

Soil Structure Interaction
Prof. Kousik Deb
Department of Civil Engineering
Indian Institute of Technology – Kharagpur

Lecture - 3
Bearing Capacity of Soil (Continued)

In the last class, I have discussed the Terzaghi's bearing capacity equation and how to incorporate the water table effect. Now, I will start from that point and then will continue with the other conditions.

(Refer Slide Time: 00:41)

Water table located at a depth b below the base of footing

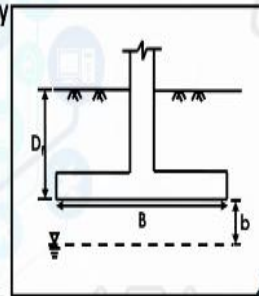
In this case, the surcharge term is not affected. However, the unit weight in the third term of bearing capacity equation is modified as

$$\bar{\gamma} = \gamma' + \frac{b}{B}(\gamma - \gamma')$$

$$q_u = c_u N_c + \gamma D_f N_q + \frac{1}{2} B \left[\gamma' + \frac{b}{B}(\gamma - \gamma') \right] N_\gamma$$

If $b = 0$, i.e., W/T at the base, $q_u = c_u N_c + \gamma D_f N_q + \frac{1}{2} B \gamma' N_\gamma$

If $b = B$, i.e., W/T at depth below B , $q_u = c_u N_c + \gamma D_f N_q + \frac{1}{2} B \gamma N_\gamma$



If water table is at a depth, b from the base of the foundation, the expression will be:

$$q_u = c_u N_c + \gamma D_f N_q + \frac{1}{2} \left[\gamma' + \frac{b}{B}(\gamma - \gamma') \right] B N_\gamma$$

If the water table is at the footing base, the γ in the term would be γ' . Now, if the water table is at a depth of B from the base of foundation, then no water table corrections are required which means the original Terzaghi's bearing capacity equation.

(Refer Slide Time: 01:19)

Ultimate bearing capacity analysis for clay soil (Skempton,1951):

$$\text{For } \phi = 0, \quad q_{mu} = c_u N_c$$

$$\text{For strip footing: } N_c = 5 \left(1 + 0.2 \frac{D_f}{B} \right) \quad \text{The maximum value of } N_c \text{ is } 7.50$$

$N_c \leq 7.5$

$$\text{For square and circular footing: } N_c = 6 \left(1 + 0.2 \frac{D_f}{B} \right)$$

The maximum value of N_c is 9

The next bearing capacity equation I will discuss is the Skempton bearing capacity equation, proposed by Skempton in 1951 which is applicable only for clay. Terzaghi's bearing capacity equation is applicable for c- ϕ soil, clay, sand and also the mixed c- ϕ soil, but this bearing capacity equation is applicable for clay. Generally, I would recommend this equation to design a foundation on clay.

As I mentioned, if this is applicable for only clays i.e., ($\phi = 0$), then the net ultimate bearing capacity will be $c_u N_c$. Skempton proposed that the value of N_c for a strip footing would be:

$$N_c = 5 \times \left(1 + 0.2 \frac{D_f}{B} \right) ; \text{where, } N_c \text{ cannot be more than } 7.5.$$

For square and circular footing, the expression for N_c is:

$$N_c = 6 \times \left(1 + 0.2 \frac{D_f}{B} \right) ; \text{where, } N_c \text{ cannot be more than } 9.$$

(Refer Slide Time: 03:06)

For rectangular footing :

$$N_c = 5.0 \left(1 + 0.2 \frac{D_f}{B} \right) \left(1 + 0.2 \frac{B}{L} \right) \quad \text{For } D_f/B \leq 2.5$$

$$N_c = 7.5 \left(1 + 0.2 \frac{B}{L} \right) \quad \text{For } D_f/B > 2.5$$

The analysis is valid for any value of D_f/B

Now for the rectangular footing, the N_c value we will be:

$$N_c = 5 \times \left(1 + 0.2 \frac{D_f}{B} \right) \left(1 + 0.2 \frac{B}{L} \right) \quad \text{For } D_f/B \leq 2.5$$

$$N_c = 7.5 \left(1 + 0.2 \frac{B}{L} \right) \quad \text{For } D_f/B > 2.5$$

Terzaghi's bearing capacity equation is valid for shallow foundation only where Skempton's bearing capacity equation is valid for any value of D_f/B .

