Soil Structure Interaction

Prof. Kousik Deb
Department of Civil Engineering
Indian Institute of Technology - Kharagpur

Lecture 40
beams on Elastic Foundation (Contd.,)

In the last class I was discussing about a finite beam subjected to UDL and resting on two parameters of soil medium. The procedure to deflections beyond the beam region as well as within the beam region was discussed.

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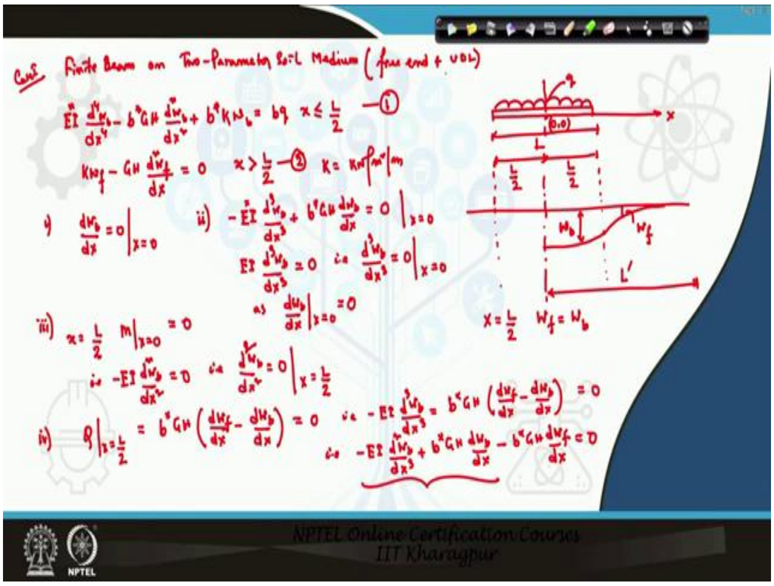

The above condition was discussed as the first case where the finite beam had both its ends free and was subjected to UDL. The previous cases considered gave only deflection profile within the beam region. All the boundary conditions required to solve both the differential equations (within the beam and beyond the beam) were evaluated.

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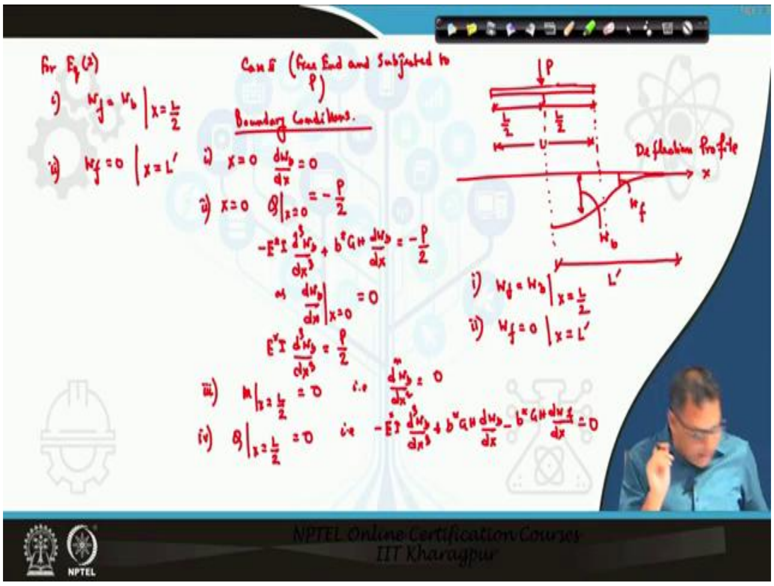

The second case in this discussion is where a finite beam is subjected to a concentrated load, P. The length of the beam is considered as L and the point load acts at the midpoint of the beam. So, distance from the point load to either of the ends is L/2. Both the ends of the beam are free. L′ is the length at which the deflection becomes zero.

