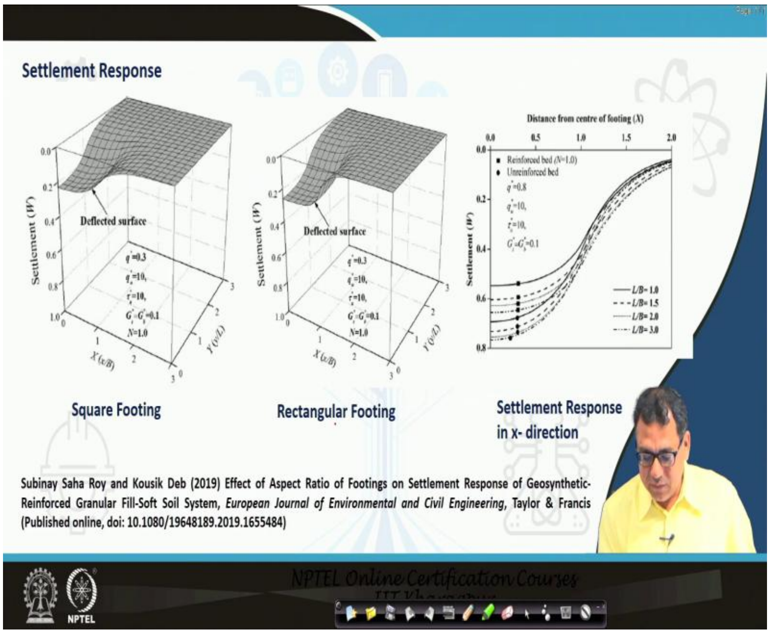

If they are solved, the deflection profile similar to that of the profiles shown above can be obtained. The deflection profiles for rectangular and square loading or footing. The first two figures show the deflection profile of the geosynthetic or the ground surface and the third figure shows the deflection profile in the x direction alone (may be under plane-strain).

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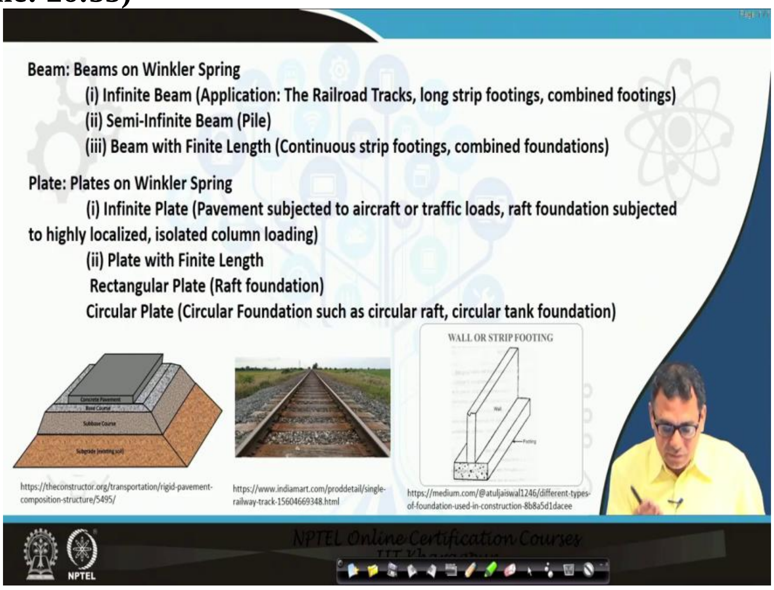

The types and applications of the beams are already discussed where infinite beam is used for railroad, strip loading and combined footing. The semi-infinite application for pile deflection will be discussed in the laterally loaded pile analysis part. Then beam with finite length will be used for the continuous footing and combined footing. The infinite plate models a pavement subjected to aircraft loading and raft foundation subjected to highly localized loads and the plate with finite length models circular foundation and also raft foundation.

