Soil Structure Interaction

Prof. Kousik Deb
Department of Civil Engineering
Indian Institute of Technology Kharagpur

Lecture – 46
Use of Finite Difference Method for Soil Structure Interaction Problems (Contd.)

In my previous lecture, I have shown you how to use the finite difference scheme in a differential equation. Then I started to solve one problem where the basic differential equation is applied to a two parameter model. I applied two boundary conditions to solve the equation**.**

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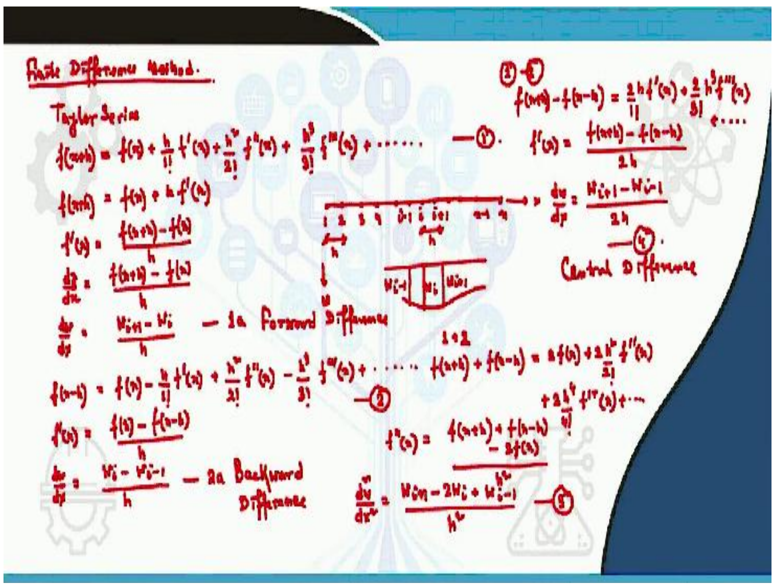

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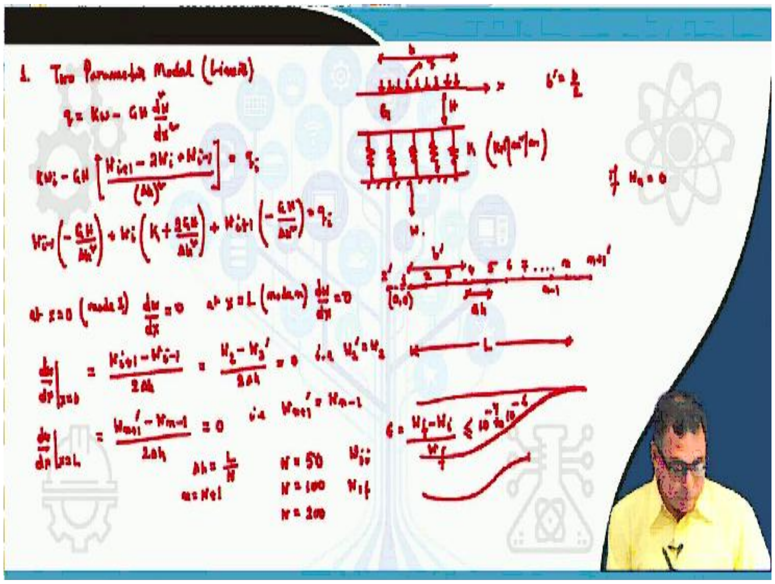

Then this relationship was obtained by considering two imaginary nodes. These imaginary node deflections were converted to real node deflections. Now this equation will be applied in all the nodes points.