(Refer Slide Time: 03:57)

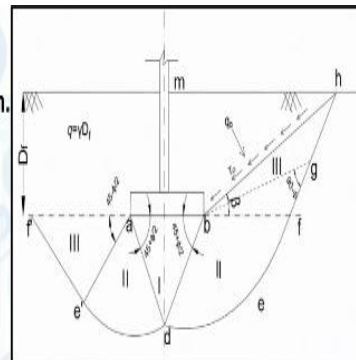
Meyerhof's Analysis :

- Bearing capacity for a strip footing at any depth.
- For shallow footing, $q_0 = \gamma D_f$

$$q_u = cN_c + q_0 N_q + \frac{1}{2} B \gamma N_\gamma$$

N_c, N_q, N_γ depends on roughness of base, depth of footing, and the shape of footing, in addition to the angle of shearing resistance ϕ'

β increases with an increase in depth D_f and is equal to 90° for deep foundation



Zone I - abd, elastic zone
 Zone II - bgd, zone of radial shear
 Zone III - bghm, zone of mixed shear in which shear varies between radial shear and plane shear



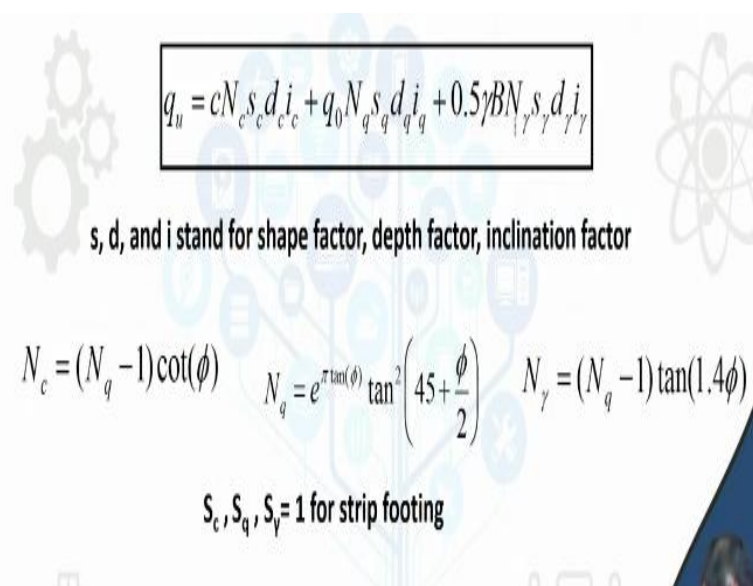
The third bearing capacity theory in our discussion is the Meyerhof's analysis okay. So, Meyerhof's analysis can be applied for. The Terzaghi's bearing capacity was initially applicable for strip footing but later on modified for circular, square and rectangular footing.

But if the loading is inclined or eccentric (not in the center), the Terzaghi's bearing capacity equation cannot be used.

The Meyerhof's bearing capacity factors: N_c , N_q , N_γ depends upon the roughness of the base, depth of footing, shape of footing in addition to angle of shearing, ϕ . These are the additional variables that are introduced in the Meyerhof's analysis. The soil below the footing according to this theory too, is divided in two 3 zones: first is the elastic zone, second is the zone of radial shear and the third is the zone of mixed shear in which shear varies from radial shear to plane shear.

Apart from this, there is one more basic difference between the failure surfaces proposed by Terzaghi and Meyerhof. Terzaghi's bearing failure surface reaches up to base of the footing only, whereas Meyerhof's failure surface reaches up to the ground surface.

(Refer Slide Time: 05:43)



$$q_u = cN_c s_c d_c i_c + q_0 N_q s_q d_q i_q + 0.5 \gamma B N_\gamma s_\gamma d_\gamma i_\gamma$$

$s, d, \text{ and } i \text{ stand for shape factor, depth factor, inclination factor}$

$$N_c = (N_q - 1) \cot(\phi) \quad N_q = e^{\pi \tan(\phi)} \tan^2 \left(45 + \frac{\phi}{2} \right) \quad N_\gamma = (N_q - 1) \tan(1.4\phi)$$

$S_c, S_q, S_\gamma = 1 \text{ for strip footing}$

The final expression proposed by Meyerhof is shown in the slide. The correction factors used are: s_c , d_c , i_c , s_q , d_q , i_q , s_γ , d_γ , i_γ . Here, s is the shape correction factor; d is the depth correction factor; i is the inclination correction factor (if the footing is inclined). The expressions for bearing capacity factors N_c , N_γ , N_q are also shown in the slide. For a strip footing, all the shape factors will be one.

