Soil Structure Interaction
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Lecture 31
Beams on Elastic Foundation (Contd.,)

In the last class I have discussed about a beam with finite length subjected to a concentrated load for free end condition and derived the expressions for deflection, slope, bending moment and shear force. Now I will solve one example problem to show how to determine that deflection for beam with finite length subjected to concentrated load.

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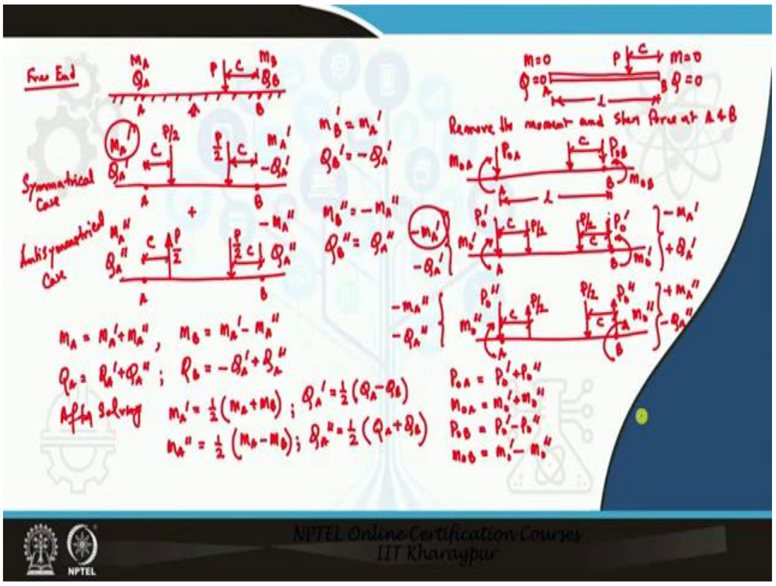

The loading condition and the expressions derived in the last class are shown in the slide above. Two loading conditions were considered which are symmetrical case and anti symmetrical case. Then the expressions for end conditioning forces, P_0', P_0'', M_0' and M_0'' to nullify the moment and shear force at the two ends A and B were determined.

Please note that though concentrated load was considered during the derivation, the expressions can be derived for any type of loading. But this derivation is for free end condition and hence the forces M_A, M_B, Q_A and Q_B which are induced due to the external loading should be determined. From these values, M_A', M_B', Q_A' and Q_B' should be determined.

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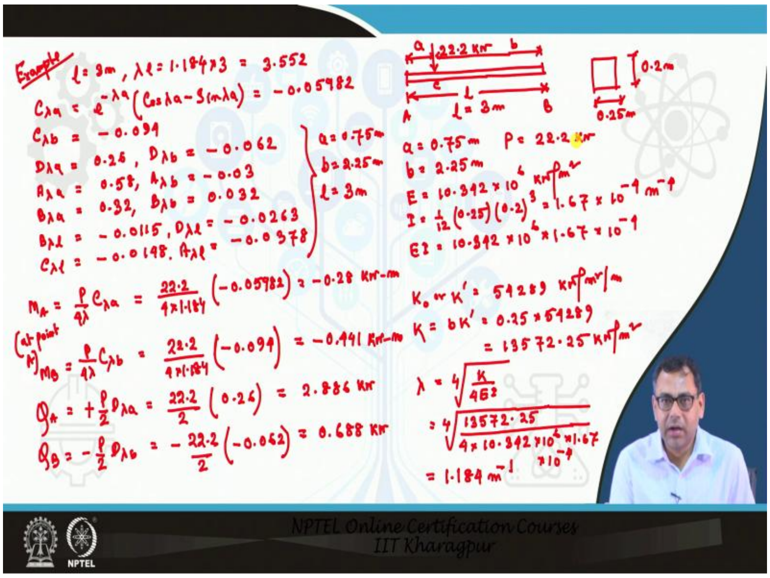

Let us solve an example problem of a beam with finite length, l = 3 m. The cross section of the beam is the same as that of the previous examples, 0.25 m × 0.2 m. A concentrated load of 22.2 kN is applied on the beam. The distance from A to the loading point is 'a' and that from B to the loading point is 'b'. Here, a = 0.75 m and b = 2.25 m. E value is 10.342×10^6 kN/m^2 and I value is 1.67×10^{-4} m^4.

