

**Soil Structure Interaction**  
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**Lecture - 4**  
**Bearing Capacity of Soil (Continued)**

In the last class, I have discussed about the various bearing capacity equations. Today I will solve two example problems to show how to determine the bearing capacity using those equations.

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Ex.1: A rectangular footing of size 3m X 6m is founded at a depth of 1m in a homogeneous sandy soil. The water table is at a great depth. The unit wt of soil 18 kN/m<sup>3</sup>. Determine net ultimate bearing capacity  $c = 0$  and  $\phi = 40^\circ$

Using Terzaghi's theory

$$q_{nm} = q_u - \gamma D_f = \gamma D_f (N_q - 1) + \frac{1}{2} \gamma B N_\gamma \left( 1 - 0.2 \frac{B}{L} \right)$$

From table  $N_q = 81.3$ ,  $N_\gamma = 100.4$  for  $\phi = 40^\circ$

$B = 3\text{m}$  and  $L = 6\text{m}$

$$q_{nm} = 18 \times 1 \times (81.3 - 1) + \frac{1}{2} \times 18 \times 3 \times 100.4 \times \left( 1 - 0.2 \times \frac{3}{6} \right) = 3885.12 \text{ kN/m}^2$$

In the first problem, a rectangular footing of size 3 meter  $\times$  6 meter is founded at a depth of 1 meter in a homogeneous sandy soil. The water table is at a great depth. The unit weight of soil is 18 kilo Newton per meter cube. Determine the net ultimate bearing capacity when  $c = 0$  and  $\phi = 40^\circ$ . Till now, I have discussed four theories: Terzaghi bearing capacity expression, Skempton bearing capacity expression, Meyerhof's bearing capacity expression and the IS method.

Out of these four, the second one i.e., Skempton's bearing capacity expression cannot be used because it is applicable only for clayey soil. So, other three methods can be used here as they are applicable for any type of soil. So, I will discuss how to determine the bearing capacity based on those three methods. First using Terzaghi's theory, the net ultimate bearing capacity is ultimate bearing capacity minus  $\gamma D_f$  and as it is a rectangular footing, corrections have to be applied.

In the bearing capacity equation the third term will be 0 because  $c = 0$ . So, there will be only the  $N_q$  term and the  $N_\gamma$  term. The second term will have  $(N_q - 1)$  because we are calculating net bearing capacity. Now, from the table of Terzaghi's bearing capacity equation or the table proposed by Terzaghi, the  $N_q$  value is 81.3 for  $\phi = 40^\circ$  and  $N_\gamma$  is 100.4 for  $\phi = 40^\circ$ . When  $\phi$  is greater than or equal to  $36^\circ$ , general shear failure should be considered.

As there is no local shear failure, no modifications are required and also, it is not in between general shear and local shear, so no interpolation is also required. So, here the general shear failure bearing capacity factor can be directly used. Now, if these values are substituted, the answer will be  $3885.12 \text{ kN/m}^2$ . This is according to Terzaghi's theory.

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Using Meyerhof's theory

$$q_{mi} = q_{ult} - \gamma D_f = \gamma D_f N_q s_q d_q + 0.5 \gamma B N_\gamma s_\gamma d_\gamma - \gamma D_f$$

$$s_q = s_\gamma = 1 + 0.1 \tan^2(45^\circ + \frac{\phi}{2}) \left( \frac{B}{L} \right) = 1.23 \quad d_q = d_\gamma = 1 + 0.1 \tan^2(45^\circ + \frac{\phi}{2}) \left( \frac{D_f}{B} \right) = 1.07$$

From table  $N_q = 64.1$ ,  $N_\gamma = 93.7$  for  $\phi = 40^\circ$

$$q_{mi} = 18 \times 1 \times 64.1 \times 1.23 \times 1.07 + 0.5 \times 18 \times 3 \times 93.7 \times 1.23 \times 1.07 - 18 \times 1 = 4830.11 \text{ kN/m}^2$$