The first boundary condition is that the slope at the centre (x = 0) because the load is a point load:

$$1.)\ \frac{\mathrm{d}w_b}{\mathrm{d}x} = 0\bigg|_{x=0}$$

The second boundary condition is that the shear force will be –P/2 at the centre (x = 0):

$$Q\big|_{x=0} = -\frac{P}{2}$$

$$\Rightarrow \text{-EI}\frac{\mathrm{d}^3 w_b}{dx^3} + b^* GH\frac{dw_b}{dx} = -\frac{P}{2}\bigg|_{x=0}$$

As the slope at the centre is 0 (1st boundary condition), the above expression reduces to:

$$2.)\ \mathrm{E}^*\mathrm{I}\frac{\mathrm{d}^3 w_b}{dx^3} = \frac{P}{2}\bigg|_{x=0}$$

The third boundary condition is that the bending moment will be 0 at the edge where, x = L/2.

$$M = 0\big|_{x=\frac{L}{2}} \Rightarrow \text{- EI}\frac{\mathrm{d}^2 w_b}{dx^2} = 0\bigg|_{x=\frac{L}{2}} \quad \Rightarrow 3.)\ \frac{\mathrm{d}^2 w_b}{dx^2} = 0\bigg|_{x=\frac{L}{2}}$$

The fourth condition is that the shear force is also 0 at x = L/2 (free end):

$$Q\big|_{x=\frac{L}{2}} = 0$$

$$\Rightarrow 4.)\ -\mathrm{E}^*\mathrm{I}\frac{\mathrm{d}^3 w_b}{dx^3} + b^* GH\frac{dw_b}{dx} - b^* GH\frac{dw_f}{dx} = 0$$

The two differential equations for which the solution should be evaluated are the same as that of given in the last lecture.

$$\text{For } x \le \frac{\mathrm{L}}{2} : E^* I\frac{d^4 w_b}{dx^4} - b^* GH\frac{d^2 w_b}{dx^2} + b^* k w_b = bq \rightarrow (1)$$

$$\text{For } x > \frac{\mathrm{L}}{2} : k w_f - GH\frac{d^2 w_f}{dx^2} = 0 \rightarrow (2)$$

These are the four boundary conditions that can be used to solve the equation-(1). Two more boundary conditions are required to solve the second equation.

For the second case (equation-2), the first boundary condition is:

$$w_f = w_b\big|_{x=\frac{L}{2}}$$

By solving the first equation, the w_b value can be determined and that value can be used in the above boundary condition.

If the region beyond the beam is considered sufficiently long then there will be a point in the soil where deflection will be zero ($w_f = 0$). Let us call that length as L′ at which the w_f becomes 0. The second boundary condition to solve equation-(2):

$$w_f = 0\big|_{x=L'}$$

These are the boundary conditions to be applied to solve the basic differential equation and determine the solutions.

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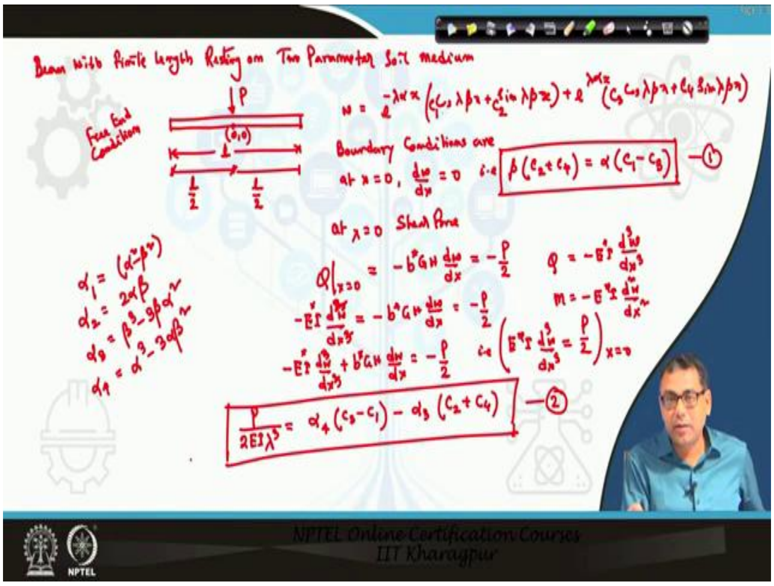

The equation for deflection, w has four unknowns and the four boundary conditions can be used to solve for these four unknowns. The equation-(2) can be solved using the other two (last two) boundary conditions.

Here, the procedure to determine the closed form solution is explained where equations will be determined to calculate the deflection and other quantities. But in the later cases, these equations will be solved numerically or using numerical methods. For this purpose, either finite element method or the finite difference method can be used. In this course, the finite difference method will be adopted. The procedure to use these boundary conditions to solve one particular case or one particular problem of beam or plate numerically will be discussed.

