Soil Structure Interaction
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Lecture 34
Beams on Elastic Foundation (Contd.,)

Hello everyone, today I will start a new topic i.e., continuity between the foundation layers in the first model proposed by Winkler.

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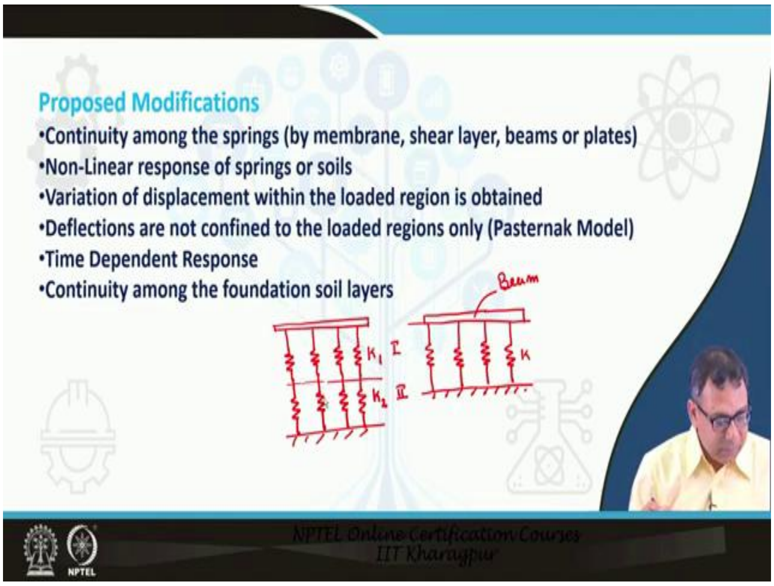

That Winkler spring had some limitations in terms of continuity issues and the deflections were observed within the loaded region only. So, that model is modified later on and the proposed modifications are that continuity among the spring is established by using membrane, shear layer or a beam or a plate which were discussed already. Then in the Winkler spring linear response was considered. So the non linearity response of the spring or the soil was introduced.

Then the variation of displacement within the loaded region is obtained because in Winkler spring the displacement was uniform whether the footing is flexible or rigid type if the modulus of subgrade reaction is uniform along the x-axis. Sometimes different modulus of subgrade reaction may be used within the loaded region to get some variation in the displacement. So, even in the Winkler model, non-uniform settlement within the loaded region can be obtained by using different springs or springs with different modulus of subgrade reaction values. But in the actual model that variation was not there.

Then in the Winkler model the displacement was confined to the loaded region. If a shear layer is provided like in the case of Pasternak model, the deflection beyond the loaded region can also be obtained. Then the time dependent response was also introduced in the Winkler model or the Pasternak model which was already discussed.

Still, the continuity among the foundation soil layers is not established. So what is the continuity among the foundation layer? For example in a model, the soil is represented by springs and these springs are connected by using beams or plate or shear layer or a membrane. For a single layer of springs with spring constant k, connection can be established with the help of a beam. But, if there is a layered soil or a soil with two different springs constants k_1 and k_2 that are resting one over other, how to incorporate this effect in the model? This is the topic that will be explained today.

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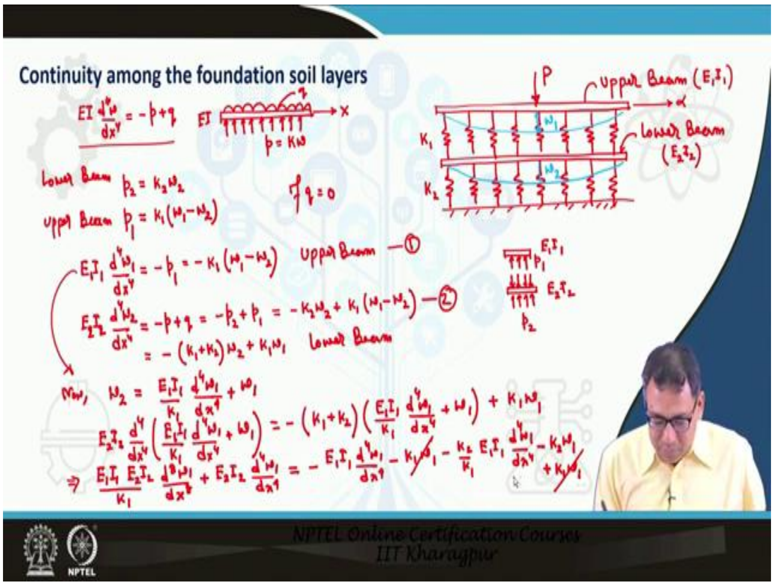