(Refer Slide Time: 06:33)

Shape, depth, inclination factor for the Meyerhof's bearing capacity equation:

Factors	Value	For
Shape	$s_c = 1 + 0.2K_p \left(\frac{B}{L} \right)$	Any ϕ
	$s_q = s_\gamma = 1 + 0.1K_p \left(\frac{B}{L} \right)$	$\phi > 10^\circ$
	$s_q = s_\gamma = 1$	$\phi = 0^\circ$
Depth	$d_c = 1 + 0.2\sqrt{K_p} \left(\frac{D_f}{B} \right)$	Any ϕ
	$d_q = d_\gamma = 1 + 0.1\sqrt{K_p} \left(\frac{D_f}{B} \right)$	$\phi > 10^\circ$
	$d_q = d_\gamma = 1$	$\phi = 0^\circ$

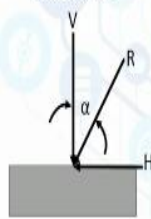
Bowles, 1997

Meyerhof has given different factors and expressions to determine them for any value of ϕ . Similarly, depth factor can also be calculated for different ϕ values.

(Refer Slide Time: 07:07)

Shape, depth, inclination factor for the Meyerhof's bearing capacity equation:

Factors	Value	For
Inclination	$i_c = i_q = \left(1 - \frac{\alpha^2}{90^\circ} \right)^2$	Any ϕ
	$i_\gamma = \left(1 - \frac{\alpha^2}{\phi^2} \right)^2$	$\phi > 0^\circ$
	$i_\gamma = 0$ For $\alpha > 0$	$\phi = 0^\circ$



$$K_p = \tan^2 \left(45^\circ + \frac{\phi}{2} \right)$$

α angle of resultant R measured from vertical

Bowles, 1997


Similarly the inclination factor expressions are also given where, α is the angle the load makes with the vertical. With the help of these equations, we can determine these inclination factors for different ϕ . The K_p used in these expressions is nothing but:

$$K_p = \tan^2 \left(45^\circ + \frac{\phi}{2} \right)$$

(Refer Slide Time: 07:49)

ϕ	N_c	N_q	N_γ
0	5.14	1.0	0.0
5	6.5	1.6	0.07
10	8.3	2.5	0.37
15	11	3.9	1.2
20	14.8	6.4	2.9
25	20.7	10.7	6.8
30	30.1	18.4	16.7
32	35.5	23.2	22.0
34	42.2	29.4	31.1
36	50.6	37.8	44.5
38	61.4	48.9	64.0

Ranjan and Rao, 1991



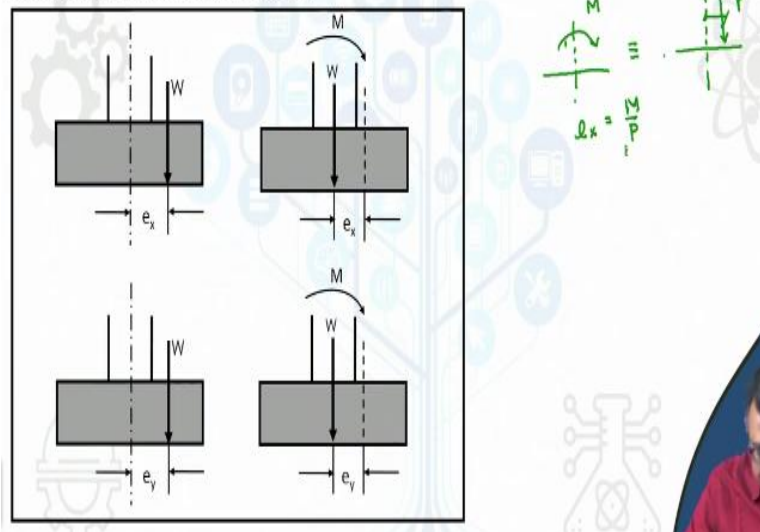
Meyerhof had also given a table for N_c , N_q , N_γ and as per this, if $\phi = 0$, N_c will be 5.14 whereas, in case of Terzaghi it was 5.7.

(Refer Slide Time: 08:10)

ϕ	N_c	N_q	N_γ
40	75.3	64.1	93.7
45	133.9	134.9	262.8
50	266.9	319.1	874.0

(Refer Slide Time: 08:15)

Eccentricity of loaded foundation:



Next in the Meyerhof's bearing capacity theory is the eccentric loading. If a load does not act upon the center of the footing, then the eccentricity it has along the width direction is taken as e_x and that along the length direction, e_y . This can also be considered as a moment acting on the footing. Also, if there is a moment, M acting upon the footing, it can be converted to eccentric loading. By this, the eccentricity, e_x or e_y can be calculated as: M/P , where P is the load.

