Soil Structure Interaction
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Lecture 32
Beams on Elastic Foundation (Contd.,)

In my previous lecture I have solved one problem where a finite beam was subjected to a concentrated load. I determined the moment and shear force that were induced due to the external load at the two ends A & B and I also determined the end conditioning forces P_{0A}, P_{0B}, M_{0A} and M_{0B} those are required to satisfy the boundary conditions. Today, I will first check whether those end conditioning forces are satisfying the boundary conditions. Then I will determine the deflection at different points along the beam.

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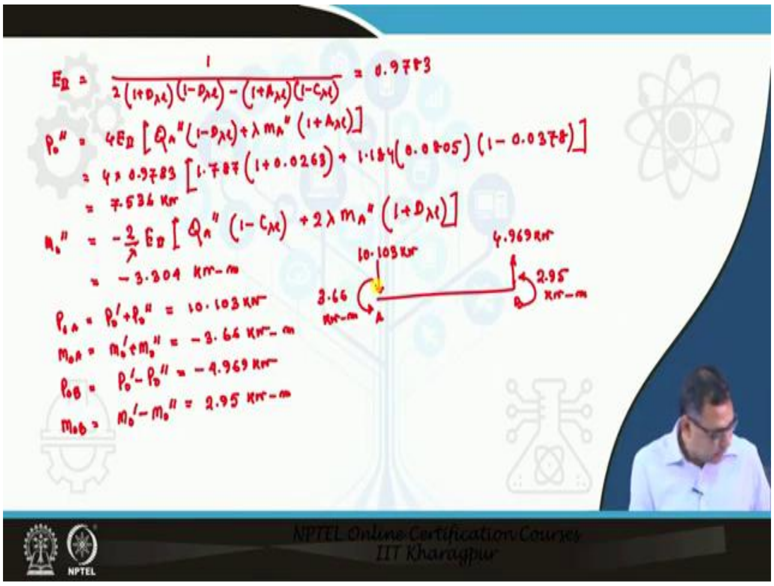

Let us rewind the example problem solved in the previous class. The moment at point A that developed due to the external load P was M_A = -0.28 kN-m, the moment at point B was - 0.441 kN-m, shear force at point A, Q_A was 2.886 kN and at point B, Q_B was 0.688 kN. Now the end conditioning forces should produce a moment of 0.28 kN-m and a shear force of -2.886 kN at point A along with a moment of 0.441 kN-m and shear force of -0.688 kN at point B.

To check the calculated results, it should be seen if the net moment and net shear force at both ends of the beam, A and B are zero. As this finite beam has free ends at both the sides, the boundary condition is that the net bending moment and shear force at both these ends are 0. So, this condition should be checked now.

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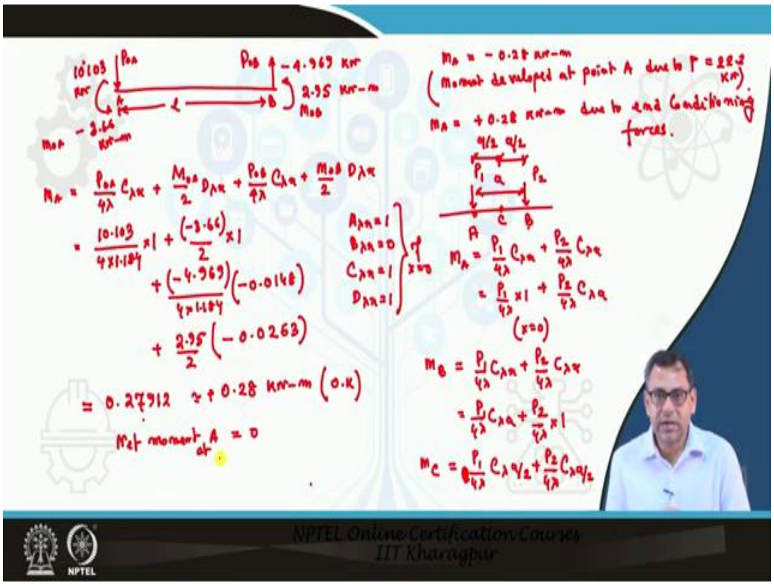

The end conditioning forces are shown in the slide above. The values of these forces as calculated are: P_{0A} = 10.103 kN, M_{0A} = -3.66 kN-m, P_{0B} = -4.969 kN and M_{0B} = 2.25 kN-m.