Similarly bearing capacity can also be calculated using Meyerhof's theory which also uses the same equation along with the shape, depth and inclination factors in addition to the previous one. The  $N_q$  term should be multiplied by  $s_q$ ,  $d_q$  and  $i_q$  generally, but here, as the load is not inclined,  $i_q$  will be 0. Similarly  $i_\gamma$  is also equal to zero. Here also the first term will be zero because  $c = 0$ . If loading is inclined,  $i_q$  and  $i_\gamma$  should also be considered but the shape factor and depth factor should be considered because it is a rectangular footing and is at a depth from ground level and there is a  $\gamma D_f$ . So, from the table we will get  $s_q = s_\gamma$  as the equation is same for both. If I put  $\phi$ , I will get 1.23. Similarly, the depth factors,  $d_q$  and  $d_\gamma$  will be 1.07. Now from the Meyerhof bearing capacity factor table, I will get  $N_q$  is equal to 64.1,  $N_\gamma$  equal to 93.7 corresponding to a  $\phi$  value of  $40^\circ$ .

So if I put there, I will get a bearing capacity, which is 4830.11 kN/m<sup>2</sup> (slightly higher as compared to Terzaghi's bearing capacity expression).

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Using IS Code Method

$$q_{nu} = \gamma D_f (N_q - 1) s_q d_q + 0.5 \gamma B N_\gamma s_\gamma d_\gamma$$

$$s_q = 1 + 0.2 \left( \frac{B}{L} \right) = 1.10 \quad s_\gamma = \left( 1 - 0.4 \frac{B}{L} \right) = 0.8$$

$$d_q = 1 + 0.1 \left( \frac{D_f}{B} \right) \tan \left( 45 + \frac{\phi}{2} \right) = 1.07 \quad d_\gamma = 1.07 \quad N_q = 64.1, N_\gamma = 109.4 \text{ for } \phi = 40^\circ$$

$$q_{nu} = 18 \times 1 \times (64.1 - 1) \times 1.10 \times 1.07 + 0.5 \times 18 \times 3 \times 109.4 \times 0.8 \times 1.07 = 3865.29 \text{ kN/m}^2$$

Now I will use the IS code method. The IS code method this expression is given in this form as I discussed and here also c is equal to 0, so the first term will be zero

$$q_u = \gamma D_f (N_q - 1) s_q d_q + \frac{1}{2} B N_\gamma s_\gamma d_\gamma$$

As done earlier,  $i_q$  and  $i_\gamma$  are not being considered as loading is perfectly vertical. The water table factor,  $w'$  is also not being considered because no water table effect is mentioned. So, that means water table is far below the base of foundation, so water table effect is not considered, in that case  $W'$  will be 1, which is why it is not considered.

Other factors can be read from the IS code table:  $s_q = 1.10$ ,  $s_\gamma = 0.8$  and  $d_q = 1.07 = d_\gamma$  ( $d_q = d_\gamma$  for  $\phi > 10^\circ$ ). As I mentioned earlier,  $N_q$  is same as the Meyerhof bearing factor, so  $N_q = 64.1$ , but  $N_\gamma = 109.4$  (different from Meyerhof theory), read from the table corresponding to a friction angle value,  $\phi = 40^\circ$ .

Substituting all the values, the bearing capacity will be equal to 3865 kN/m<sup>2</sup> which is similar to the Terzaghi's bearing capacity value. From the IS code method, Meyerhof's theory and Terzaghi's theory, the values of bearing capacity respectively are: 3865, 4830 and 3885. In this case, Meyerhof's theory gave the maximum value and then Terzaghi's theory, 3885 followed by the IS code method, 3865. IS code is giving slightly lower value compared to the Terzaghi's bearing capacity expression.

So, IS code is giving the lowest value, but this is only for this particular condition. There is no guarantee that this trend will follow for all the cases. If the  $c$ - $\phi$  values change, then this trend may also change. Now, if you want to determine the net safe bearing capacity, then divide this net ultimate bearing capacity by a factor of safety 2.5 or 3.