So, eccentricity comes into the picture due to external moment, or sometimes, loading itself is eccentric. The eccentricity can either be e_x or e_y depending upon the direction of the moment.

(Refer Slide Time: 10:26)

Shape, depth, inclination factor for the Meyerhof's bearing capacity equation:

Factors	Value	For
Shape	$s_x = 1 + 0.2K_p \left(\frac{B'}{L} \right)$	Any ϕ
	$s_q = s_\gamma = 1 + 0.1K_p \left(\frac{B}{L} \right)$	$\phi > 10^\circ$
	$s_q = s_\gamma = 1$	$\phi = 0^\circ$
Depth	$d_c = 1 + 0.2\sqrt{K_p} \left(\frac{D_f}{B} \right)$	Any ϕ
	$d_q = d_\gamma = 1 + 0.1\sqrt{K_p} \left(\frac{D_f}{B} \right)$	$\phi > 10^\circ$
	$d_q = d_\gamma = 1$	$\phi = 0^\circ$

Bowles, 1997

$B' = B - 2e_x$
 $L' = L - 2e_y$
 $e = \frac{M}{P}$

When the load is eccentric, the full width of the footing cannot be considered, as the effective width or length which actually bears the load will be lesser than the original footing

dimensions. In that case, width should be considered as B' and not B (where, $B' = B - 2e_x$). Similarly for a rectangular footing, the dimensions will be B' and L' (where, $L' = L - 2e_y$). Finally, the effective area of the footing will reduce to $B' \times L'$. So, in the event of eccentric loading, if any, the corrected dimensions, B' and L' should be used to determine the correction factors proposed by Meyerhof.

Then if the moment is known, e_y can be calculated by dividing the moment with the load. The value of eccentricity obtained by this can be referred to as e' and depending upon the direction of the moment, it can either be e_x or e_y .

Sometimes while solving numerical problems, it may be mentioned in the question that the P or the vertical load is acting at a distance e_x or e_y from the center. It can be either e_x or e_y or both. But it should be kept in mind that B' and L' should be used to calculate the bearing capacity factors.

(Refer Slide Time: 13:32)

For strip footing: $B' = B - 2e_x$

For rectangular footing: $B' = B - 2e_x$
 $L' = L - 2e_y$

The effective area of footing $A' = B' \times L'$

The ultimate load bearing capacity of footing can be expressed as:

$$Q_u = q_u \times A'$$

$$q_u = cN_c s_c d_c i_c + q N_q s_q d_q i_q + 0.5 \gamma B N_\gamma s_\gamma d_\gamma i_\gamma$$

The diagrams illustrate a rectangular footing of width B and length L subjected to an eccentric load. The effective width B' and length L' are shown as the portion of the footing where the load is applied, with eccentricities e_x and e_y from the center.

Even in the bearing capacity equation, B' should only be used for eccentric loading. So that is also another important change okay. Another important aspect to be considered is that this equation can only be used if: $e_x \leq B/6$ and $e_y \leq L/6$. This restriction can be marked in form of a zone on the footing as shown in the slide and the bearing capacity equation is valid only if the load acts within this zone. This condition needs to be checked when the values of e_x and e_y are calculated.

(Refer Slide Time: 16:06)

IS code method (6403-1981)

$$q_{nu} = cN_c s_c d_c i_c + q(N_q - 1) s_q d_q i_q + 0.5 \gamma B N_{\gamma} s_{\gamma} d_{\gamma} i_{\gamma} W'$$

q = effective pressure at base

N_c, N_q, N_{γ} are the same as those given by Vesic

W' factor for water table

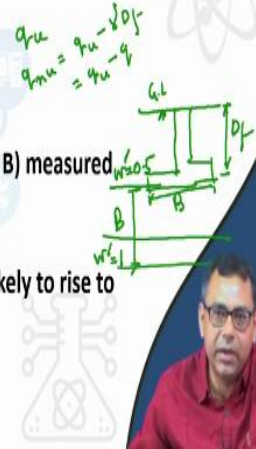
$W' = 1$, when water table is at or below a depth of $(D_f + B)$ measured

from the GL

$W' = 0.5$, when water table is located at a depth D_f or likely to rise to

the base of footing or above

W' can be linearly interpolated when $D_f < D_w < D_f + B$



The IS code, 6403-1981 is available which explains the process of determining bearing capacity. This method, indeed is similar to the Meyerhof theory, but the only difference is that here, net ultimate bearing capacity is given directly unlike that in the Meyerhof equation. In the Meyerhof theory, if the net ultimate bearing capacity is to be determined, the ultimate bearing capacity has to be calculated first and then q should be subtracted from that equation. But in IS code, this term itself is given in that form $(N_q - 1)$.

