Soil Structure Interaction
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Lecture-47
Use of Finite Difference Method for Soil Structure Interaction Problems (Contd.,)

In this class I will show you more applications of finite difference method for soil structure interaction problem. I will apply the method now, to a beam resting on Winkler model.

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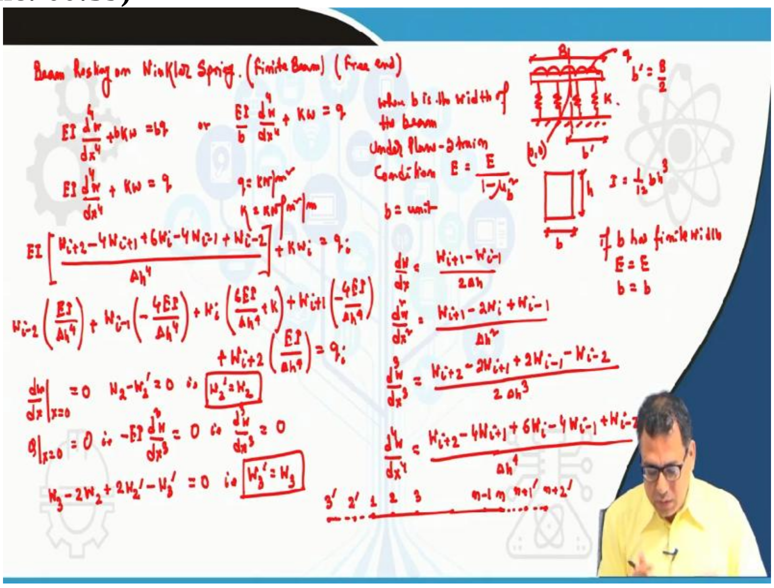

Now let us start with the application of finite difference scheme to a finite beam resting on Winkler springs. This method can also be used for the infinite beam. Consider a finite beam of width B, with free ends. The beam is subjected to UDL and is resting on Winkler springs.

The basic governing differential equation is:

$$EI\frac{d^4w}{dx^4} + bkw = bq \qquad \Rightarrow \frac{EI}{b}\frac{d^4w}{dx^4} + kw = q$$

Under plane strain conditions E should be replaced with $\frac{E}{\left(1-\mu_b^{\,2}\right)}$ and only unit width should be considered (i.e. b = 1). So, under plane strain condition the above equation can be written as:

$$\Rightarrow EI\frac{d^4w}{dx^4} + kw = q$$

where, q is in kN/m^2 and k is in kN/m^2/m.

For central difference scheme:

$$\frac{dw}{dx}=\frac{w_{i+1}-w_{i-1}}{2\Delta h}$$

$$\frac{d^2w}{dx^2}=\frac{w_{i+1}-2w_i+w_{i-1}}{\Delta h^2}$$

$$\frac{d^3w}{dx^3}=\frac{w_{i+2}-2w_{i+1}+2w_{i-1}-w_{i-2}}{2\Delta h^3}$$

$$\frac{d^4w}{dx^4}=\frac{w_{i+2}-4w_{i+1}+6w_i-4w_{i-1}+w_{i-2}}{\Delta h^4}$$

These are the 4 finite difference forms that will be used here. Using the fourth order derivative value in finite difference form in the differential equation:

$$EI\left[\frac{w_{i+2}-4w_{i+1}+6w_i-4w_{i-1}+w_{i-2}}{\Delta h^4}\right]+kw_i=q_i$$

$$\Rightarrow w_{i-2}\left(\frac{EI}{\Delta h^4}\right)+w_{i-1}\left(-\frac{4EI}{\Delta h^4}\right)+w_i\left(\frac{6EI}{\Delta h^4}+k\right)+w_{i+1}\left(-\frac{4EI}{\Delta h^4}\right)+w_{i+2}\left(\frac{EI}{\Delta h^4}\right)=q_i$$

For the analysis, the half portion of the beam will be considered as it is the same for the other half too. Considering that the b′ region of beam is divided into n segments, two imaginary nodes on both sides of this region are required to obtain the solution. The nodes 2′ & 3′ are considered towards the side of node-1 and similarly nodes (n+1)′ & (n+2)′ towards the side of node-n.