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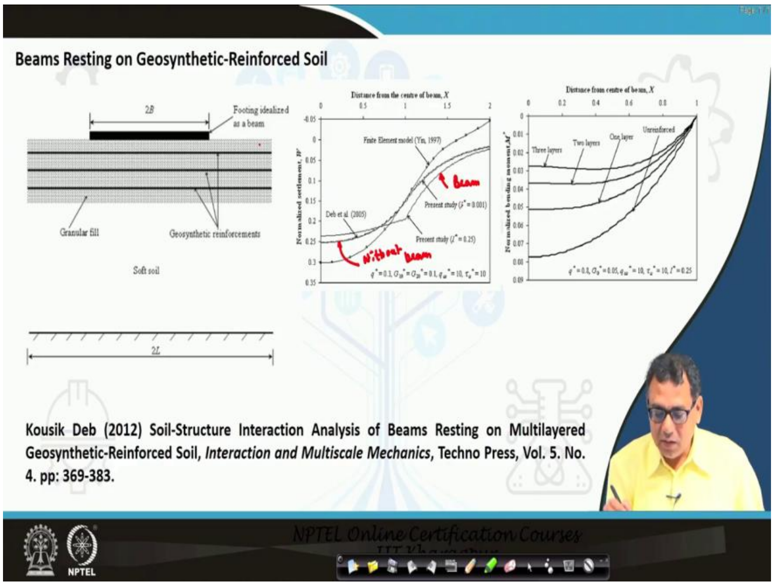

Another example is that a beam resting on a reinforced soil. The footing rests on a reinforced granular soil that is underlain by soft soil stratum. The settlement response and bending moment response are shown in the graphs above. The settlement responses the case with beam and without beam are plotted in the graph. Because of the rigidity of the beam the settlement is more or less uniform compared to the case when there is no beam.

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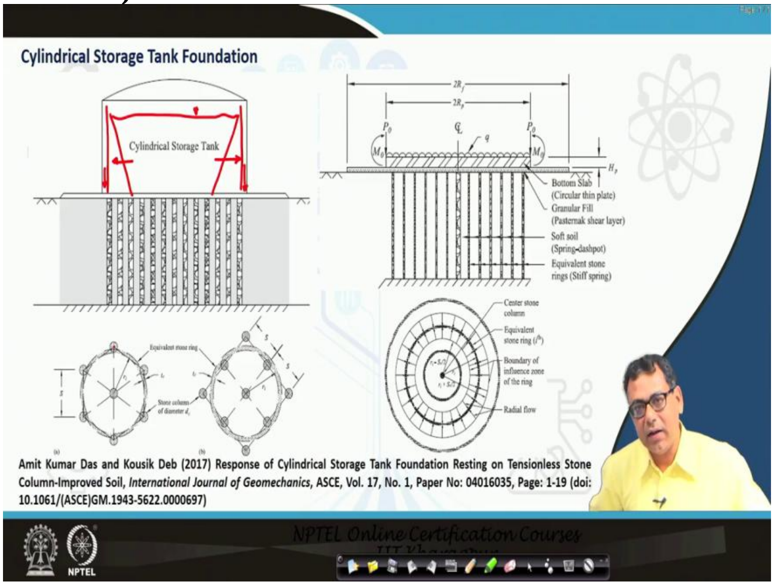

The next example is of a circular tank which is a very interesting problem. Nowadays circular tanks are being placed over stone columns. The above figure shows a circular tank placed over the stone column and the different loads acting in this case. The circular tank can be empty or can be full. If it is a full tank, a uniform circular UDL will act on the base. The base of the tank or basically the foundation of the tank is modeled as a circular plate and this tank is generally placed over a sand layer. First, the soft soil is improved by stone column, then the sand layers are placed and over that, the circular tank is placed. The soft soil is modeled by spring and dashpot to incorporate the time effect and stone column is modeled by stiffer spring compared to that of the soft soil.

Stone column is a type of ground improvement technique by which the stiffness and rate of consolidation of the soil can be increased. Stone column is made with a material that is stiffer when compared to the soft soil. This is why the stone column will be represented by a stiffer spring and soft soil by the spring & dashpot. The spring constant value may be k which is much lesser than that of the stone columns. The granular layer is represented by the Pasternak shear layer and then the tank foundation by a circular plate.

In the top figure to the left, a section of the tank along with the foundation soil is shown. As there is a periphery wall around the footing, it will act like concentrated load. So, two concentrated loads at the edges and a UDL of q_o within the point loads will act. If the tank is full of water or any liquid the liquid pressure acts laterally on the walls which in turn produce a moment at the edges.