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Example:

$$q_u = c_u N_c + [\gamma' D_f + (\gamma - \gamma') D_w] N_q + \frac{1}{2} \gamma' B N_\gamma$$

Now, let us see how to incorporate water table effect in the bearing capacity equation proposed by Terzaghi. In this problem, the depth of foundation is given 1.5 meter.  $D_w$  is given 2.5 meter (let this be first case). So, the water table position is such that, it is 1 meter ( $=b$ ) below the footing base which is at a depth of 1.5 meter ( $=D_f$ ) from ground surface. In this case, what would be the bearing capacity as per Terzaghi's theory? To understand the water table effect properly, let us determine bearing capacity assuming water table at different locations in different cases. Let the first case considered be such that  $\bar{\gamma}$  will be equal to:

$$\bar{\gamma} = [\gamma' D_f + (\gamma - \gamma') D_w] N_q$$

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$\phi = 40^\circ, c = 0$   
 $N_c = 81.3, N_q = 100.4$   
 $\gamma_{sat} = 20 \text{ kN/m}^3, \gamma_w = 10 \text{ kN/m}^3, \gamma_{bulk} = 18 \text{ kN/m}^3$   
 $\bar{\gamma} = \gamma' + \frac{b}{B} (\gamma - \gamma') = 10 + \frac{1}{3} (18 - 10) = 12.67 \text{ kN/m}^3$   
 $\gamma' = \gamma_{sat} - \gamma_w = 20 - 10 = 10 \text{ kN/m}^3$   
 $\frac{1}{2} \times 3 \times 12.67 \times 100.4 = ?$  (shallow footing)

$q_u = c N_c + \gamma D_f N_q + \frac{1}{2} B \left[ \gamma' + \frac{b}{B} (\gamma - \gamma') \right] N_\gamma$

For the second case, let us consider a condition where  $D_w$  is 2.5 meter,  $D_f$  is 1.5 meter. So,  $b = 1$  meter,  $B = 3$  meter.  $\phi$  value is given as  $40^\circ$  and  $c$  value is 0. So, as per Terzaghi:  $N_c = 0$ ,  $N_q = 81.3$  from the table and  $N_\gamma$  will be 100.4 for  $\phi = 40^\circ$ . Here we are considering only Terzaghi's equation.

If I consider another equation (theory), the above procedure should be followed, but the shape factor, depth factor and bearing capacity factors will be different. So here we will get, our value is this term, first we will calculate this term.

$$q_u = \gamma D_f N_q + \frac{1}{2} B \left[ \gamma' + \frac{b}{B} (\gamma - \gamma') \right] N_\gamma$$

where,  $\bar{\gamma} = \left[ \gamma' + \frac{b}{B} (\gamma - \gamma') \right]$

The saturated unit weight,  $\gamma_{sat} = 20 \text{ kN/m}^3$ ; unit weight of water,  $\gamma_w = 10 \text{ kN/m}^3$  and the bulk unit weight,  $\gamma_{bulk}$  or sometimes written simply as  $\gamma = 18 \text{ kN/m}^3$ . The difference between  $\gamma_{sat}$  and  $\gamma_{bulk}$  is that  $\gamma_{sat}$  is the unit weight of the soil below water table and  $\gamma_{bulk}$  is the unit weight of the soil above water table.

So, the submerged unit weight,  $\gamma'$  is equal to  $(\gamma_{sat} - \gamma_w)$ . As,  $\gamma_{sat}$  is 20 -  $\gamma_w$  is 10,  $\gamma'$  will be 10  $\text{kN/m}^3$ . So, I can write:

$$\bar{\gamma} = \left[ 10 + \frac{1}{3} (18 - 10) \right]$$

because  $b$  is 1,  $B$  is 3,  $\gamma$  or  $\gamma_{bulk}$  is 18 and  $\gamma'$  is 10. So, this value will be  $12.67 \text{ kN/m}^3$ .

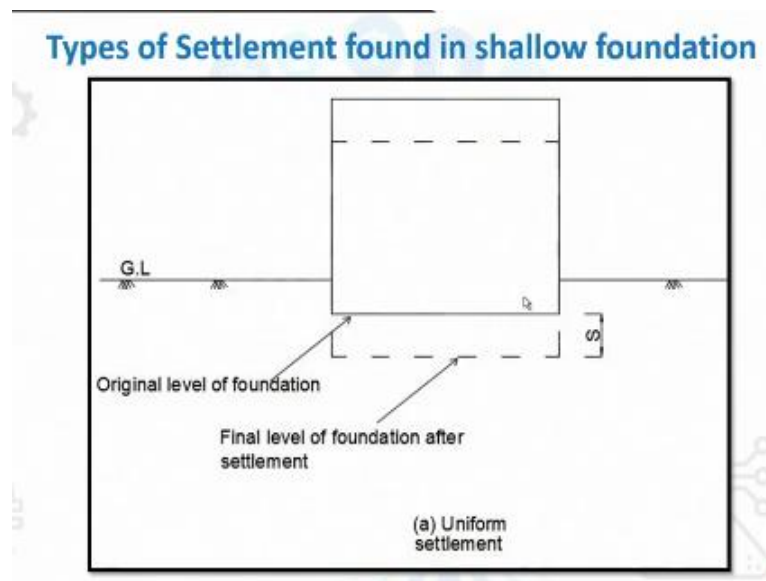
If I substitute these values, then  $q_{ultimate}$  will be:

$$q_u = 18 \times 1.5 \times 81.3 + \frac{1}{2} \times 3 \times 12.67 \times 100.4$$

As  $\gamma = \gamma_{\text{bulk}} = 18$  as this is above water table,  $D_f$  is 1.5;  $N_q$  is 81.3;  $B$  is 3;  $\bar{\gamma}$  is 12.67; and  $N_\gamma$  is 100.4. Remember that the footing considered here is a strip footing.

Similarly, if the position of the water table changes, the unit weight or the surcharge value should be modified accordingly and the load carrying capacity or bearing capacity of the foundation should be determined. Also, if the foundation is rectangular, correction factors should be applied to get the bearing capacity. So, this way you can determine the bearing capacity by incorporating the water table effect okay.

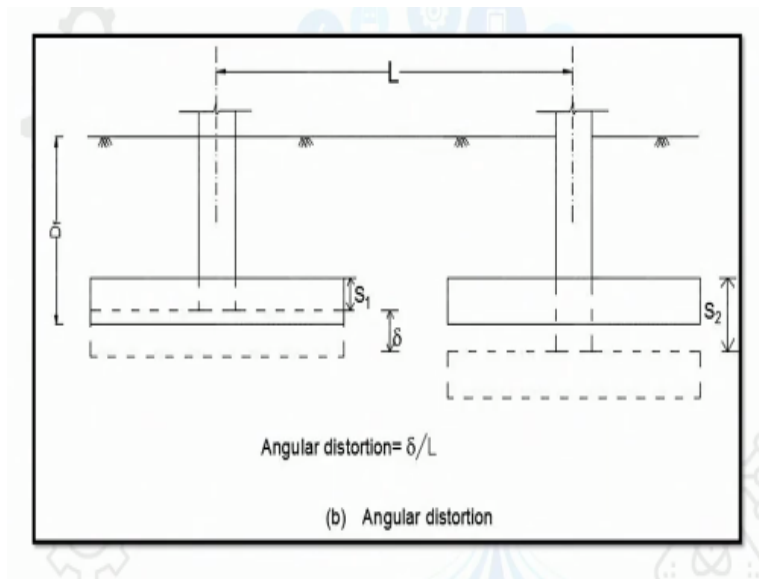
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The next concept that I will discuss is the settlement. Till now I have discussed about the bearing capacity part, but this is not the total design. Instead, it is only half portion of the design and the other half is the settlement criteria which should also be satisfied. So, now I will discuss the settlement criteria and then finally, I will design the foundation satisfying both bearing capacity as well as the settlement criteria.

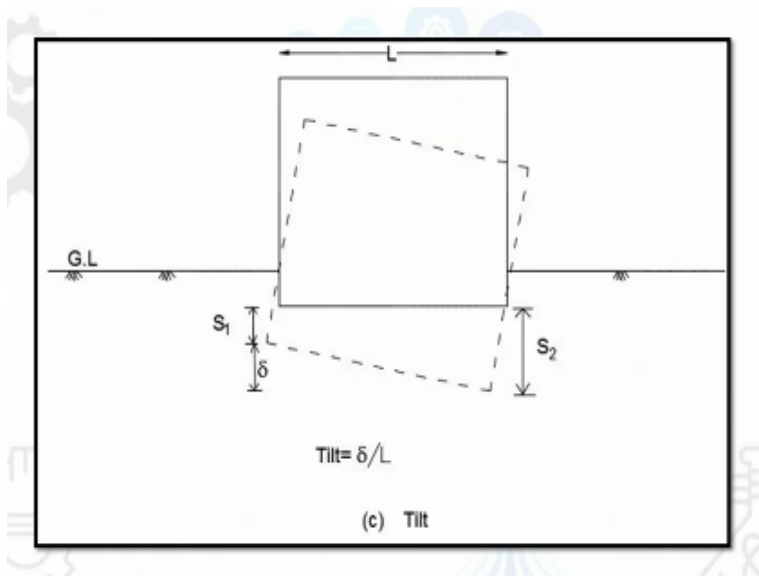
Settlement can be in different forms. So, let us investigate the different types of settlement found in a shallow foundation. Uniform settlement which means that the structure settles down uniformly. The foundation / structure uniformly sinks from the original level and the foundation level after settlement may be, say  $S$  units below the original position. So,  $S$  is the settlement of the structure and it is uniform. The uniform settlement is often limited depending upon in the type of foundation and soil stratum which is called the maximum settlement.