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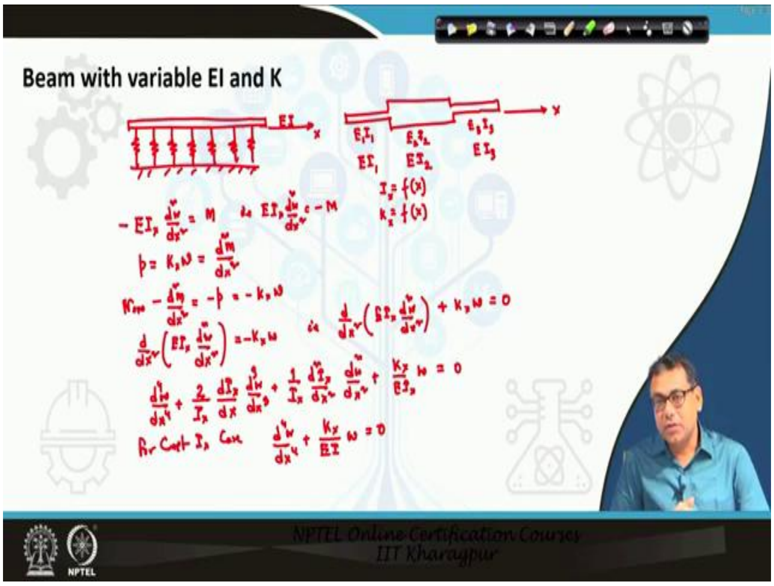

The next concept is about the beam with variable EI and k. Till now all the problems solved are where the flexural rigidity of the beam and the k value are constant. But there may be some possibility that the beam may have different EI values at different locations and also all the springs may not have the same k value.

So in that case as mentioned earlier, if different k values are used for different springs, non uniform settlement can be obtained in the Winkler's model also. One of the limitations of the Winkler's model was that for the flexible or rigid footing, always the settlement will be uniform along the entire loading region. But that limitation can be overcome by considering different k values for different springs.

Suppose there is a beam which has a uniform elastic modulus throughout the beam, but varying cross section. If the cross section varies, the I value varies and I value can be considered as a function of x. If the k value is also varying, it can also be considered as a function of x. That means the k is also varying along with the x.

Considering both I and k are varying with x, let us see how to analyse such a condition. The basic expression for bending moment can be given below.

$$EI_x \frac{d^2w}{dx^2} = -M$$

When a beam is resting on springs, the reaction force from the springs is:

$$p = k_x w$$

Both I and k values are denoted with suffix x to indicate that they are functions of x.
Now, if the moment is differentiated twice with respect to x, then the reaction can be determined. So the above expression can be equated to:

$$p = k_x w = \frac{d^2M}{dx^2}$$

$$\Rightarrow -\frac{d^2M}{dx^2} = -k_x w = -p$$

Substituting the M value from the basic equation, we get:

$$\frac{d^2}{dx^2}\left(EI_x \frac{d^2w}{dx^2} \right) = -k_s w$$

$$\Rightarrow \frac{d^2}{dx^2}\left(EI_x \frac{d^2 w}{dx^2}\right) + k_s w = 0$$

$$\Rightarrow \frac{d^4 w}{dx^4} + \frac{2}{I_x}\frac{d^2 I_x}{dx^2}\frac{d^2 w}{dx^2} + \frac{k_x}{EI_x} w = 0$$

This expression is obtained after differentiating the moment twice and simplifying. But, if the I is constant, but k is changing:

$$\Rightarrow \frac{d^4 w}{dx^4} + \frac{k_x}{EI} w = 0$$

These equations have to be solved now to determine the required quantities. This type of equations can be solved numerically. Though it is always preferable to have a closed form solution, it is sometimes very difficult and complicated to get a closed form solution. In such case we can go for the numerical solution methods. The numerical methods, how to solve or how to determine the deflection and the other quantities for this type of problems will be discussed.

In the next few classes I will discuss about the plates resting on elastic foundation because till now I have discussed only about the beams resting on elastic Foundation. There also I will derive the basic differential equation for plate (rectangular and circular). Then I will discuss about the boundary conditions under different loading and end condition.

Then I will show you how to numerically solve these soil-structure interaction problems. I have given the closed form solutions for the infinite and finite beam resting on Winkler springs and two parameter model. Also for the finite beam resting on elastic springs, but if the springs upon which the beam is resting on are nonlinear, then getting the closed form solution is very difficult. So I will solve those cases numerically.

I will show you how to solve these types of problems numerically. Even the problems for which I have given the closed form solutions, I will show how to use the finite difference method to solve soil-structure interaction problem. Thank you.