To begin with, the continuity among the foundation layers can be established by introducing another beam between the two springs or the two different layers. The beam which is on the upper layer springs and that which supports the external load, is called the upper beam. The beam in between the two layers of springs is called the lower beam. These beams may be finite or infinite.

The flexural rigidity for the upper beam will be considered as E_1I_1 and that of the lower beam will be considered as E_2I_2. Similarly spring constant for the upper springs is k_1 and for the lower springs is k_2. If there is a deflection in the beams, the deflection of the upper beam would be indicated as w_1 and for the lower beam as w_2. The upper beam represents a normal foundation and the lower beam is used to provide connectivity between the springs. Now let us start with the derivation of the basic differential equation for this condition. Consider that a UDL of intensity q acts on the upper beam. The governing differential equation for this beam will be:

$$EI\frac{d^4w}{dx^4} = -p + q$$

where, p is the reaction from the springs on which the upper beam is resting. Usually the reaction from the springs, p equals (k × w) where k is the spring constant and w is the deflection. Here, as two layers of springs with two different spring constants are there, the p value for lower springs or lower beam will be:

$$p_2 = k_2 \times w_2.$$

Similarly, p value for the upper springs or upper beam will be:

$$p_1 = k_1 (w_1 - w_2)$$

If q = 0, then the governing differential equation for the upper beam would be:

$$E_1I_1\frac{d^4w}{dx^4} = -p_1$$

$$\Rightarrow E_1I_1\frac{d^4w}{dx^4} = -k_1(w_1 - w_2) \rightarrow (1)$$

Similarly, the governing differential equation for the upper beam would be:

$$E_2I_2\frac{d^4w}{dx^4} = -p + q = -p_2 + p_1 = -k_2w_2 + k_1(w_1 - w_2)$$

$$\Rightarrow E_2I_2\frac{d^4w}{dx^4} = -(k_1 + k_2)w_2 + k_1w_1 \rightarrow (2)$$

Here, q was replaced with p1 because for the lower beam, the load would be the reaction from the upper springs.

Now, from equation-(1):

$$w_2 = \frac{E_1I_1}{k_1}\frac{d^4w_1}{dx^4} + w_1$$

By substituting this value of w_2 in equation-(2), we get:

$$E_2I_2\frac{d^4}{dx^4}\left(\frac{E_1I_1}{k_1}\frac{d^4w_1}{dx^4} + w_1\right) = -(k_1 + k_2)\left(\frac{E_1I_1}{k_1}\frac{d^4w_1}{dx^4} + w_1\right) + k_1w_1$$

$$\Rightarrow \frac{E_1I_1 \times E_2I_2}{k_1}\frac{d^8w_1}{dx^8} + E_2I_2\frac{d^4w_1}{dx^4} = -E_1I_1\frac{d^4w_1}{dx^4} - k_1w_1 - \frac{k_2}{k_1}E_1I_1\frac{d^4w_1}{dx^4} - k_2w_1 + k_1w_1$$

