Soil Structure Interaction
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Lecture 36
Beams on Elastic Foundation (Contd.,)

In the last class I discussed an example problem and determined the coefficient value to determine the deflection at x equal to 0 for upper beam as well as the lower beam. In this class I will determine the deflection for the upper beam.

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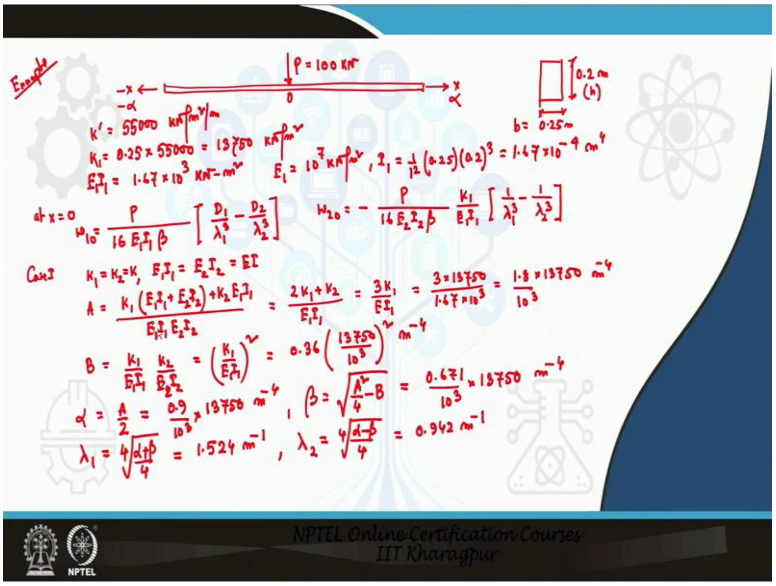

This was the problem and the equations for deflection of upper beam & lower beam at x = 0 are also shown. Using all the coefficients determined in the last class, the deflection of upper and lower beams at x = 0 will now be calculated for case-1 ($k_1 = k_2$ & $E_1I_1 = E_2I_2$).

The deflection for the upper beam at x = 0:

$$w_{1o} = \frac{P}{16E_1I_1\beta}\left[\frac{D_1}{\lambda_1^3} - \frac{D_2}{\lambda_2^3}\right]$$

$$w_{1o} = -\frac{100\times10^3}{16\times1.67\times10^3\times0.671\times13750}\left[\frac{0.37}{10^3}\times13750\times\frac{1}{1.524^3} + \frac{0.972}{10^3}\times13750\times\frac{1}{0.942^3}\right] = 7.1mm$$

The deflection for the lower beam at x = 0:

$$w_{2o} = \frac{P}{16E_2I_2\beta}\frac{k_1}{E_1I_1}\left[\frac{1}{\lambda_1^3} - \frac{1}{\lambda_2^3}\right]$$

$$w_{1o} = -\frac{100\times10^3}{16\times1.67\times10^3\times0.671\times13750}\times\frac{13750}{1.67\times10^3}\left[\frac{1}{1.524^3}-\frac{1}{0.942^3}\right] = 3.05mm$$

The deflection of the lower beam is lower than that of the upper beam which is expected because the loading is applied on the upper beam itself. This is how the deflection will be if $k_1 = k_2$ and $E_1I_1 = E_2I_2$.

Now for the case-2, consider that the stiffness of the lower springs is 5 times to the stiffness of the upper soil which is a valid assumption as the lower soil will usually be the denser soil. The flexural rigidity of both the beams is assumed to be equal. So, for case-2: $k_2 = 5k_1$ and $E_1I_1 = E_2I_2$. Considering these assumptions, the coefficients should be determined now.

Case-2: $k_2 = 5k_1$ and $E_1I_1 = E_2I_2$

$$A = \frac{k_1(E_1I_1+E_2I_2)+k_2(E_1I_1)}{E_1I_1\times E_2I_2} = \frac{2k_1+k_2}{EI} = \frac{2k_1+5k_1}{EI} = \frac{7k_1}{EI} = \left(\frac{4.2}{10^3}\times13750\right)m^{-4}$$

$$B = \frac{k_1k_2}{E_1I_1\times E_2I_2} = \left(\frac{k_1}{E_1I_1}\times\frac{5k_1}{E_1I_1}\right) = 1.8\times\left(\frac{13750}{10^3}\right)^2 m^{-4}$$

$$\alpha = \frac{A}{2} = \frac{2.1}{10^3}\times13750m^{-4}; \quad \beta = \sqrt{\frac{A^2}{4}-B} = \frac{1.62}{10^3}\times13750m^{-4}$$

$$\lambda_1 = \sqrt[4]{\frac{\alpha+\beta}{4}} = 1.89m^{-1}; \quad \lambda_2 = \sqrt[4]{\frac{\alpha-\beta}{4}} = 1.13m^{-1}$$

$$D_1 = \frac{k_1}{E_1I_1}-(\alpha-\beta) = \frac{0.12}{10^3}\times13750m^{-4}$$

$$D_1 = \frac{k_1}{E_1I_1}-(\alpha+\beta) = -\frac{3.12}{10^3}\times13750m^{-4}$$

