Soil Structure Interaction

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Lecture-48
Use of Finite Difference Method for Soil Structure Interaction Problems (Contd.,)

In this class I will continue the problem of determining the solution for a finite beam resting on two parameter soil medium under plane strain condition. In the last class, the governing differential equations were represented in the finite difference form.

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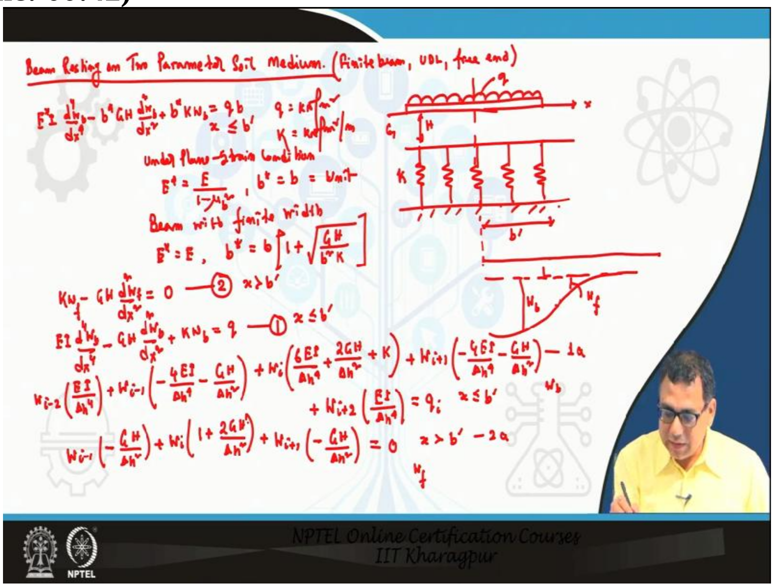

The above slide shows both the governing differential equations in finite difference form. Now, let us use the boundary conditions to solve these equations. When the beam rests on springs and is subjected to UDL, the shear force at the centre of the beam and also at the free end is zero. But here the beam is resting on a two parameter soil medium.

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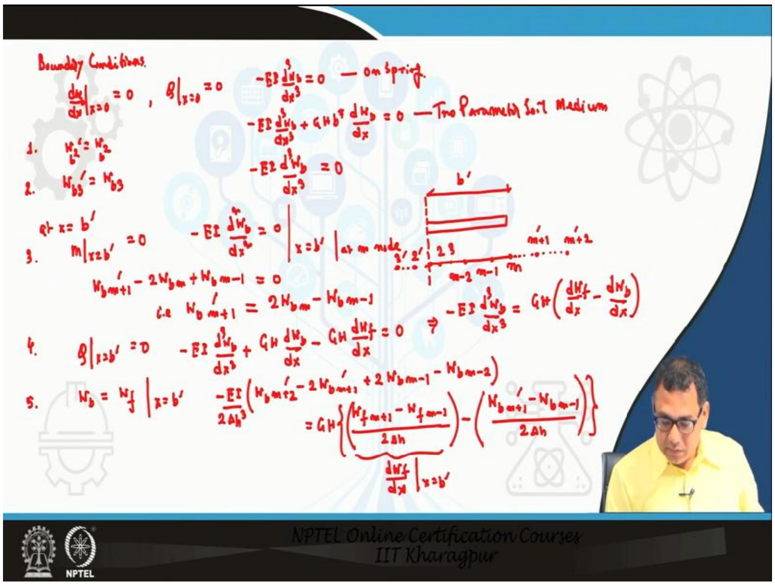

Remember that if the beam is resting on Winkler springs, the shear force under UDL at the beam centre will be zero and the expression for that will be:

$$-EI\frac{d^3w_b}{dx^3}=0$$

But, the expression for similar conditions on the two parameter model will be:

$$-EI\frac{d^3w_b}{dx^3}+GHb^*\frac{dw_b}{dx}=0$$

The first boundary condition is that the slope at the centre is zero (x = 0) because the load is symmetric:

$$\frac{\mathrm{dw_b}}{\mathrm{dx}}=0\Big|_{x=0} \qquad \Rightarrow 1.) w'_{b2}=w_{b2}$$

By adopting the finite difference scheme in the above boundary condition, the relation written can be found just like that of showed in the previous lecture.