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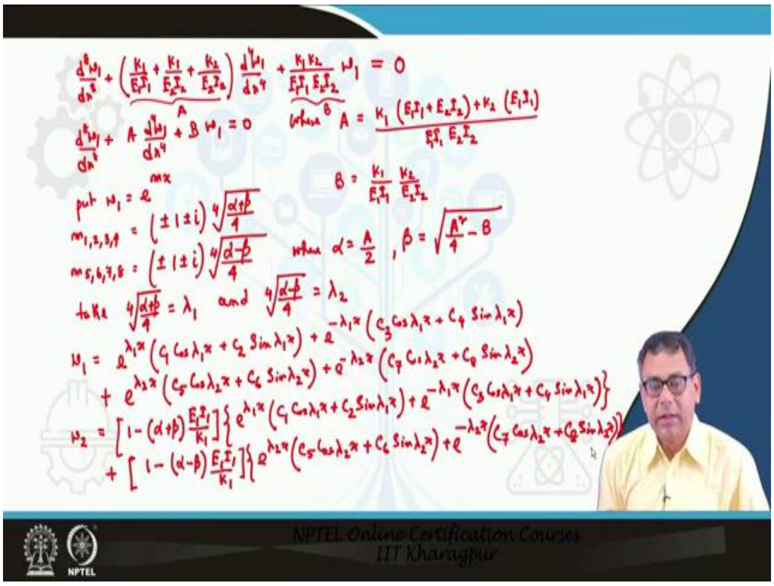

Dividing the above equation with $\frac{E_1 I_1 \times E_2 I_2}{k_1}$ and simplifying, we get:

$$\Rightarrow \frac{d^8 w_1}{dx^8} + \left(\frac{k_1}{E_1 I_1} + \frac{k_1}{E_2 I_2} + \frac{k_2}{E_2 I_2} \right) \frac{d^4 w_1}{dx^4} + \frac{k_1 k_2}{E_1 I_1 \times E_2 I_2} w_1 = 0$$

$$\Rightarrow \frac{d^8 w_1}{dx^8} + A \frac{d^4 w_1}{dx^4} + B w_1 = 0$$

$$\text{where, } A = \frac{k_1 (E_1 I_1 + E_2 I_2) + k_2 (E_1 I_1)}{E_1 I_1 \times E_2 I_2} \text{ and } B = \frac{k_1 k_2}{E_1 I_1 \times E_2 I_2}$$

Let $w_1 = e^{mx}$: then, the roots for m will have the values:

$$m_{1,2,3,4} = (\pm 1 \pm i) \sqrt[4]{\frac{\alpha + \beta}{4}} \text{ and } m_{5,6,7,8} = (\pm 1 \pm i) \sqrt[4]{\frac{\alpha - \beta}{4}}$$

$$\text{where, } \alpha = \frac{A}{2} \text{ and } \beta = \sqrt{\frac{A^2}{4} - B}$$

$$\text{Now consider: } \sqrt[4]{\frac{\alpha + \beta}{4}} = \lambda_1 \text{ and } \sqrt[4]{\frac{\alpha - \beta}{4}} = \lambda_2$$

$$w_1 = \{ e^{\lambda_1 x} (C_1 \cos \lambda_1 x + C_2 \sin \lambda_1 x) + e^{-\lambda_1 x} (C_3 \cos \lambda_1 x + C_4 \sin \lambda_1 x)$$
$$+ e^{\lambda_2 x} (C_5 \cos \lambda_2 x + C_6 \sin \lambda_2 x) + e^{-\lambda_2 x} (C_7 \cos \lambda_2 x + C_8 \sin \lambda_2 x) \}$$

If this value of w1 is substituted in the expression for w2, we get:

$$w_2 = \left[1 - (\alpha + \beta) \frac{E_1 I_1}{k_1} \right] \left\{ e^{\lambda_1 x} (C_1 \cos \lambda_1 x + C_2 \sin \lambda_1 x) + e^{-\lambda_1 x} (C_3 \cos \lambda_1 x + C_4 \sin \lambda_1 x) \right\}$$
$$+ \left[1 - (\alpha - \beta) \frac{E_1 I_1}{k_1} \right] \left\{ e^{\lambda_2 x} (C_5 \cos \lambda_2 x + C_6 \sin \lambda_2 x) + e^{-\lambda_2 x} (C_7 \cos \lambda_2 x + C_8 \sin \lambda_2 x) \right\}$$

So, these are the expressions of deflection for the upper beam and lower beam. In the next class I will discuss how to determine these 8 constant values for infinite beam as well as the finite beam. Thank you.