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Then the next one is the differential settlement. Differential settlement occurs when a structure has two foundations under two different columns and the settlement of these two foundations are not same. This may happen because of the difference in load on these two columns or the difference in soil properties below these two foundations. This means that there will not be uniform settlement in the building. So, at one place there will be less settlement and at another place there will be more settlement. So that is why there will be differential settlement in the structure and that is called angular distortion also.  $\delta$  is basically the differential settlement because  $S_1$  is the settlement of one foundation,  $S_2$  is the settlement of another foundation and  $\delta$  is the difference of these two, which is the differential settlement.

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Then the third type of settlement is tilt, which also happens because of the differential settlement. If there is a difference in settlements between two sides of a structure i.e.,

differential settlement, it may lead to tilting of the entire structure. The difference between tilt and differential settlement is that, in the later only one column or a small portion of the structure only settles differently or in other words, the differential settlement is restricted to a small part of a structure. But in tilt, the entire structure is settled in such a way that one particular corner / side settles more compared to the other corner / side throughout the structure, this differential settlement is uniform. So, that means the total structure tilts because of the differential settlement to one side.

These three cases should be checked during the design. But mainly the maximum settlement will be checked. It should be kept in mind that these two, angular distortion or differential settlement and tilt should also be checked for.

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**Settlement of shallow foundation**

Total Settlement  $S_t = S_i + S_c + S_s$

$S_i$  = Immediate or elastic settlement (<7 days). It takes place during the application of loading. In clays, the settlement is due to the change in the shape of the soil without a change in volume or water content. It is neglected as compared to long term settlement.

$S_c$  = Primary consolidation settlement. It is due to the consolidation.

$S_s$  = Secondary Compression Settlement. It occurs because of volume change occurring due to rearrangement of soil particles.



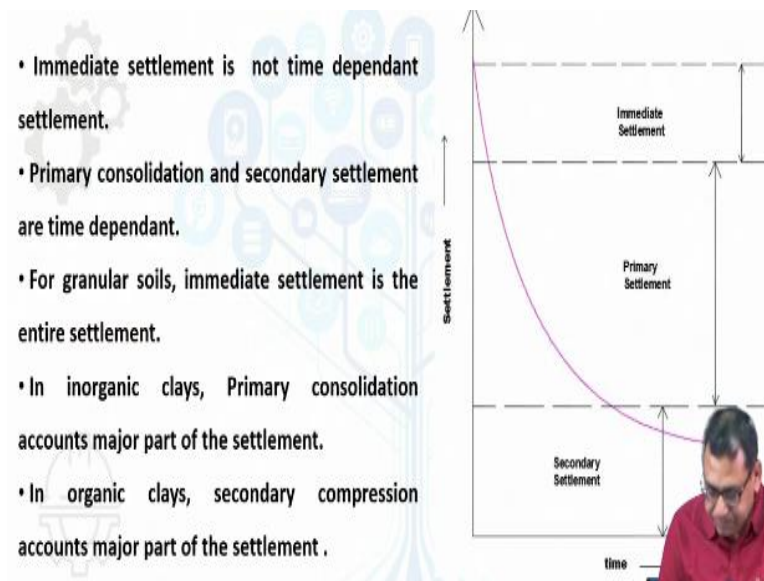
The settlement of a shallow foundation, i.e., the total or maximum settlement ( $S_t$ ) can be expressed as a summation of three types of settlement. So,  $S_t = S_i + S_c + S_s$  where  $S_i$  is the immediate or the elastic settlement that will generally occur within a very small duration of time. So, this takes place during the application of load.