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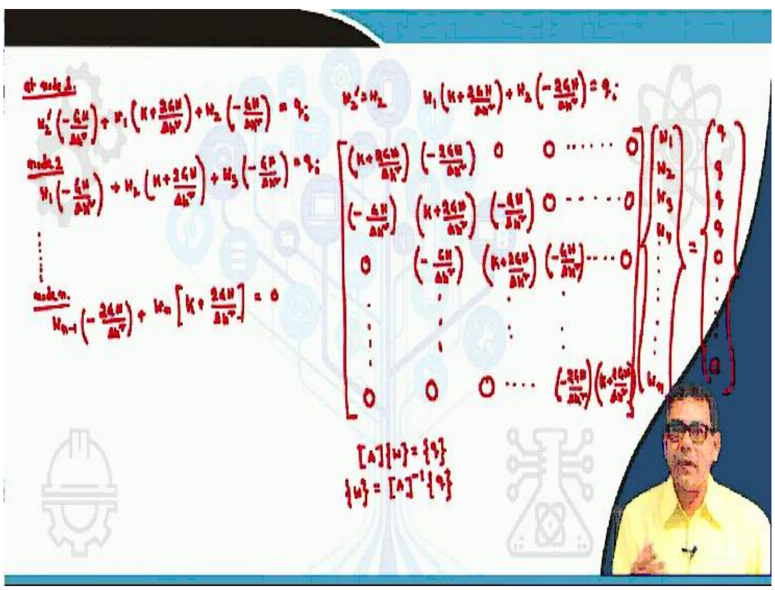

When the equation is applied to node-1:

$$w_2'\left(-\frac{GH}{\Delta h^2}\right)+w_1\left(k+\frac{2GH}{\Delta h^2}\right)+w_2\left(-\frac{GH}{\Delta h^2}\right)=q_i$$

$$\text{As } w_2' = w_2$$

$$w_1\left(k+\frac{2GH}{\Delta h^2}\right)+w_2\left(-\frac{2GH}{\Delta h^2}\right)=q_i$$

Similarly, when the equation is applied to node-2:

$$w_1\left(-\frac{GH}{\Delta h^2}\right)+w_2\left(k+\frac{2GH}{\Delta h^2}\right)+w_3\left(-\frac{GH}{\Delta h^2}\right)=q_i$$

The 1st and nth nodes are the extremes and hence will have the same form of equation. All the other nodes will have similar form which is shown in the node-2 equation above. So, now if the equation is applied to node-n:

$$w_{n-1}\left(k+\frac{2GH}{\Delta h^2}\right)+w_n\left(-\frac{2GH}{\Delta h^2}\right)=0$$

In the current problem, the load is assumed to be applied only till node-4 and so beyond the node-4, there will be no load and hence the right hand side of the above equation is 0.

If all the n equations are written in the matrix form:

$$\begin{bmatrix} \left(k+\frac{2GH}{\Delta h^2}\right) & \left(-\frac{2GH}{\Delta h^2}\right) & 0 & 0 & \cdots & 0 & 0 \\ \left(-\frac{GH}{\Delta h^2}\right) & \left(k+\frac{2GH}{\Delta h^2}\right) & \left(-\frac{GH}{\Delta h^2}\right) & 0 & \cdots & 0 & 0 \\ 0 & \left(-\frac{GH}{\Delta h^2}\right) & \left(k+\frac{2GH}{\Delta h^2}\right) & \left(-\frac{GH}{\Delta h^2}\right) & \cdots & 0 & 0 \\ . & . & . & . & & . & . \\ . & . & . & . & & . & . \\ . & . & . & . & & . & . \\ . & . & . & . & & . & . \\ 0 & 0 & 0 & 0 & \cdots & \left(-\frac{2GH}{\Delta h^2}\right) & \left(k+\frac{2GH}{\Delta h^2}\right) \end{bmatrix} \begin{Bmatrix} w_1 \\ w_2 \\ w_3 \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ w_n \end{Bmatrix} = \begin{Bmatrix} q \\ q \\ q \\ q \\ 0 \\ 0 \\ 0 \\ . \\ . \\ . \\ . \\ 0 \end{Bmatrix}$$

This matrix is called the stiffness matrix. If the coefficients are multiplied with the deflection vector {w_1, w_2, w_3, …. w_n}, the result will be the load vector {q, q, q, q, 0, 0, ….. 0}.

$$[A]\{w\}=\{q\}$$

$$\{w\}=[A]^{-1}\{q\}$$

This can be solved using the software, MATLAB or by writing a code specifically for the equation. The further process will not be detailed here. So, the procedure to form these finite difference equations (set of n number of equations) is given. Then this type of matrix can be formed and this matrix should be solved to get the deflection, w as it is unknown. From the solution, the w value will be determined at all the n points and if those values are plotted against x value, it makes the deflection profile.