Using all these values and substituting them in the deflection expression of upper beam:

$$w_{1o} = \frac{P}{16E_1I_1\beta}\left[\frac{D_1}{\lambda_1^3}-\frac{D_2}{\lambda_2^3}\right]$$

$$w_{1o} = 5.04mm$$

Similarly, the deflection of the lower beam is:

$$w_{2o} = \frac{P}{16E_2I_2\beta}\frac{k_1}{E_1I_1}\left[\frac{1}{\lambda_1^3}-\frac{1}{\lambda_2^3}\right]$$

$$w_{2o} = 0.754mm$$

It can be observed that introducing a stronger layer reduced the deformation to 5.04 mm which was initially 7.1 mm. This reduction is more significant for the lower beam as it reduced from 3.05 mm to 0.754 mm.

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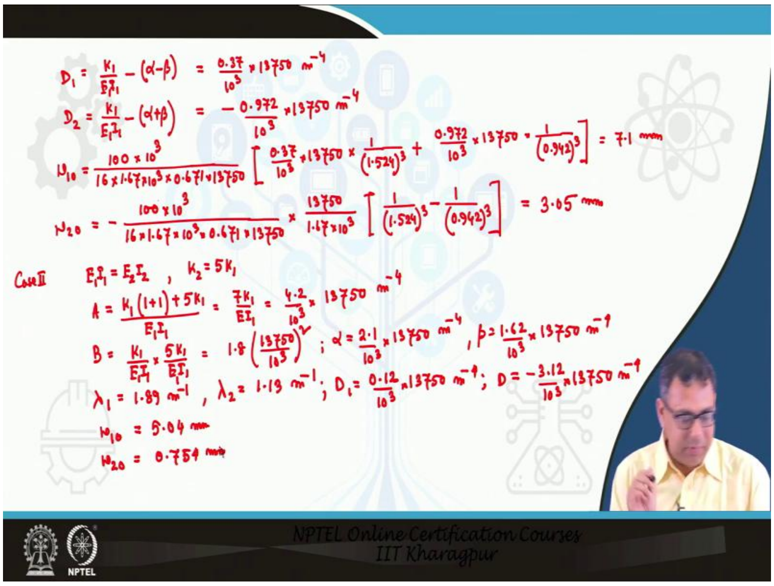

In the next case, let us consider the lower layer is five times stronger than the upper soil layer (k_2 = 5k_1) and the flexural rigidity of the lower beam is half that of the upper beam (E_2I_2 = 0.5 E_1I_1). It is already mentioned to consider a lesser value of EI for the lower beam because in reality there is no beam between the soil layers. So, a low value of flexural rigidity for the lower beam (E_2I_2) will both provide connectivity within the springs and will not tamper with the value of deflection or other required quantities. The deflection of both the beams for this case will be directly given.

To understand the effect of various factors on the settlement, it is better to consider different case and if all the deflection values are tabulated, it would be easy to compare all of them.

Case			w_{1o} (mm)	w_{2o} (mm)
1.	$k_2 = k_1$	$E_2I_2 = E_1I_1$	7.1	3.05
2.	$k_2 = 5k_1$	$E_2I_2 = E_1I_1$	5.04	0.754
3.	$k_2 = 5k_1$	$E_2I_2 = 0.5\ E_1I_1$	4.98	0.77
4.	$k_2 = 5k_1$	$E_2I_2 = (1/20)E_1I_1$	4.98	0.82
5.	$k_2 = (1/5)k_1$	$E_2I_2 = E_1I_1$	14.6	11.0
6.	$k_2 = (1/5)k_1$	$E_2I_2 = (1/20)E_1I_1$	15.85	13.0
7.	$k_2 = (1/5)k_1$	$E_2I_2 = (1/40)E_1I_1$	16.37	13.5
8.	$k_2 = (1/5)k_1$	$E_2I_2 = (1/100)E_1I_1$	16.54	13.7

It can be observed that there is no much difference of the upper beam deflection if the flexural rigidity of the lower beam is reduced but there is a slight change in the lower beam deflection. This phenomenon can be observed by comparing the cases 3 with 4 and also by observing the trend of the last four cases.

In the last four cases, the upper layer is assumed to be stronger than the lower layer. General trend is that the lower layer is stronger than the upper layer, but it is possible that if very soft soil is overlain by a granular soil which will be softer than the lower layer. So, ideally the settlement also increases because a weaker layer has been introduced. In the fifth case, where the lower layer is considered 5 times weaker than the upper layer, the settlement increased drastically showing the significant effect of stiffness on the settlement of the upper beam.