While calculating any quantity at a point due to some force if x is considered 0, it means that the force is acting at the point itself. For example, on a beam if a load P_1 acts at point A and load P_2 acts at point B, while calculating moment induced at A due to P_1, the value of x will be 0. Say that the distance between both the loads is 'a' and the midpoint between them is at point C. The total moment at A due to both the loads will be:

$$M_A = \frac{P_1}{4\lambda} C_{\lambda x} + \frac{P_2}{4\lambda} C_{\lambda x}$$

$$\Rightarrow M_A = \frac{P_1}{4\lambda} \times 1 + \frac{P_2}{4\lambda} C_{\lambda a} \ (\because x = 0) \text{ for load } P_1$$

The total moment at B due to both the loads will be:

$$M_B = \frac{P_1}{4\lambda} C_{\lambda x} + \frac{P_2}{4\lambda} C_{\lambda x}$$

$$\Rightarrow M_B = \frac{P_1}{4\lambda} C_{\lambda a} + \frac{P_2}{4\lambda} \times 1$$

If the point of interest is C, the expression for bending moment at C would be:

$$M_C = \frac{P_1}{4\lambda} C_{\lambda a/2} + \frac{P_2}{4\lambda} C_{\lambda a/2}$$

The forces developed due to the load P are already discussed in this class. Let us see the value of moment that develops at point A due to the end conditioning forces (this should balance the moment, M_A = -0.28 kN-m). Each of the four end conditioning forces contributes to the development of some moment at point A which can be calculated by:

$$M_A = \frac{P_{OA}}{4\lambda} c_{\lambda x} + \frac{M_{OA}}{2} D_{\lambda x} + \frac{P_{OB}}{4\lambda} C_{\lambda x} + \frac{M_{OB}}{2} D_{\lambda x}$$

$$\Rightarrow M_A = \frac{10.103}{4 \times 1.184} \times 1 + \frac{(-3.66)}{2} \times 1 + \frac{(-4.969)}{4 \times 1.184}(-0.0148) + \frac{2.95}{2}(-0.0148)$$

$$\therefore M_A = 0.27912 kN - m \approx 0.28 kN - m$$

So, the net moment at point A is 0.

This means that when the end conditioning forces are applied, the boundary conditions are being satisfied. Similarly the shear force at A (Q_A), bending moment at B (Q_B) and shear force at B (Q_B) that are induced due to the end conditioning forces can be determined. These can be check too, in the similar way to know if they are satisfying the boundary conditions. Here only M_A is checked for and if one is satisfied all others will also satisfy.

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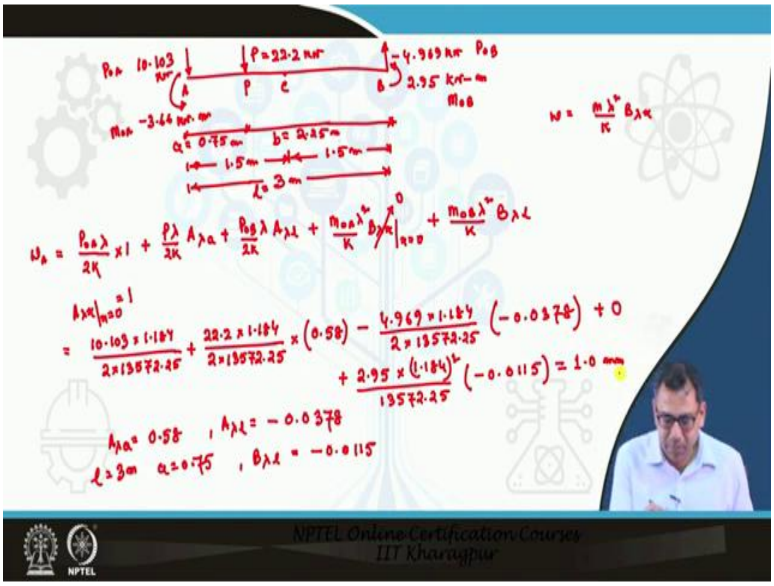