The second boundary condition is that the shear force will be 0 at the centre (x = 0):

$$-\mathrm{EI}\frac{\mathrm{d}^3w_b}{dx^3}+b^*GH\frac{dw_b}{dx}=0\Big|_{x=0}$$

As the slope at the centre is 0 (1st boundary condition), the above expression reduces to:

$$\Rightarrow -\mathrm{EI}\frac{\mathrm{d}^3w_b}{dx^3}=0\Big|_{x=0}$$

$$\frac{\mathrm{d}^3w_b}{dx^3}=0\Big|_{x=0} \qquad \Rightarrow 2.) w'_{b3}=w_{b3}$$

By adopting the finite difference scheme in the above boundary condition, the relation written can be found just like that of showed in the previous lecture.

The third boundary condition is that the bending moment will be 0 at the edge where, x = b′.

$$-\mathrm{EI}\frac{\mathrm{d}^2 w_b}{dx^2}=0\Big|_{x=b'} \quad \Rightarrow \frac{\mathrm{d}^2 w_b}{dx^2}=0\Big|_{x=b'}$$

Now writing the above expression in the finite difference form:

$$w'_{b(m+1)}-2w_{b(m)}+w_{b(m-1)}=0$$

$$\Rightarrow 3.)w'_{b(m+1)}=2w_{b(m)}-w_{b(m-1)}$$

The fourth boundary condition is that the shear force is also 0 at x = b′ (free end)

$$Q\big|_{x=b'}=0$$

The second boundary condition used was also for shear force, but this expression is a bit different because this is the edge of the beam. So, here both the beam and foundation soil should be considered to evaluate the shear force. So:

$$\Rightarrow -\mathrm{EI}\frac{\mathrm{d}^3 w_b}{dx^3}+GH\frac{dw_f}{dx}-GH\frac{dw_b}{dx}=0$$

$$\Rightarrow -\mathrm{EI}\frac{\mathrm{d}^3 w_b}{dx^3}=GH\left(\frac{dw_f}{dx}-\frac{dw_b}{dx}\right)$$

$$-\frac{EI}{2\Delta h^3}\left[w'_{b(m+2)}-2w'_{b(m+1)}+2w_{b(m-1)}-w_{b(m-2)}\right]=GH\left\{\underbrace{\left[\frac{w_{f(m+1)}-w_{f(m-1)}}{2\Delta h}\right]}_{\frac{dw_f}{dx}\big|_{x=b'}}-\left[\frac{w'_{b(m+1)}-w_{b(m-1)}}{2\Delta h}\right]\right\}$$

In the above expression, the deflections at the nodes in the foundation soil are not denoted with the imaginary symbol. This is because beyond the beam region, the imaginary nodes should be assumed only with respect to the beam deflection (w_b). But as per the deflection of foundation soil (w_f) is concerned, these nodes really exist and so the imaginary symbol (prime or dash) is not been used for this. For now, it is better to keep the foundation soil part in the dw_f/dx form as it will not be substituted.

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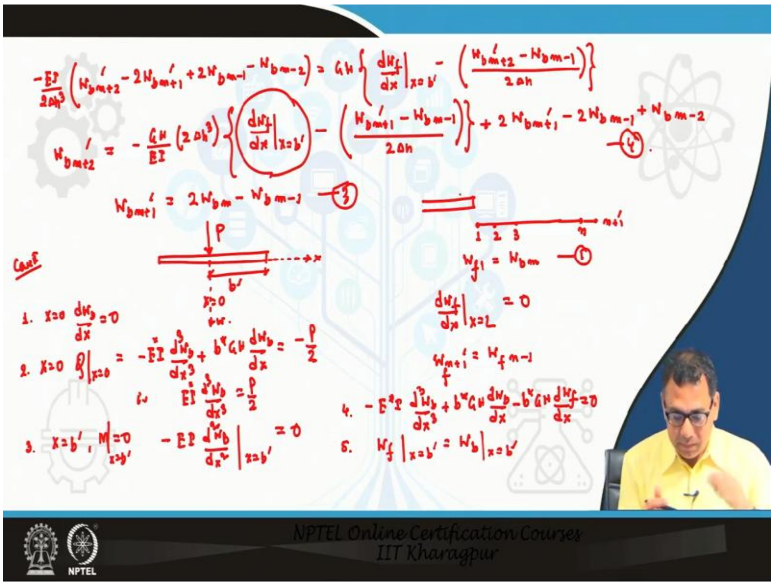