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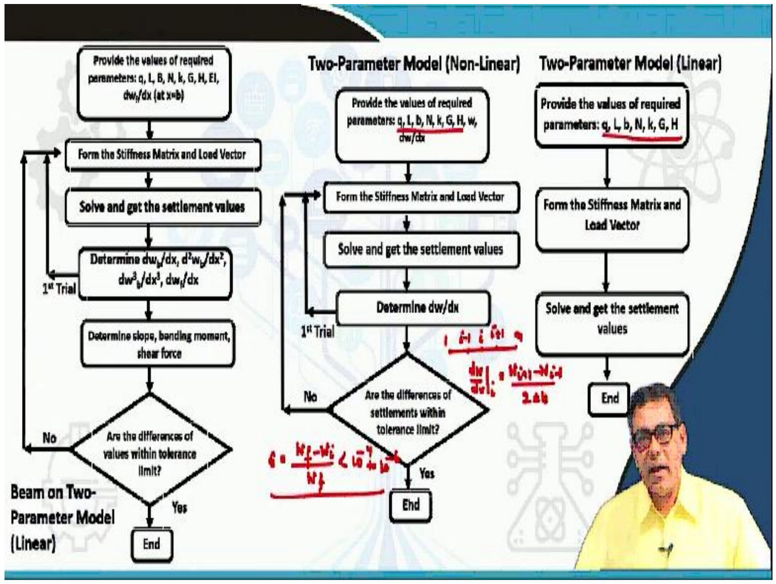

The above slide shows the flow charts for the procedure to be followed to obtain the solution in case of a two parameter linear and also non linear model. Studying the flow chart, the first step is to provide all the required values like the load, length of the zone (l), width of the loaded region (b), number of elements or zones the loaded region is divided into (n). Note that if n is the number of zones then the number of nodes will be (n + 1).

Along with these, the modulus of subgrade reaction (k), the shear modulus (G), the thickness of the shear layer (H) should also be given as input. Then the stiffness matrix and load vector should be formed. The load vector will have a value of some load, q_i until the loaded region and beyond that it will be zero. After forming the load vector and stiffness matrix they should be solved to get the settlement value. This is the flow chart for linear model.

Now, how to decide the Δh value? The possible best possible way to decide this is to take sufficiently large n value. For example, for the first case, divide the region with say, 50 nodes. By dividing with 50 nodes, the solution should be determined and the deflection value should be calculated. Then the number of nodes should be increased to say, 100. If the length of the region is L and the number of segments is n, the Δh value will be L/n. So, the higher the n value is the lesser the Δh value will be.

After calculating for two cases, the rate of change observed in the deflection should be calculated with the formula:

$$\varepsilon = \frac{w_f - w_i}{w_f} \leq 10^{-4} \text{ to } 10^{-6}$$

where, w_f is the deflection found out from the latest trial and w_i is the deflection determined from just the previous trial. If the above value is greater than the given range, the n value should be increased further and checked again. After making few trials, say that the difference was found to be in the acceptable limit when checked for the n values of 1000 and 1200. So, an n value of 1000 can be adopted as it is not useful to increase further. This process is called the mess convergence study.

It is also important to decide the L value because if it is taken very higher than the required value, it will consume a lot of time in computation as the number of segments (n) will increase. On the other hand, if it is taken lesser than the required, the deflections may not become zero within the model itself which may tamper with the results. So to avoid this, initially the L value should be taken sufficiently large and keep on reducing this while calculating the deflections. Like the previous case, when the change is within the acceptable limit, that L value should be considered for the model. This process can be done in the reverse order by starting with a very small L value. By increasing the L value, the deflections should be calculated. This difference will be very high for the initial trials and tend to converge after some time. When the difference falls within the acceptable range, that L value can be considered.

Most of the cases, it is enough to check the tolerance limit for center node but it is recommended to check with all the nodes. Another way to decide the L value and the Δh value is that first the Δh value should be kept constant and for that value, the L value can be kept on increasing to check for the convergence. While following this procedure, the convergence will be checked for the centre node alone.

