Soil Structure Interaction
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Lecture 30
Beams on Elastic Foundation (Contd.,)

In the last class, I was discussing about the two different loading conditions considered to analyse a finite beam subjected to concentrated load. The two different loading conditions are symmetrical loading condition and anti symmetrical loading condition which are shown in the slide below.

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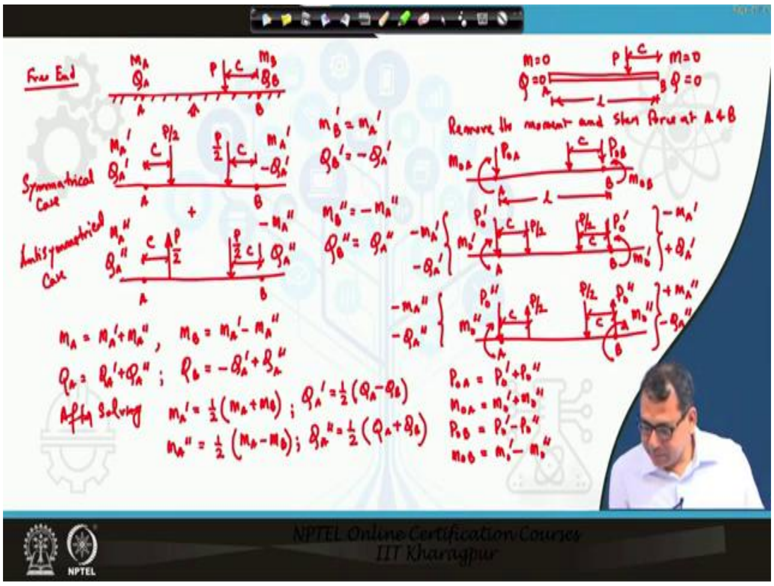

The bending moment and shear force developed at both the ends are named as per the loading condition and for the symmetric condition:

$$M_B' = M_A' \text{ and } Q_B' = -Q_A'$$

Similarly for the anti-symmetric condition, the below is valid:

$$M_B'' = -M_A'' \text{ and } Q_B'' = Q_A''$$

These two cases are considered in such a way that if they both are added or considered together, the original loading condition can be obtained. So:

$$M_A = M_A' + M_A'', \qquad M_B = M_A' - M_A''$$

$$Q_A = Q_A' + Q_A'', \qquad Q_B = -Q_A' + Q_A''$$

After solving the above equations, we get:

$$M_A' = \frac{1}{2}(M_A + M_B); \quad Q_A' = \frac{1}{2}(Q_A - Q_B)$$

$$M_A'' = \frac{1}{2}(M_A - M_B); \quad Q_A'' = \frac{1}{2}(Q_A + Q_B)$$

Now, the end conditioning forces should be applied to nullify this bending moment and shear force. So, in the original condition, the end conditioning forces are P_{oA} & M_{oA} at the point A and P_{oB} & M_{oB} at the point B. The directions of these end conditioning forces are just an assumption and after calculation, if the values are positive, it means that the assumed directions are correct.

The end conditioning forces to be applied in the original condition were assumed. As the original loading condition is split into two cases, the end conditioning forces in each case should be formulated. The end conditioning forces in the symmetrical condition are: P_o' & M_o'. Similarly, the end conditioning forces in the symmetrical condition are: P_o'' & M_o''. As the original condition is split into two cases, the end conditioning forces considered in the original condition will also be split for the two conditions. It means that the force P_{oA} will be split into P_o' and P_o''.

$$P_{oA} = P_o' + P_o''; \qquad M_{oA} = M_o' + M_o''$$

$$P_{oB} = P_o' - P_o''; \qquad M_{oB} = M_o' - M_o''$$

Now these end conditioning forces in each case should produce the moment and shear force in such a way that they will nullify both moment and shear force for that particular case. So, the end conditioning forces in each case should satisfy the boundary conditions. For example, in the symmetrical case, as the bending moment at point A induced due to the load is M_A', the end conditioning forces of that case (P_o' & M_o' at both ends) should produce a moment of $-M_A'$ at point A.