By substituting the value of $w'_{b(m+1)}$ in the last expression, we get:

$$\Rightarrow -\frac{EI}{2\Delta h^3}\left[w'_{b(m+2)} - 2w'_{b(m+1)} + 2w_{b(m-1)} - w_{b(m-2)}\right] = GH\left\{\left.\frac{dw_f}{dx}\right|_{x=b'} - \left[\frac{w'_{b(m+1)} - w_{b(m-1)}}{2\Delta h}\right]\right\}$$

$$4.)\; w'_{b(m+2)} = -\frac{GH}{EI}\left(2\Delta h^3\right)\left\{\left.\frac{dw_f}{dx}\right|_{x=b'} - \left[\frac{w'_{b(m+1)} - w_{b(m-1)}}{2\Delta h}\right]\right\} + 2w'_{b(m+1)} - 2w_{b(m-1)} + w_{b(m-2)}$$

In the third boundary condition, the value of $w'_{b(m+1)}$ was found out and if that value is substituted in the above expression, $w'_{b(m+2)}$ can be expressed in terms of known values.

Till now all the boundary conditions and expressions are related to the region within the beam. The equation (1a) should be applied to all the nodes within the beam region only and similarly for the nodes beyond the beam region, equation (2a) should be used. Note that the beam region ends at x = b′ and from that point, the nodes will be numbered again from 1 to n. Where, node-1 in the second case is at the beam edge.

For the first node beyond beam, the boundary condition is that the deflection of the beam and the foundation soil are equal at the edge of the beam (x=b′):

$$w_f = w_b\big|_{x=b'}$$

$$w_{f1} = w_{bm}$$

As the m^{th} node for the beam region and the first node for the foundation soil region are the same.

Now, if the length of the zone beyond the beam is denoted as L, it is evident that the slope at x = L should be zero. As already mentioned, L should be considered large enough to be free from the deflections due to loading on the beam. So, if the deflection at x = L is zero, the slope is also zero.

$$\left.\frac{dw_f}{dx}\right|_{x=L} = 0$$

Applying the finite difference for to the above expression, we get:

$$w'_{f(n+1)} = w_{f(n-1)}$$

For the beam region 4 boundary conditions are needed and for the field region 2 boundary conditions are needed. So, using all the boundary conditions, first the imaginary nodes can be converted to the real node values. Then, by forming the stiffness matrix, the solution can be determined.

In the fourth equation, there is an unknown term (dw_f/dx). The value of this term can be determined by following the similar method used for the non linear case already discussed.

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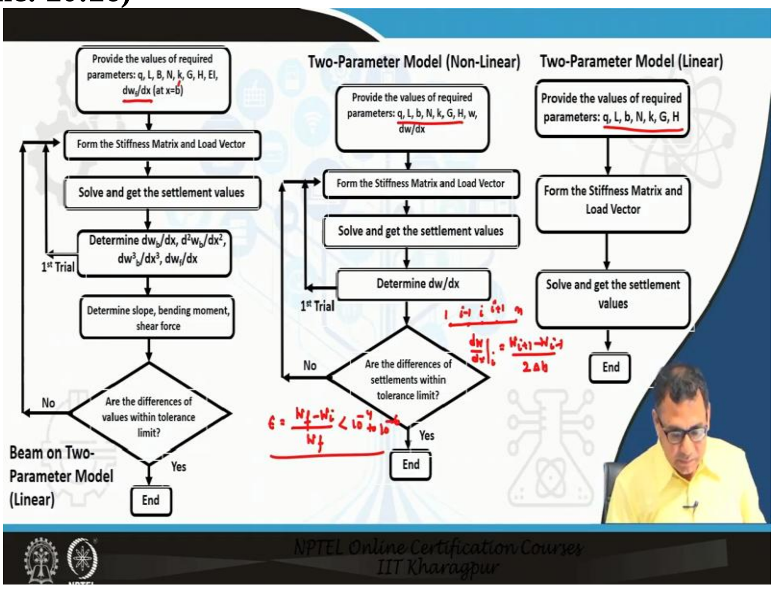

This is the flow chart which is already shown. In the solution for nonlinear behaviour in two parameter soil, it was mentioned to assume the values of w and dw/dx for the first trial. The problem should be solved with the arbitrary values and then slowly after some iterations, the solution converges indicating the approximate solution. Similarly here, the dw_f/dx value should be assumed first and calculate the deflections. From those values, the dw_f/dx value for the next iteration should be calculated.

For each trial, the deflections can be determined and checked for the tolerance limit. If the difference is within the acceptable range, all other quantities can be determined.

Now the next loading condition considered as case 2 here, is the point load acting on the beam at its centre.