Now let us calculate the deflection and the other quantities due to these forces. The loading condition of the beam is shown in the slide above. The deflection at A can be found by:

$$w_A = \left(\frac{P_{OA}\lambda}{2k} A_{\lambda x}\Big|_{x=0} \right) + \frac{P\lambda}{2k} A_{\lambda a} + \frac{P_{OB}\lambda}{2k} A_{\lambda l} + \left(\frac{M_{OA}\lambda^2}{k} B_{\lambda x}\Big|_{x=0} \right) + \frac{M_{OB}\lambda^2}{k} B_{\lambda l}$$

As a = 0.75 m, l = 3 m, $A_{\lambda a}$=0.58, $A_{\lambda l}$=-0.0378, $B_{\lambda l}$=-0.0115

$$\Rightarrow w_A = \frac{10.103 \times 1.184}{2 \times 13572.5} + \frac{22.2 \times 1.184}{2 \times 13572.5}(0.58) - \frac{4.969 \times 1.184}{2 \times 13572.5}(-0.0378) + 0 + \frac{2.95 \times (1.184)^2}{13572.5}(-0.0115)$$

$$\therefore w_A = 1.0 mm$$

So, the deflection at point A is 1 mm. Now let us determine the deflection at another point say, P for now and then other points too.

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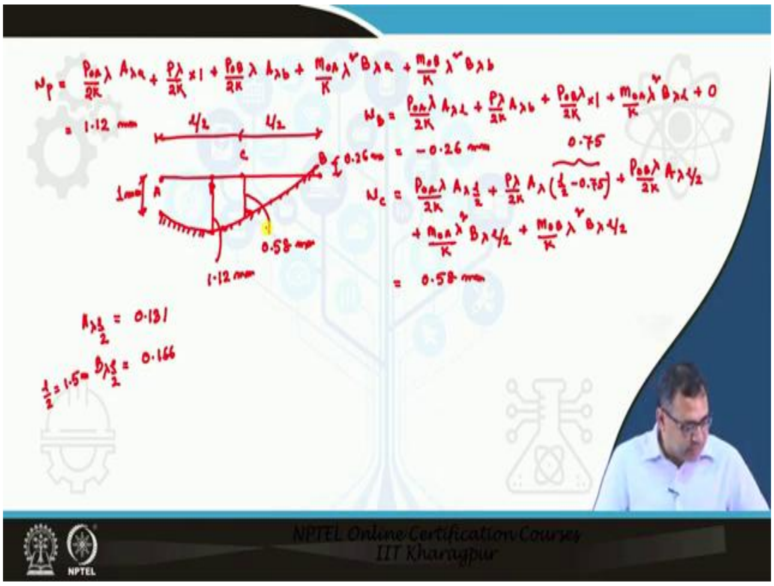

$$w_A = \frac{P_{OA}\lambda}{2k} A_{\lambda a} + \frac{P\lambda}{2k} \times 1 + \frac{P_{OB}\lambda}{2k} A_{\lambda b} + \frac{M_{OA}\lambda^2}{k} B_{\lambda a} + \frac{M_{OB}\lambda^2}{k} B_{\lambda b}$$

By substituting all the values in the above expression, we get w_A = 1.12 mm.