Here, the water table effect is incorporated here in the third term with W' , a factor for water table which is equal to 1 when water table is at or below a depth of $(D_f + B)$ measured from the ground level. W' will be 0.5 when water table is located at a depth D_f or likely to rise to the base of the footing or above. W' can also be linearly interpolated if it is located in between D_f and $(D_f + B)$.

So, if the water table is considered to be at the ground level, the bearing capacity roughly reduces by 50%.

(Refer Slide Time: 20:38)

ϕ	N_γ	ϕ	N_γ
0	0	40	109.4
5	0.4	45	271.3
10	1.2	50	762.84
15	2.6		
20	5.4		
25	10.9		
30	22.4		
32	30.2		
34	41		
36	56.2		
38	77.9		

The bearing capacity factor, N_γ used in the IS code is originally proposed by Vesic, and the other two factors: N_c , N_q are same as the Meyerhof bearing capacity factors.

(Refer Slide Time: 21:21)

Shape Factor:

S_c	$\left(1 + 0.2 \frac{B}{L}\right)$	Rectangular footing
	1.3	Square and Circular
S_q	$\left(1 + 0.2 \frac{B}{L}\right)$	Rectangular footing
	1.2	Square and Circular
S_γ	$\left(1 - 0.4 \frac{B}{L}\right)$	Rectangular footing
	0.8	Square
	0.6	Circular

IS code also gave the shape factors (S_c , S_q , S_γ) for rectangular, square and circular footings.

(Refer Slide Time: 21:32)

Depth Factor:

d_c	$1 + 0.2 \frac{D_f}{B} \tan\left(45^\circ + \frac{\phi}{2}\right)$	For any ϕ
d_q	$1 + 0.1 \frac{D_f}{B} \tan\left(45^\circ + \frac{\phi}{2}\right)$	$\phi > 10^\circ$
	1	$\phi < 10^\circ$
d_γ	$1 + 0.1 \frac{D_f}{B} \tan\left(45^\circ + \frac{\phi}{2}\right)$	$\phi > 10^\circ$
	1	$\phi < 10^\circ$

For different values of ϕ , values of different depth factors (d_c , d_q , d_γ) can be calculated.

(Refer Slide Time: 21:38)

Inclination Factor:

i_c	$i_c = i_q = \left(1 - \frac{\alpha^\circ}{90^\circ}\right)^2$
$i_q = i_\gamma$	$i_\gamma = \left(1 - \frac{\alpha^\circ}{\phi^\circ}\right)^2$

The inclination factors i_c , i_q , i_γ are also given in terms of α where α is the angle the inclined load makes with the vertical.

There are other bearing capacity equations available that are proposed by Vesic, Hanson, Peck. These were detailed in my previous course, foundation engineering.

So far we discussed that if we know the shear strength parameters, c or ϕ or both depending upon the type of soil, we can determine the load carrying capacity or bearing capacity of the foundation.

(Refer Slide Time: 21:50)

Bearing capacity of granular soils based on SPT (Standard Penetration Test)

Teng (1962)

$$q_{nu} = \frac{1}{6} [3N^2 BR'_w + 5(100 + N^2) D_f R_w] \quad \text{For strip footing}$$

$$q_{nu} = \frac{1}{3} [N^2 BR'_w + 3(100 + N^2) D_f R_w] \quad \text{For square and circular footing}$$

q_{nu} = net ultimate bearing capacity in kN/m²

N = average N value corrected for overburden pressure

D_f = depth of footing in m; if $D_f > B$ take $D_f = B$

Now, let us see how to calculate the bearing capacity if the SPT value is known. The expression proposed by Teng in 1962 is frequently used to determine the net ultimate bearing capacity by the equation:

$$q_{nu} = \frac{1}{3} [N^2 BR'_w + 3(100 + N^2) D_f R_w] \quad \text{For square and circular footing}$$