On the other hand, the flexural rigidity of the lower beam has a very negligible effect on the settlement of both the beams. So a very small value of flexural rigidity for the lower beam can be considered just to maintain the continuity between the two layers.

Here the calculations were made only at the x = 0 point but it can be calculated at any x value. This way, even by introducing two different layers of soil with different properties the continuity between the two layers (springs) can be maintained.

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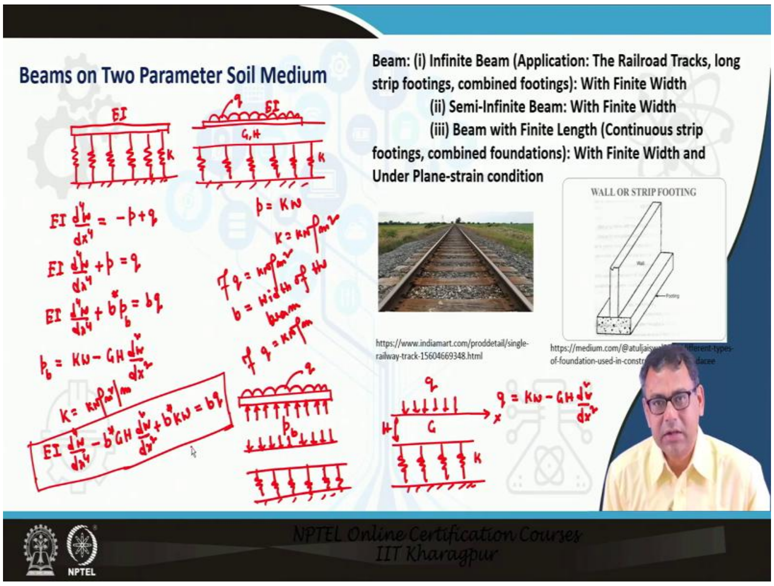

The next concept is about the beams on two parameter soil medium. Till now the discussion has been about the beams resting on springs only. The main advantage of the beam on two parameter soil medium is that here there would be continuity among the springs. If there is no continuity within the springs, no deformation will occur beyond the loaded region or say beyond the beam region. If it is an infinite beam there is no question of deformation beyond the loaded region because all the springs will be connected by the beam (foundation) itself.

But if the beam is of finite length, then there would be no deformation beyond the beam region or the loaded region or the foundation region when it rests on springs. But in the real case there will be deformation even beyond the foundation region. So to get that deformation if a beam is placed over the two parameter soil medium, the continuity will be maintained within the springs and we can get the deformation beyond the foundation beam.

So, in a two parameter model or soil medium the shear layer acts as an infinite beam that provides connectivity within the springs there by enabling the model to simulate the deformations beyond the loaded region too. The shear modulus of the shear layer can be considered as G and the height of the layer as H. The flexural rigidity of the beam is EI and the spring constant is k.

The basic equation for a beam is:

$$EI\frac{d^4w}{dx^4} = -p + q$$

$$\Rightarrow EI\frac{d^4w}{dx^4} + p = q$$

The reaction from the springs, p can be written as a product of k and w. So, if the beam is subjected to a UDL of q, this is the basic equation of the beam. The above equation can be written as:

$$\Rightarrow EI\frac{d^4w}{dx^4} + b^* p_b = bq$$

The b^* will be explained later on. If the UDL acting is in the units of kN/m^2, then it should be multiplied with the width of the beam, b. If the units of q are kN/m, then it need not be multiplied by the width of the beam, b. The term, p_b is nothing but $k' \times w$. Usually the k value we use to find the reaction p ($p = k \times w$) will be in kN/m^2. But the k value involved in the calculation of p_b would have the units $kN/m^2/m$ and so there is a width term in multiplication of the p_b value.

Now if the beam is considered to be subjected to a UDL (q), there would be a reaction acting on the lower part of the beam, p_b. This reaction p_b acts on the shear layer. Now, considering the shear layer, the p_b value will be:

$$p_b = kw - GH\frac{d^2w}{dx^2}$$

In the above expression, the k is in units $kN/m^2/m$ which is why p_b is multiplied with width.

This was already derived as the p_b is just a UDL acting on the two parameter model for which the equation was derived as: $q = kw - GH\frac{d^2w}{dx^2}$. The only difference here is that the UDL acting on the shear layer is p_b.

If the p_b value is substituted in the expression, the final equation of this beam on two parameter soil medium will be:

$$\Rightarrow EI\frac{d^4w}{dx^4} - b^*GH\frac{d^2w}{dx^2} + b^*kw = bq$$

where, b is the width of the beam, G is the shear modulus of the shear layer, h is the thickness of the shear layer, k is the modulus of subgrade reaction or the spring constant. (*p_b will be explained in the next class*)

In the next class I will discuss two different end conditions for the beams on two parameter soil medium and then I will solve the differential equation for infinite beam and for the finite beam. Thank you.