In clay, the settlement is due to the change of shape of soil without a change in volume or water content. So,  $S_i$  is neglected in clay as it is very small compared to the long-term settlement. But, if the soil is sand, the immediate settlement is significant, and it contributes to the major portion of the settlement. The second one is  $S_c$  which is the primary consolidation settlement which occurs due to consolidation.



For a clayey soil, major part of the settlement is due to consolidation and not due to immediate settlement.  $S_s$  is the secondary compression settlement which occurs because of the volume change occurring due to the rearrangement of the soil particles. First one is the immediate, immediately which occurs immediately after the application of load, second one is due to the consolidation, and third one is due to the rearranging of the soil particle due to the application of the load or volume change. Now, the summation of all the three settlements will give the total settlement.

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From the graph, it is evident that the immediate settlement is not time dependent indicating it happens within no time, most of the times during the construction only. The primary consolidation and secondary consolidation are time dependent. Both time dependent and time independent settlements are significant only in some types of soil which we will study in-detail. For granular soil or sandy soil, immediate settlement is almost the entire settlement. For inorganic clay, primary consolidation settlement is the major part of the settlement and for organic clay, secondary compression is the major part of the settlement. This should be kept in mind, so that during the calculation of total settlement, the insignificant part according to the soil type can be neglected.

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## Settlement Calculation

Immediate Settlement (for clay)

$$S_i = qB \left( \frac{1 - \mu^2}{E} \right) I_f$$

Consolidation Settlement (for clay)

$$S_c = \sum \frac{C_c}{1 + e_0} H \log_{10} \left( \frac{p_0 + \Delta p}{p_0} \right)$$

$$\text{or } S_c = \sum m_v H_v \Delta p$$

Settlement (granular soil or sand) (all Immediate Settlement)

(a) Plate load test method (IS-1888-1982)

(b) Method based on SPT (IS 8009-Part 1-1976)

(c) Method based on SCPT

$$S = 2.3 \frac{H}{C} \log \left( \frac{\bar{\sigma}_v + \Delta \sigma}{\bar{\sigma}_v} \right)$$

De Beer and Martens (1957)

$$\text{where } C = 1.5 \left( \frac{q_c}{\bar{\sigma}_v} \right) \text{ or } C = 1.9 \left( \frac{q_c}{\bar{\sigma}_v} \right)$$

Meyerhof(1965)

(d) Semi-empirical Method (Buisman, 1948)

$$S = \sum 2.3 \frac{\bar{\sigma}_v}{E} H \log \left( \frac{\bar{\sigma}_v + \Delta \sigma}{\bar{\sigma}_v} \right)$$

This is the consolidated chart of the settlement formulae which may aid in the process of designing a footing. Here, only the steps for design or the summary is been provided. If the soil is inorganic clay, though most of the settlement is the consolidation settlement, we can calculate the immediate settlement. Immediate settlement for clay can be calculated using this equation:

$$S_i = qB \left( \frac{1 - \mu^2}{E} \right) I_f$$

where, q is the load intensity acting on the soil base, B is width of foundation,  $\mu$  is the Poisson ratio, E is the elastic modulus or modulus of elasticity of the soil, I is the influence factor.

I will discuss how I can use this equation to get the immediate settlement. Then the consolidation settlement for the clay can be calculated by:

$$S_c = \sum \frac{C_c}{1 + e_0} H \log_{10} \left( \frac{p_0 + \Delta p}{p_0} \right)$$

The above formula shows that the consolidation can be calculated as the summation of settlement of different layers if the soil is layered.

The conditions and soil properties effect which equation should be used. The usage of this equation to determine the consolidation settlement will be discussed in-detail. Let  $\bar{p}_0$  be the effective overburden pressure and  $\Delta p$  is the additional stress coming on that particular point due to the application of external load, H is the thickness of each layer, Cc is the compression index, and  $e_0$  is the initial void ratio of the soil.

The consolidation settlement can be determined by another expression:

$$S_c = \sum m_v H_0 \Delta p$$

$\Delta p$  is the additional stress acting on the soil at that point due to the application of external load okay and  $m_v$  is the coefficient of volume change.