Now, four boundary conditions are needed to proceed with the solution as there are four imaginary nodes (additional nodes).

The first boundary condition is that the slope at x = 0 will be as the load is symmetric:

$$\text{At x = 0: } \left.\frac{dw}{dx}\right|_{x=0}=0$$

Using this boundary condition, we can get: w_2 - w_2' = 0 and so, w_2 = w_2'.

$$\text{At x = 0: } -EI\left.\frac{d^3w}{dx^3}\right|_{x=0}=0 \qquad \Rightarrow \left.\frac{d^3w}{dx^3}\right|_{x=0}=0$$

The finite difference form of this condition is:

$$w_3-2w_2+2w_2'-w_2'=0 \qquad \Rightarrow w_3'=w_3$$

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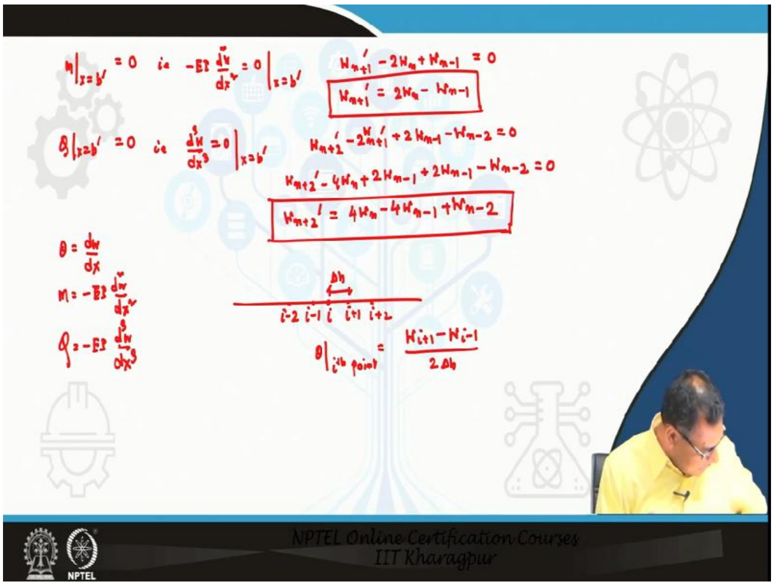

Now, 2 more boundary conditions are needed and the third condition is that the bending moment at x = b′ i.e., at the edge is zero.

$$\text{At x = 0:} \; -EI\left.\frac{d^3w}{dx^3}\right|_{x=b'} = 0 \qquad \Rightarrow \left.\frac{d^3w}{dx^3}\right|_{x=b'} = 0$$

The finite difference form of this condition is:

$$w'_{n+1} - 2w_n + w_{n-1} = 0 \quad \Rightarrow w'_{n+1} = 2w_n - w_{n-1}$$

The next and fourth boundary condition is also at the edge of the beam, but now the shear force at the edge will be considered. As the edge is free, both bending moment and shear force will be zero at x = b′.

$$\text{At x = 0:} \; -EI\left.\frac{d^2w}{dx^2}\right|_{x=b'} = 0 \qquad \Rightarrow \left.\frac{d^2w}{dx^2}\right|_{x=b'} = 0$$

The finite difference form of this condition is:

$$w'_{n+2} - 2w'_{n+1} + 2w_{n-1} - w_{n-2} = 0$$

$$\Rightarrow w'_{n+2} - 4w_n + 2w_{n-1} + 2w_{n-1} - w_{n-2} = 0$$

$$\Rightarrow w'_{n+2} = 4w_n - 4w_{n-1} + w_{n-2}$$

Now, relations are established within the four imaginary unknowns and real unknowns. So, the solution can be obtained and deflections can be calculated. If the deflections at each point are known, all other quantities can also be calculated form that. As all the quantities are developed in terms of deflections in the finite difference scheme, it is very easy to calculate all the quantities. Remember that whenever an imaginary node comes into picture, it should be replaced with the real node (s). Here the solution given is similar to that of the linear case already discussed. But, non linearity can also be introduced in this problem.