So two concentrated moments will act on the walls due to the liquid present in the tank, two concentrated loads, due to the wall of the tank and a UDL due to the liquid weight and the weight of the foundation will act over this system.

The stone columns may be placed in a rectangular or square pattern but are converted into circular rings. The conversion is done in such a way that the ring area is equal to the number of stone columns area which is within one ring. For example, in case of a square pattern for a particular ring, there are 9 columns. So, these 9 columns area is equal to the area of one stone ring. This is the procedure to determine the thickness of the stone column. Similarly for triangular pattern, there are a total of 7 columns within a ring. So, the area of the 7 columns will be equal to the area of that one ring used for replacement.

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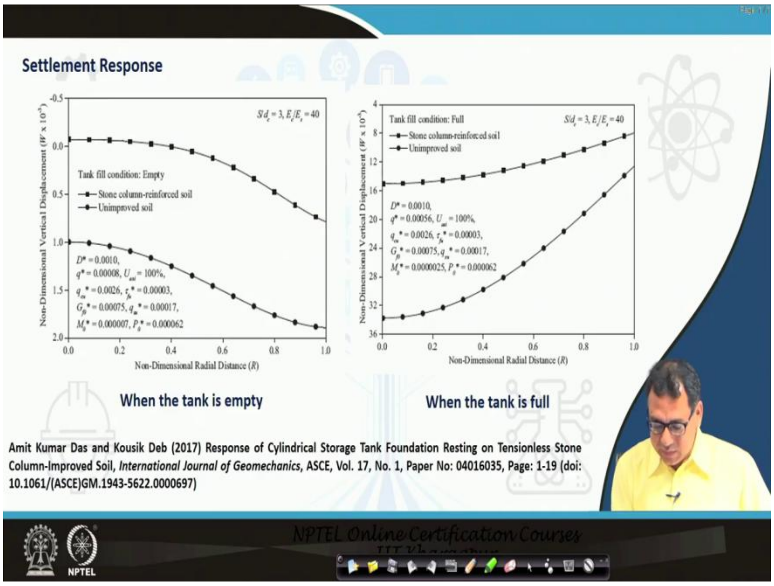

After modeling the system in this way, the deflections can be obtained. The deflection profile for full empty condition empty of the tank is shown in the left side graph above. Empty condition means that there will be no moment, only concentrated load will act from the tank wall and a UDL due to the soil and weight of the foundation. So, the deformation at the edges will be more when compared to the center. But when the tank is full, there will be moments and UDL due to the liquid pressure. So, the deformation of the center will be more compared to that of the edges.

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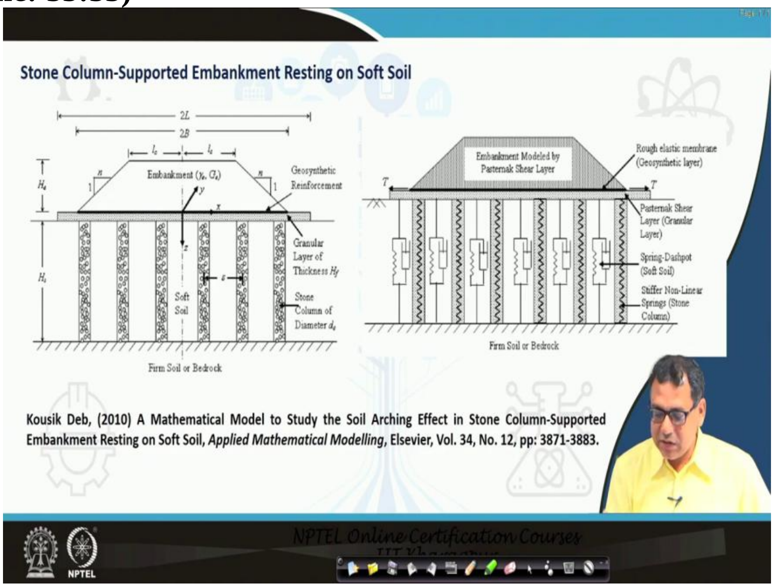

There is another application area, which is the embankment. An embankment can also be modeled using Pasternak shear layer. The above figure shows an embankment resting on stone columns.

In the next class, I will start the pile foundation part. But before that I will quickly give an idea about how to use the Pasternak model to model the embankment. Thank you