The settlement of granular soil, which is mostly immediate can be determined by plate load test as per IS 1888-1982 or the method based on SPT as per IS 8009-Part 1 – 1976. The method based on static cone penetration test is a semi-empirical method and is similar to that of the consolidation settlement for clay.

The equation for settlement according to the SCPT based method is:

$$S_c = 2.3 \frac{H}{C} \log \left( \frac{\bar{\sigma}_0 + \Delta\sigma}{\bar{\sigma}_0} \right)$$

where H is the thickness of each layer,  $\bar{\sigma}_0$  is the effective overburden pressure,  $\Delta\sigma$  is the increase in pressure at that point due to the application of external load and C is a constant = 1.5 ( $q_c/\bar{\sigma}_0$ ) as per De Beer and Martens or = 1.9 ( $q_c/\bar{\sigma}_0$ ) as per Meyerhof. C can be calculated separately for different layers to calculate the settlement and summed up later.

In the design procedure, the calculation of immediate settlement and consolidation settlement will be discussed along with the settlement calculation based on SPT and plate load test. Remember that any field test is suitable for granular type of soil or sandy soil, but not for clayey soil because any field test in site is a short term test. Whereas the clay properties are long term because of the very less permeability of clay and it takes very long term for clay to settle clay.

So the long term behavior may not be captured during the short duration of the field test. Another reason is that in the clay, obtaining undisturbed soil sample is very easy which is not the case for granular type of soils. So, usually for a granular type of soil, the field test data will be used mainly.

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### Immediate or elastic settlement

$$S_i = qB \left( \frac{1 - \mu^2}{E} \right) I_f$$

where  $q$  = Net foundation pressure

$\mu$  = Poisson's ratio

$E$  = Elastic Modulus of soil

$I_f$  = Influence factor

Types of corrections: 1. Depth correction

2. Rigidity correction for raft foundation

$$S_i = qB \left( \frac{1 - \mu^2}{E} \right) I_f$$

This is the expression for immediate settlement. Here,  $q$  is the net foundation pressure,  $\mu$  is the Poisson's ratio,  $E$  is the elastic modulus of soil,  $I_f$  is the influence factor. I will discuss how we can get the influence factor and then depending upon the type of foundation in immediate settlement, we will apply two corrections. One is the depth correction because these expressions are basically given for the footing on surface. If the foundation or footing is on the surface, then depth correction is not required. Another correction is the rigidity correction which is to be applied for rigid type of foundations because this expression is generally for the flexible type of foundation. An isolated footing is usually treated as a flexible foundation and a raft foundation, as a rigid type of foundation. As the raft foundation supports the entire structure on it, it should be treated a rigid kind of foundation.

For a rigid foundation, lesser settlement is expected as compared to the flexible foundation. So, a rigidity correction of 0.8 should be applied if foundation is rigid (generally raft).

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Shape	I <sub>f</sub> for Flexible Foundation			I <sub>f</sub> for Rigid Foundation
	Centre	Corner	Average	
Circle	1.0	0.64	0.85	0.86
Square	1.12	0.56	0.95	0.82
Rectangle				
L/B= 1.5	1.36	0.68	1.2	1.06
L/B= 2	1.52	0.76	1.3	1.2
L/B= 5	2.10	1.05	1.83	1.70
L/B= 10	2.52	1.26	2.25	2.10
L/B= 100	3.38	1.69	2.96	3.40

Ranjan and Rao, 1991

The influence factor values depend upon the type of foundation or footing (circular, square, rectangular), L by B ratio and also the position (center or corner) below which the factor is to be determined. The influence factor is more at the center which is why it is safe to calculate and design the footing for the value at the center.

To find the influence factor for a rigid foundation at the center, take the I<sub>f</sub> value at the center of flexible footing and multiply it with 0.8. I<sub>f</sub> values for the rigid foundation is more or less 80% of the flexile foundation values. That is why during the rigid foundation design, we will take the I<sub>f</sub> value at the center of flexible foundation, then we will apply the rigidity correction of 0.8.

In the next class, I will discuss how to determine the other types of settlement that is the consolidation settlement and settlement as per plate load test and by SPT for granular soils, through example problems. Thank you.