Similarly deflection at B can be given by:

$$w_B = \frac{P_{OA}\lambda}{2k} A_{\lambda l} + \frac{P\lambda}{2k} A_{\lambda b} + \frac{P_{OB}\lambda}{2k} \times 1 + \frac{M_{OA}\lambda^2}{k} B_{\lambda l} + \left(\frac{M_{OB}\lambda^2}{k} \times 0 \right) = -0.26mm$$

Similarly deflection at C can be given by:

$$w_C = \frac{P_{OA}\lambda}{2k} A_{\lambda l/2} + \frac{P\lambda}{2k} A_{\lambda(l/2 - 0.75)} + \frac{P_{OB}\lambda}{2k} A_{\lambda l/2} + \frac{M_{OA}\lambda^2}{k} B_{\lambda l/2} + \frac{M_{OB}\lambda^2}{k} B_{\lambda l/2} = 0.58mm$$

A settlement profile can be drawn by the obtained values along the beam as shown in the slide above for a clear visualisation. The problem solved here is for a finite beam with free ends subjected to concentrated load and the deflections were determined at 4 different points.

The other quantities like bending moment, shear force and slope at this 4 points can also be determined and this problem can be solved for any other loading condition also. The expressions given are general and so are valid for any other loading condition. The next concept involves in a change in the boundary condition.

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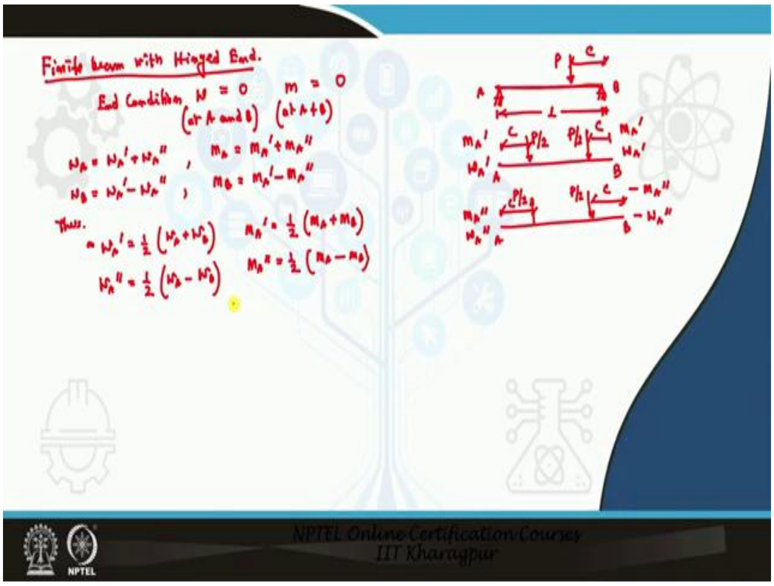

A finite beam of length, l and with hinge ends on both sides is considered. Points A and B are the ends which are hinged and a point load, P acts on the beam at a distance of ‘c’ from point B. The goal is to develop a general expression for this beam and for any loading condition. Boundary conditions are that both deflection and bending moment at both ends A and B are 0. Split the load into two equal loads and develop two cases: symmetrical case and anti symmetrical case as shown in the figure above.

Now, the beam is considered as an infinite beam and if it is infinite, moment and deflection will exit at both A and B. Let the moment and deflection for the symmetrical case be M_A' and w_A' respectively. Since this is a symmetrical case, the same moment and deflection exists at both the ends (A & B). In the anti symmetrical case, the load nearer to A should act in the reverse direction to balance the imaginary load considered in the symmetrical case.

So, here now M_A'' and w_A'' will be induced due to the forces at A but $-M_A''$ and $-w_A''$ will be induced at point B.

$$w_A = w_A' + w_A'', M_A = M_A' + M_A''$$

$$w_B = w_A' - w_A'', M_B = M_A' - M_A''$$

By solving the above equations, we get:

$$w_A' = \frac{1}{2}(w_A + w_B);\ M_A' = \frac{1}{2}(M_A + M_B)$$

$$w_A'' = \frac{1}{2}(w_A - w_B);\ M_A'' = \frac{1}{2}(M_A - M_B)$$

In the next class first from here I will start from here and determine the end conditioning forces which will be required to make deflection and the bending moment 0 at both the ends and then based on that will determine the P_0', P_0'', M_0' and M_0''. After determining the end conditioning forces, any required quantities at any point along the beam can be determined. I will show those derivations in the next class. Thank you.