So, for the symmetrical case the end conditioning forces should produce a moment and shear force of: $-M_A'$ & $-Q_A'$ at point A and $-M_A'$ & $+Q_A'$ at point B. Similarly, for the anti-symmetrical case, the end conditioning forces should produce a moment and shear force of: $-M_A''$ & $-Q_A''$ at point A and $+M_A''$ & $-Q_A''$ at point B.

Ultimately the net moment and shear force at both the ends of the beam should be 0.

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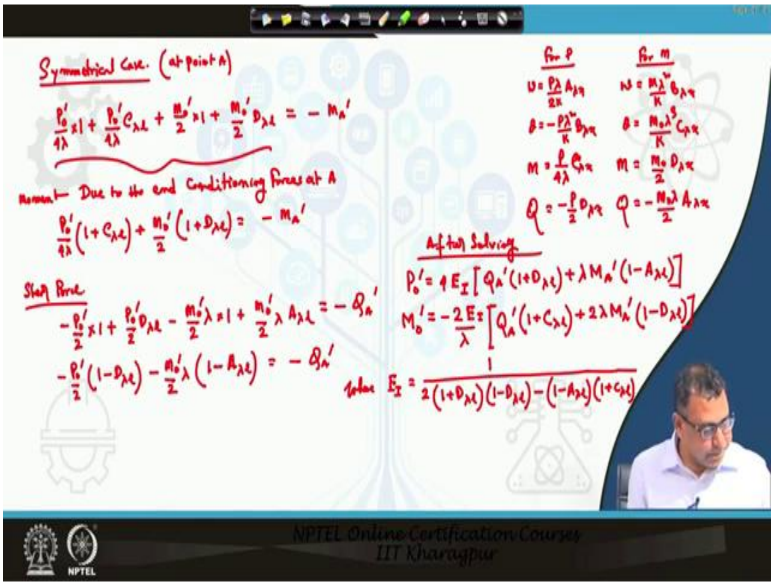

Before going any further, let us remind the formulae for all the four quantities when the infinite beam is subjected to: 1.) Point load 2.) Concentrated moment. These eight formulae are very important and are useful to analyse the cases of complex loading conditions.

Under Point load	Under Concentrated Moment
$w = \frac{P\lambda}{2k} A_{\lambda x}$	$w = \frac{M_0 \lambda^2}{k} B_{\lambda x}$
$\theta = -\frac{P\lambda^2}{k} B_{\lambda x}$	$\theta = \frac{M_0 \lambda^3}{k} C_{\lambda x}$
$M = \frac{P}{4\lambda} C_{\lambda x}$	$M = \frac{M_0}{2} D_{\lambda x}$
$Q = -\frac{P}{2} D_{\lambda x}$	$Q = -\frac{M_0 \lambda}{2} A_{\lambda x}$

For the symmetrical case, the forces acting on the beam are: end conditioning forces at both ends (P_o' & M_o'), two external point loads each of P/2. A point load acts at a distance of ‘c’ from B and the other is at a distance of ‘c’ from A. Now, if only the end conditioning forces are considered, they should produce a moment of $-M_A'$ at point A:

$$\frac{P_o'}{4\lambda}\times 1+\frac{P_o'}{4\lambda}\times C_{\lambda l}+\frac{M_o'}{2}\times 1+\frac{M_o'}{2}\times D_{\lambda l}=-M_A'$$

In the above expression, the first term is the effect of P_o' acting at A. As the moment is being calculated at the same point, for this force x = 0 and $A_{\lambda x} = 1$. This is the reason that one M_o' term also has the coefficient value equal to one as one M_o' is acting at point A. The other end conditioning forces are acting at point B, l m away from point A and hence two coefficients should be calculated for x = l.

$$\frac{P'_o}{4\lambda}\times(1+C_{\lambda l})+\frac{M'_o}{2}\times(1+\times D_{\lambda l})=-M'_A$$

