Soil Structure Interaction
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Lecture 33
Beams on Elastic Foundation (Contd.,)

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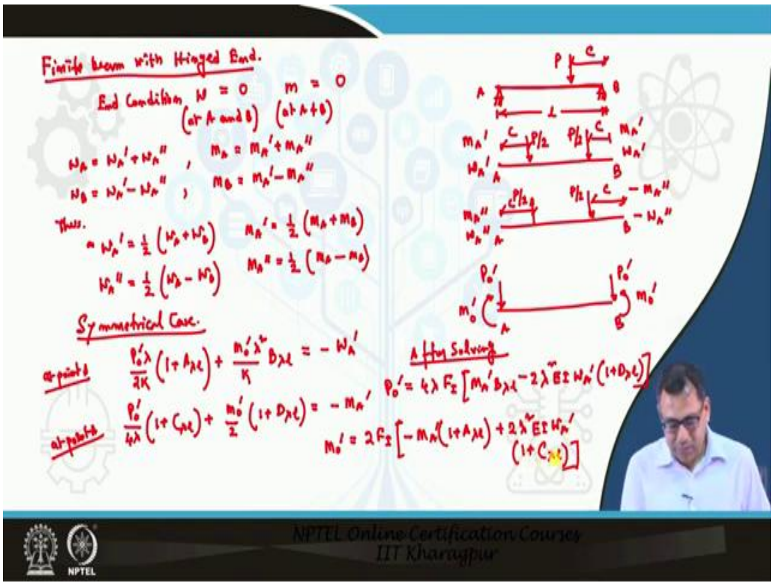

In the last class I was discussing about finite beam with hinged ends. I have derived the expressions for M_A', w_A', M_A'' and w_A''. The moment M_A' and deflection w_A' are induced due to the load in symmetric case and in anti-symmetric case, the moment and deflection induced will be M_A'' and w_A''.

Let us start with the symmetric case, where P_0' and M_0' should be applied as end conditioning forces. The deflection at A, w_A' due to the end conditioning forces will be:

$$\frac{P_\circ'\lambda}{2k}\left(1+A_{\lambda l}\right)+\frac{M_\circ'\lambda^2}{k}B_{\lambda l}=-w_A'$$

The effect due to the P_0' force at both the sides is in the first term. As one P_0' acts at A, x is 0 for it and $A_{\lambda x}$ will be 1 (it will show full effect). Another P_0' acts at B, l units away from A and hence the $A_{\lambda l}$. Similarly M_0' also is acting at both ends, but the moment at A will not have any effect on the deflection at A ($\because x=0; B_{\lambda x}=0$) and the moment at B should be multiplied by $B_{\lambda l}$. As a deflection of w_A occurs due to the external load, the end conditioning forces should be able to produce a deflection of $-w_A$ to nullify the effect of the former.

Similarly, the bending moment at A, M_A' due to the end conditioning forces will be:

$$\frac{P_{\circ}'}{4\lambda}(1+C_{\lambda l})+\frac{M_{\circ}'}{2}(1+D_{\lambda l})=-M_A'$$

The only unknowns in the above two equations are P_0' and M_0' which can be solved as there are two equations. After solving:

$$P_{\circ}' = 4\lambda F_1\left[M_A' B_{\lambda l} - 2\lambda^2 EIw_A'(1+D_{\lambda l})\right]$$

$$M_{\circ}' = 2F_1\left[-M_A'(1+A_{\lambda l})+2\lambda^2 EIw_A'(1+C_{\lambda l})\right]$$