So, EI = $(1.67 \times 10^{-4}) \times (10.342 \times 10^6) = 1.73 \times 10^3$ kN-m^2. The value of k_0 or k' is given as 54289 kN/m^2/m. But, to obtain k value:

$$k = b \times k' = 0.25 \times 54289 = 13572.25 kN/m^2$$

Now, determine the Lambda value:

$$\lambda = \sqrt[4]{\frac{k}{4EI}}$$

$$\Rightarrow \lambda = \sqrt[4]{\frac{13572.25}{4 \times 1.73 \times 10^3}} = 1.184 m^{-1}$$

As l = 3 m, λl = 1.184 × 3 = 3.552

As a = 0.75 m, λa = 1.184 × 0.75 = 0.888

As b = 2.25 m, λb = 1.184 × 2.25 = 2.664

Calculate all the required coefficients:

$$C_{\lambda a} = e^{-\lambda a}(\cos \lambda a - \sin \lambda a) = -0.05982$$

$$C_{\lambda b} = \text{-}0.094$$

$$D_{\lambda a} = 0.26;\ D_{\lambda b} = -0.062$$

$$A_{\lambda a} = 0.58;\ A_{\lambda b} = -0.03$$

$$B_{\lambda a} = 0.32;\ B_{\lambda b} = 0.032$$

$$A_{\lambda l} = -0.0378;\ B_{\lambda l} = -0.0115$$

$$C_{\lambda l} = -0.0148;\ D_{\lambda l} = -0.0263$$

Moment induced at point A due to the point load, P:

$$M_A = \frac{P}{4\lambda} C_{\lambda a} = \frac{22.2}{4 \times 1.184} (-0.05982) = -0.28 kN - m$$

Moment induced at point B due to the point load, P:

$$M_B = \frac{P}{4\lambda} C_{\lambda b} = \frac{22.2}{4 \times 1.184} (-0.094) = -0.441 kN - m$$

Shear force induced at point A due to the point load, P:

$$Q_A = +\frac{P}{2} D_{\lambda a} = \frac{22.2}{2} (0.26) = 2.886 kN$$

Note that the point A is to the left side of the loading point, C and hence the expression should be negative of the usual. The usual expression for shear force has a negative sign and so, the expression for Q_A here would be positive.

Shear force induced at point B due to the point load, P:

$$Q_B = -\frac{P}{2} D_{\lambda b} = -\frac{22.2}{2} (-0.062) = 0.688 kN$$

The procedure is being done for a concentrated load, but can be followed for any type of loading. If a UDL acts on the beam, use the expression for UDL to determine the bending moment and shear force. Now, calculate M_A', M_A'', Q_A' and Q_A'' using the expressions already derived.

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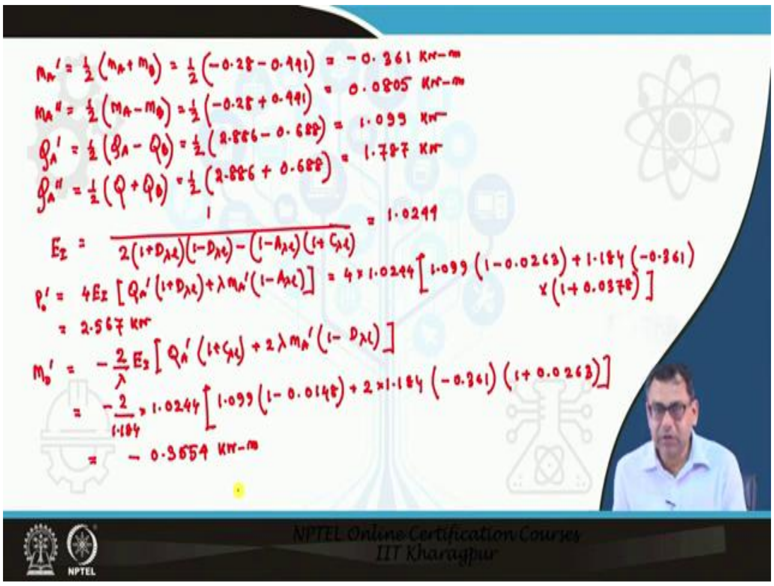

$$M_A{}' = \frac{1}{2}(M_A + M_B) = \frac{1}{2}(-0.28 - 0.441) = -0.861 kN - m$$

$$M_A{}'' = \frac{1}{2}(M_A - M_B) = \frac{1}{2}(-0.28 + 0.441) = 0.0805 kN - m$$

$$Q_A{}' = \frac{1}{2}(Q_A - Q_B) = \frac{1}{2}(2.886 - 0.688) = 1.099 kN$$

$$Q_A{}'' = \frac{1}{2}(Q_A + Q_B) = \frac{1}{2}(2.886 + 0.688) = 1.787 kN$$

$$E_1 = \frac{1}{2(1+D_{\lambda l})(1-D_{\lambda l}) - (1-A_{\lambda l})(1+C_{\lambda l})} = 1.0244$$