Similarly, the shear force at point A due to the end conditioning forces is:

$$-\frac{P'_o}{2}\times 1+\frac{P'_o}{2}\times D_{\lambda l}-\frac{M'_o\lambda}{2}\times 1+\frac{M'_o\lambda}{2}\times A_{\lambda l}=-Q'_A$$

The second term in the above expression is positive because it is due to the P_o' acting at point B. So, this load is to the right of the point of interest and hence for shear force, its effect will be negative to the formula. As the expression for shear force has a negative sign by default, this term is positive. Similar explanation applies to the next two terms of the expression.

$$-\frac{P'_o}{2}\times(1+D_{\lambda l})-\frac{M'_o\lambda}{2}\times(1-A_{\lambda l})=-Q'_A$$

In the above two equations (one for moment & one for shear force), there are only two unknowns: P_o' and M_o'. So, by solving the two equations, these two unknowns can be found out. After solving:

$$P'_o=4E_1\left[Q'_A(1+D_{\lambda l})+\lambda M'_A(1-A_{\lambda l})\right]$$

$$M'_o=\frac{-2E_1}{\lambda}\left[Q'_A(1+C_{\lambda l})+2\lambda M'_A(1-D_{\lambda l})\right]$$

$$\text{where, } E_1=\frac{1}{2(1+D_{\lambda l})(1-D_{\lambda l})-(1-A_{\lambda l})(1+C_{\lambda l})}$$

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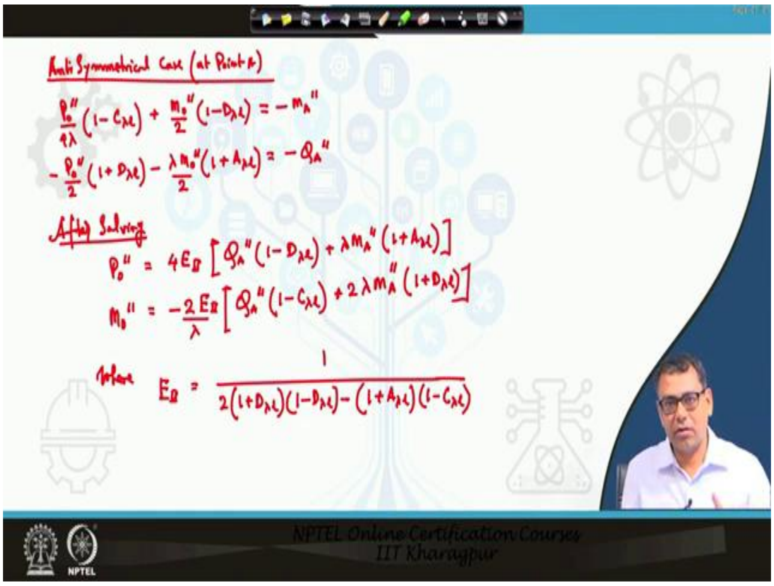

The expressions for the symmetric case were derived. Now for the anti symmetrical case, the similar procedure can be adopted. So, the expressions will be given directly.

$$\frac{P_o''}{4\lambda}\times(1-C_{\lambda l})+\frac{M_o''}{2}\times(1-D_{\lambda l})=-M_A''$$

$$-\frac{P_o''}{2}\times(1+D_{\lambda l})-\frac{M_o''\lambda}{2}\times(1+A_{\lambda l})=-Q_A''$$

After solving:

$$P_o''=4E_2\left[Q_A''(1-D_{\lambda l})+\lambda M_A''(1+A_{\lambda l})\right]$$

$$M_o''=\frac{-2E_2}{\lambda}\left[Q_A''(1-C_{\lambda l})+2\lambda M_A''(1+D_{\lambda l})\right]$$

where, $E_2=\dfrac{1}{2(1+D_{\lambda l})(1-D_{\lambda l})-(1+A_{\lambda l})(1-C_{\lambda l})}$

In the anti-symmetric case, as the end conditioning forces at the point B are acting in the opposite direction, the terms related to these forces will be different from that of the symmetrical case.

In the next class I will solve one example problem for the beam with finite length subjected to a concentrated load. Then we will see how to use these expressions and determined all the required quantities. Thank you.