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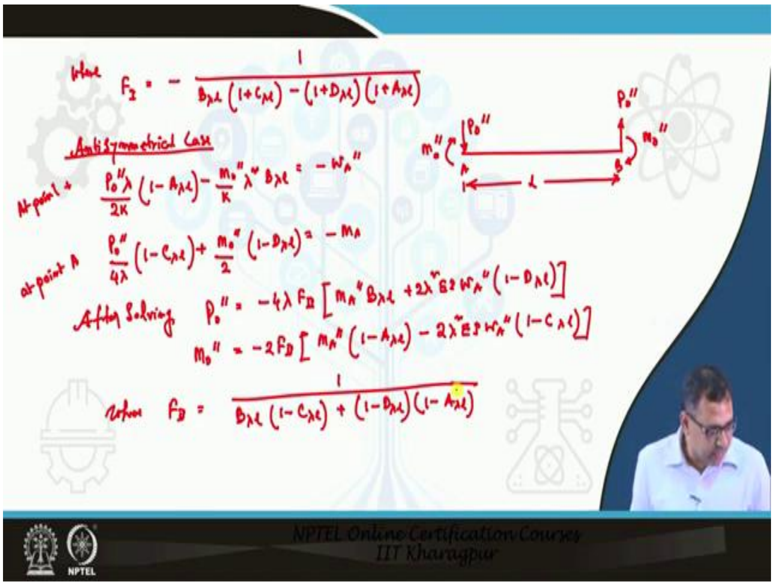

where, $F_1 = -\frac{1}{B_{\lambda l}(1+C_{\lambda l})-(1+D_{\lambda l})(1+A_{\lambda l})}$

Following the procedure and formulation adopted for the symmetrical case for the anti symmetrical case, the following equations can be developed:

At point A: $\frac{P_{\circ}''\lambda}{2k}(1-A_{\lambda l})-\frac{M_{\circ}''\lambda^2}{k}B_{\lambda l}=-w_A''$

At point B: $\frac{P_{\circ}''}{4\lambda}(1-C_{\lambda l})+\frac{M_{\circ}''}{2}(1-D_{\lambda l})=-M_A''$

After solving the above equations:

$$P_{\circ}'' = -4\lambda F_2\left[M_A'' B_{\lambda l} + 2\lambda^2 EIw_A''(1-D_{\lambda l})\right]$$

$$M_{\circ}'' = -2F_2\left[M_A''(1-A_{\lambda l})-2\lambda^2 EIw_A''(1-C_{\lambda l})\right]$$

$$\text{where, } F_2 = \frac{1}{B_{\lambda l}(1 - C_{\lambda l}) + (1 - D_{\lambda l})(1 - A_{\lambda l})}$$

These are the expressions for a finite beam with hinged ends.

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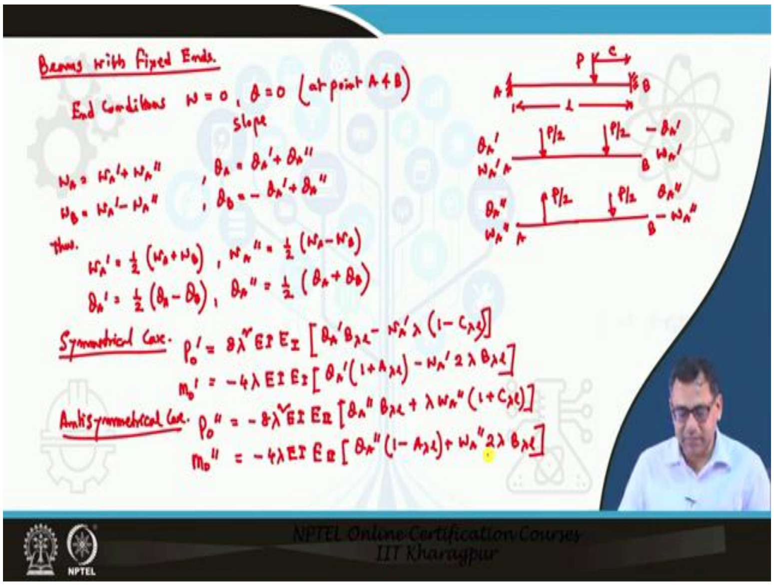

The next one in our discussion is the finite beam with fixed ends. The beam now is of length l with fixed ends, supporting a point load, P. Now the end conditions or boundary conditions will be zero deflection and zero slope at point A and point B. There is a possibility that both the ends have different boundary conditions (one end in is free and other end is fixed). In that case, different boundary conditions should be used for the two different ends.