$$P_{\circ}{}' = 4E_1\left[Q_A{}'(1+D_{\lambda l}) + \lambda M_A{}'(1-A_{\lambda l})\right]$$

$$\Rightarrow P_{\circ}{}' = 4 \times 1.0244[1.099(1-0.0263) + 1.184(-0.861)(1+0.0363)] = 2.567 kN$$

$$M_{\circ}{}' = -\frac{2}{\lambda}E_1\left[Q_A{}'(1+C_{\lambda l}) + 2\lambda M_A{}'(1-D_{\lambda l})\right]$$

$$\Rightarrow M_{\circ}{}' = -\frac{2}{1.184} \times 1.0244[1.099(1-0.0148) + 2 \times 1.184(-0.361)(1+0.0263)] = -0.3554 kN - m$$

So, for the symmetrical case, P_0' and M_0' are found out. Now, the values for anti-symmetric case should be determined.

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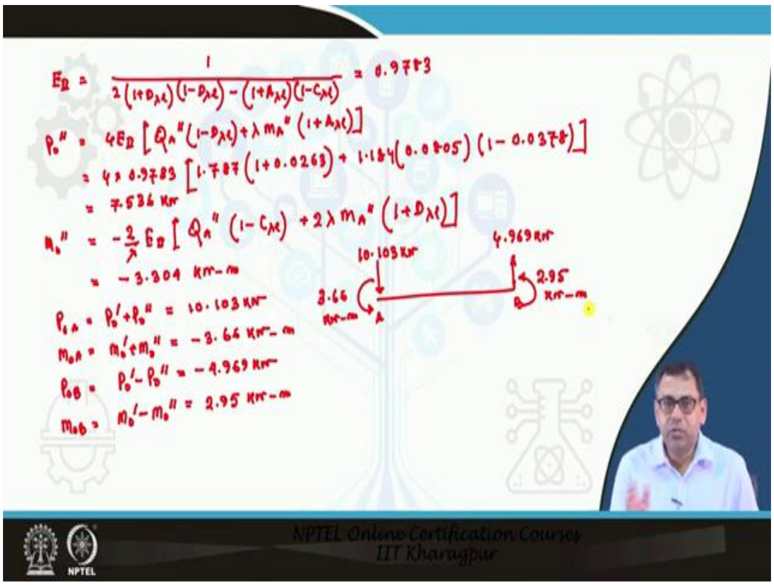

$$E_2 = \frac{1}{2(1+D_{\lambda l})(1-D_{\lambda l}) - (1+A_{\lambda l})(1-C_{\lambda l})} = 0.9783$$

$$P_{\circ}{}'' = 4E_2\left[Q_A{}'(1-D_{\lambda l}) + \lambda M_A{}''(1+A_{\lambda l})\right]$$

$$\Rightarrow P_{\circ}'' = 4 \times 0.9783[1.787(1+0.0263)+1.184(0.0805)(1-0.0378)] = 7.536kN$$

$$M_{\circ}'' = -\frac{2}{\lambda}E_2\left[Q_A''(1-C_{\lambda l})+2\lambda M_A''(1+D_{\lambda l})\right] = -3.304kN-m$$

P_0' and M_0' are the end conditioning forces that are to be applied for the symmetrical case. P_0'' and M_0'' are the end conditioning forces that are to be applied for the anti-symmetrical case. The final end conditioning forces P_{0A}, M_{0A}, P_{0B} and M_{0B} can be found out by the expressions:

$$P_{oA} = P_o' + P_o'' = 10.103kN$$

$$M_{OA} = M_O' + M_O'' = -3.66kN-m$$

$$P_{oB} = P_o' - P_o'' = -4.969kN$$

$$M_{OB} = M_O' - M_O'' = 2.95kN-m$$

Here, P_{0A} and M_{0B} are positive indicating that the direction assumed for these two forces initially was correct. But, P_{0B} and M_{0A} turned out to be negative which means the direction assumed for these two forces in the initial stages was wrong. These are the end conditioning forces and their directions that are to be applied to nullify the bending moment and shear force at points A and B. Remember that M_A and M_B are the bending moments induced at A and B respectively due to external load. Also, that Q_A and Q_B are the shear forces induced at A and B respectively due to external load.

In the next class, I will first check that the values of end conditioning forces are satisfying the boundary conditions and then will determine the deflection at different points along the beam. Thank you.