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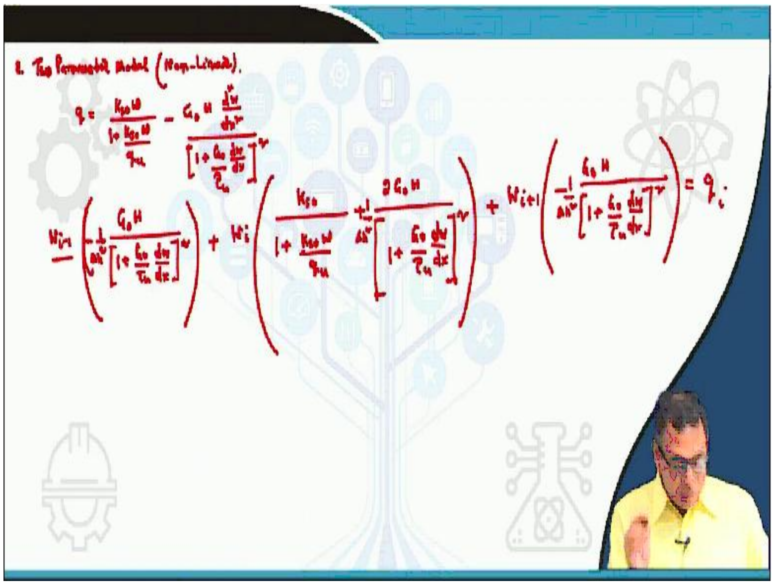

Now, a problem will be solved for the nonlinear behavior in two parameter model. So in the nonlinear case the equation is:

$$q = \frac{k_{so} w}{1 + \dfrac{k_{so} w}{q_u}} - \frac{G_o H \dfrac{d^2 w}{dx^2}}{\left[1 + \dfrac{G_o}{\tau_u} \dfrac{dw}{dx}\right]^2}$$

If the finite difference scheme is applied to this equation:

$$w_{i-1}\left\{-\frac{1}{\Delta h^2}\frac{G_o H \dfrac{d^2 w}{dx^2}}{\left[1+\dfrac{G_o}{\tau_u}\dfrac{dw}{dx}\right]^2}\right\} + w_i\left\{\frac{k_{so} w}{1+\dfrac{k_{so} w}{q_u}} - \frac{1}{\Delta h^2}\frac{G_o H \dfrac{d^2 w}{dx^2}}{\left[1+\dfrac{G_o}{\tau_u}\dfrac{dw}{dx}\right]^2}\right\} + w_{i+1}\left\{-\frac{1}{\Delta h^2}\frac{G_o H \dfrac{d^2 w}{dx^2}}{\left[1+\dfrac{G_o}{\tau_u}\dfrac{dw}{dx}\right]^2}\right\} = q_i$$

For this equation the boundary condition should be applied as discussed in the previous case. Then n number of equation sets will be obtained that are to be solved. But the problem here is that in the coefficient of w_{i-1} or w_i, there is another w term. In the equation, the values of G, H, k, q_u, τ_u, Δh and G_o are known. Some values, though are not known initially can be determined. So, the only unknowns in the above expression are the deflection related terms.

To determine these coefficients, iterative techniques should be followed. For the first iteration, the w and dw/dx values should be assigned to all the nodes with some arbitrary values. This is why even in the flow chart for two parameter model with non linear behavior, the inputs include w and dw/dx. So initially the input given may not be correct. But, with those values, the solution should be determined and with that, a stiffness matrix can be formed. From the stiffness matrix and the load vector, the displacement matrix can be determined. This is nothing but the displacement value at each node. In the linear behavior case, these are the final deflection values, but here it is not the case because there are arbitrary values (of w & dw/dx) in the input.

Now, there are deflection values at each node and so the dw/dx values at each node can also be found out. Now these w and dw/dx values should be used as the input for the next trial and the entire procedure should be repeated. After each trial, the difference between the settlement values should be checked and if it is within the tolerance limit (10^{-4} to 10^{-6}), the iteration can be stopped and those values can be considered as the final values or the solution. But, if the difference is not within the tolerance limit, this entire procedure should be repeated again and again till the difference between the consecutive results falls within the permissible limit.

This is the procedure to be followed to solve the nonlinear problem for any two parameter model. In the next class I will show more uses of these finite difference techniques for other problems like beam resting on Winkler model or two parameter model and then finally for plate resting on Winkler model. Thank you.