Here again the actual condition is idealised as a sum of a symmetrical loading case and an anti symmetrical loading case. As the finite beam is considered infinite in these two cases, deflection & slope will be induced and in the symmetrical case they will be w_A' & θ_A' whereas in the anti symmetrical case they will be w_A'' & θ_A'' respectively. The directions and signs of both deflection and slope in both the cases are shown in the slide above.

$$w_A = w_A' + w_A'', \theta_A = \theta_A' + \theta_A''$$

$$w_B = w_A' - w_A'', \theta_B = -\theta_A' + \theta_A''$$

By solving the above equations, we get:

$$\mathrm{w_A}' = \frac{1}{2}(w_A + w_B);\ \theta_{\mathrm{A}}' = \frac{1}{2}(\theta_A - \theta_B)$$

$$\mathrm{w_A}'' = \frac{1}{2}(w_A - w_B);\ \theta_{\mathrm{A}}'' = \frac{1}{2}(\theta_A + \theta_B)$$

Following the similar procedure followed in the last two cases, for the symmetrical case:

$$P_{\circ}^{'} = 8\lambda^2 EIE_1\left[\theta_A^{'} B_{\lambda l} - w_A^{'}\lambda\left(1 - C_{\lambda l}\right)\right]$$

$$M_{\circ}^{'} = -4\lambda EIE_1\left[\theta_A^{'}\left(1 + A_{\lambda l}\right) - w_A^{'} 2\lambda B_{\lambda l}\right]$$

Similarly, for the anti symmetrical case:

$$P_{\circ}^{''} = -8\lambda^2 EIE_2\left[\theta_A^{''} B_{\lambda l} + w_A^{''}\lambda\left(1 + C_{\lambda l}\right)\right]$$

$$M_{\circ}^{''} = -4\lambda EIE_2\left[\theta_A^{''}\left(1 - A_{\lambda l}\right) + w_A^{''} 2\lambda B_{\lambda l}\right]$$

This way the end conditioning forces for a finite beam with fixed ends can be determined. The example problem was solved only for the free end because it is a common boundary condition for any foundation. But the deflection or any other quantities can be determined for the other two cases also.

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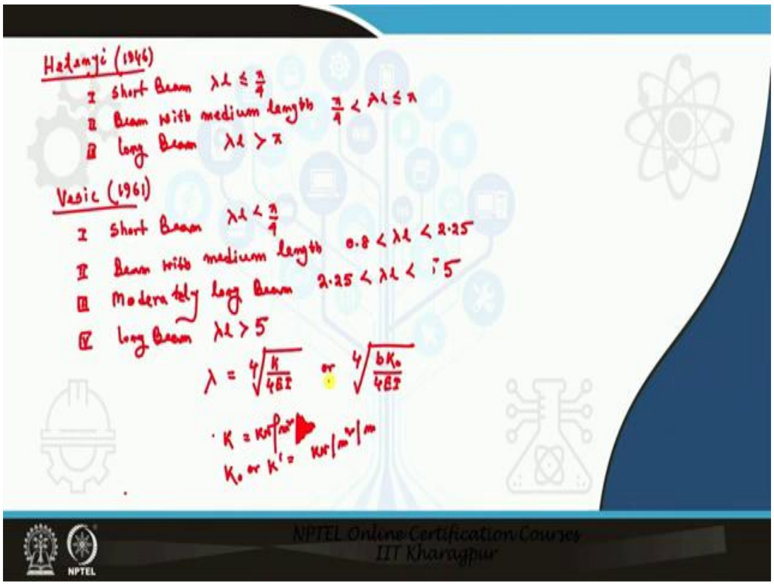

Hetenyi (1946) and Vesic (1961) classified beams according to their lengths which will be discussed now. As per the Hetenyi, beams are classified into three categories:

1. Short Beam (if $\lambda l \leq \pi/4$)
2. Beam with medium length (if $\pi/4 < \lambda l \leq \pi$)
3. Long Beam (if $\lambda l > \pi$)

According to Vesic (1961), the classification is:

1. Short Beam (if $\lambda l < \pi/4$)
2. Beam with medium length (if $0.8 < \lambda l < 2.25$)

3. Moderately long beam (if $2.25 < \lambda l < 5$)
4. Long beam (if $\lambda l > 5$)

These are the classifications of the beam based on the length as per Hetenyi and Vesic.

Till now, I have discussed about the infinite beam, the semi infinite beam and the beam with finite length. In the next class I will start the discussion of a new topic which is very interesting where there is a two layer system. The connectivity between two different layers has not been dealt with yet. I will discuss what happens if there is a two layer system and how to establish connection between these two layers in the next class. Thank you.