Now let us what happens if this beam is resting on a two parameter soil medium.

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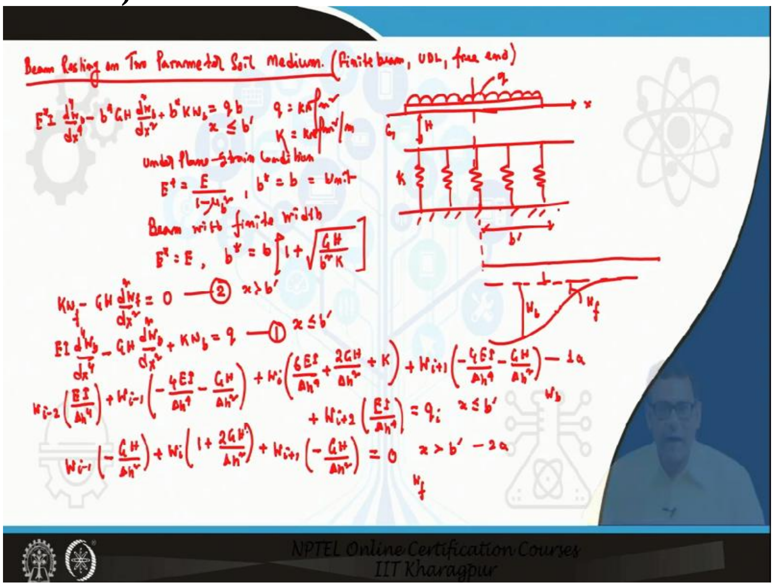

Consider a finite beam resting on 2 parameter soil medium. The beam is subjected to a UDL, q and the end condition at both the ends is free. This condition is also symmetric and hence the analysis can be done only for half the portion, b′. The basic differential equation is:

For x ≤ b′: $$E^*I\frac{d^4w_b}{dx^4} - b^*GH\frac{d^2w_b}{dx^2} + b^*kw_b = qb$$

where, q is in kN/m^2 and k is in kN/m^2/m

Under Plane-Strain condition: $E^* = \dfrac{E}{1-\mu_b^2}$ and b* = b = unit

Beam with finite width: E* =E and $b^* = b\left[1+\sqrt{\dfrac{GH}{b^2k}}\right]$

Now, the solution will be derived for the finite beam under plane strain condition and hence:

For x ≤ b′: $$EI\frac{d^4w_b}{dx^4} - GH\frac{d^2w_b}{dx^2} + kw_b = q \rightarrow (1)$$

The differential equation for the region beyond the beam, x > b′ is:

For x > b′: $$kw_f - GH\frac{d^2w_f}{dx^2} = 0 \rightarrow (2)$$

The finite difference form of the first equation or for the region within the beam is (x ≤ b′):

$$w_{i-2}\left(\frac{EI}{\Delta h^4}\right) + w_{i-1}\left(-\frac{4EI}{\Delta h^4} - \frac{GH}{\Delta h^2}\right) + w_i\left(\frac{6EI}{\Delta h^4} + \frac{2GH}{\Delta h^2} + k\right) + w_{i+1}\left(-\frac{4EI}{\Delta h^4} - \frac{GH}{\Delta h^2}\right) + w_{i+2}\left(\frac{EI}{\Delta h^4}\right) = q_i \rightarrow (1a)$$

The finite difference form of the second equation or for the region beyond the beam (x > b′):

$$w_{i\text{-}1}\left(-\frac{GH}{\Delta h^2}\right)+w_i\left(1+\frac{2GH}{\Delta h^2}\right)+w_{i+1}\left(-\frac{GH}{\Delta h^2}\right)=0\rightarrow(2a)$$

Note that the deflections in both the equations (1) & (1a) considering the region within the beam are denoted by w_b representing deflection of beam and in the equations (2) & (2a) it is w_f representing the deflection of foundation soil.

In the next class I will apply the boundary conditions again and then I will show how to solve the set of equations. Then I will discuss the procedure solving them to get the deflection and other quantities. Thank you.