The first boundary condition is that the slope at the centre (x = 0) because the load is a point load:

$$1.)\ \frac{\mathrm{dw_b}}{\mathrm{dx}} = 0\Big|_{x=0}$$

The second boundary condition is that the shear force will be –P/2 at the centre (x = 0):

$$Q\big|_{x=0} = -\frac{P}{2}$$

$$\Rightarrow \text{-EI}\frac{\mathrm{d}^3 w_b}{dx^3} + b^* GH\frac{dw_b}{dx} = -\frac{P}{2}\Big|_{x=0}$$

As the slope at the centre is 0 (1st boundary condition), the above expression reduces to:

$$2.)\ \mathrm{E^*I}\frac{\mathrm{d}^3 w_b}{dx^3} = \frac{P}{2}\Big|_{x=0}$$

The third boundary condition is that the bending moment will be 0 at the edge where, x = b′.

$$M = 0\big|_{x=b'} \Rightarrow \text{- EI}\frac{\mathrm{d}^2 w_b}{dx^2} = 0\Big|_{x=b'} \quad \Rightarrow 3.)\ \frac{\mathrm{d}^2 w_b}{dx^2} = 0\Big|_{x=b'}$$

The fourth condition is that the shear force is also 0 at x = L/2 (free end):

$$Q\big|_{x=b'} = 0$$

$$\Rightarrow 4.)\ \text{-}\mathrm{E^*I}\frac{\mathrm{d}^3 w_b}{dx^3} + b^* GH\frac{dw_b}{dx} - b^* GH\frac{dw_f}{dx} = 0$$

The fifth boundary condition is:

$$w_f = w_b\big|_{x=b'}$$

The finite difference method applied to the problem of finite beam was discussed in this class and in the similar way, the infinite beam can also be solved for.

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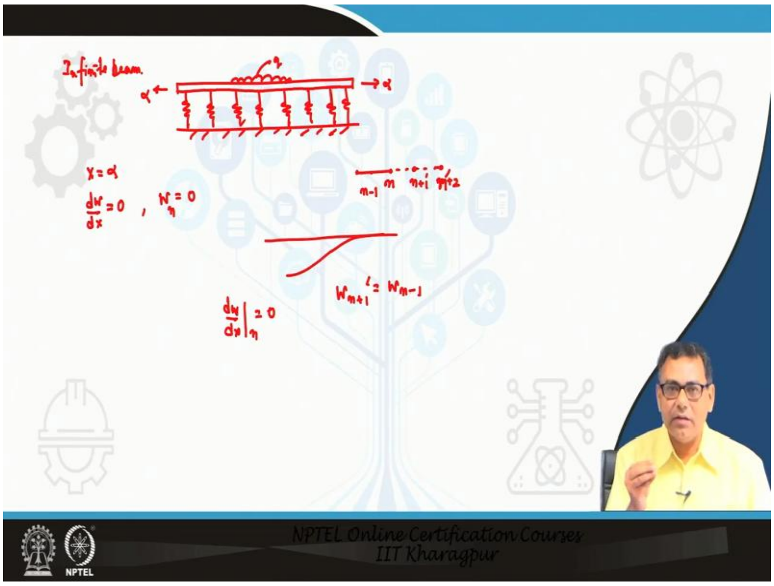

Now consider an infinite beam keeping all the other conditions same. As the UDL, q is acting on the beam, the governing differential equation for infinite beam is the same as that of the finite beam. The UDL acts on the beam for a certain finite length only. So, when the load vector is being developed, the q value should be assigned only to the nodes beneath the UDL. The load vector for the nodes beyond the UDL will have zero value.

Here, the loading condition is symmetric and hence it is sufficient if only half portion is analysed. When $x = \infty$, the deflection and slope will be zero because the effect of the load would not be present till infinity. As usual, the last node i.e., at $x = \infty$ is named as node-n and the deflection at node-n is 0: $w_n = 0$. Till now, imaginary nodes are considered to solve for the n^{th} and $(n-1)^{th}$ nodes. But here the value of deflection for the n^{th} node is already obtained and so, the $(n+2)^{th}$ node need not be assumed. But still, to calculate the value at the $(n-1)^{th}$ node, the $w'_{(n+1)}$ is required.

As the deflection at n is zero, the slope will also be zero and if dw_n/dx is zero, we can write: $w'_{(n+1)} = w'_{(n-1)}$. So, this way the problem of infinite beam can be solved. The infinite beam problem can be solved in a similar way that of the finite beam problem, because the governing differential equation is same. The only difference is in the usage and application of the boundary condition.

In the next class I will discuss how to express this solution in a non dimensional form and then I will show you how to apply the final difference technique to solve the plate problem. Thank you.