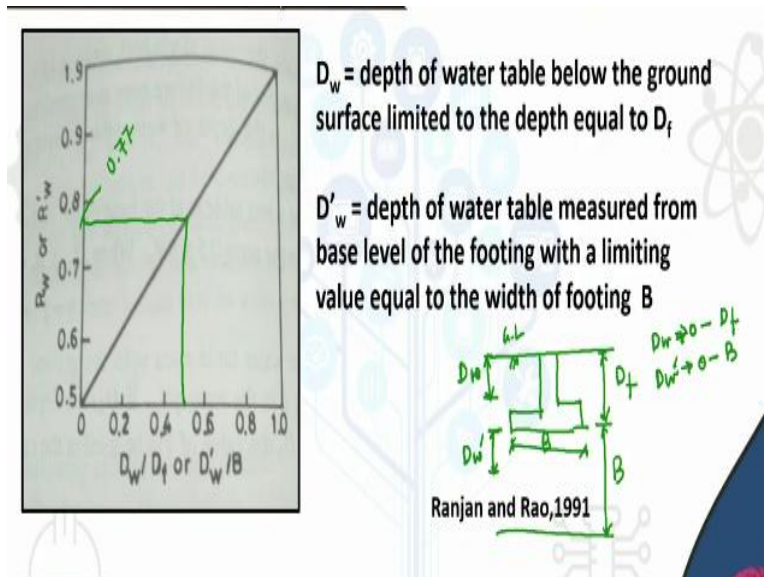
$$q_{nu} = \frac{1}{6} [3N^2 BR'_w + 5(100 + N^2) D_f R_w] \quad \text{For strip footing}$$

Where, N is the corrected SPT value, D_f is the depth of foundation, R_w' and R_w are the two correction factors for water table.

The above expression is empirical and hence the units should be remembered. (q_{nu} in kN/m²)

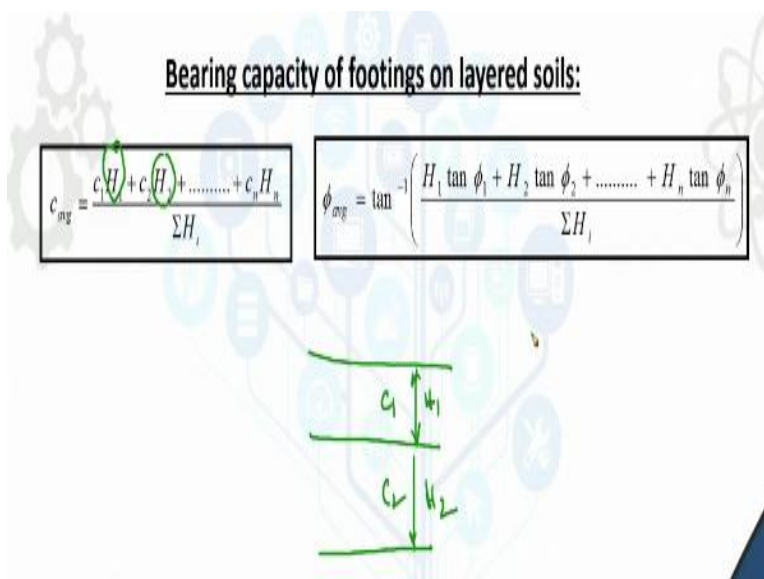
The N values used in the expression should have been corrected for overburden pressure. D_f is the depth of footing in meter and if $D_f > B$, take $D_f = B$.

(Refer Slide Time: 24:20)



The two water table correction factors (R_w and R'_w) depend upon two depth values D_w and D'_w defined as: D_w - depth of water table below ground surface, limited to D_f and D'_w - depth of water table measured from footing base limited to B . So, D_w varies from 0 to D_f and D'_w from 0 to B . The values of R_w and R'_w can be obtained by calculating D_w/D_f and D'_w/B respectively using the graph shown in the slide above.

(Refer Slide Time: 26:57)



The equations mentioned so far can be easily applied for homogeneous soil. But if the soil is layered, these equations cannot be directly used. In general, the weighted average of cohesion and friction values will be calculated. c_1 is the cohesion of the first layer, H_1 is the thickness of the first layer; c_2 is the cohesion of the second layer and H_2 is the thickness of the second layer.

All the layers which are within the influence zone of the footing should be considered for this weighted average for calculating both cohesion and friction angle.

(Refer Slide Time: 28:21)

Factors influencing bearing capacity :

i) For $c_u = 0$

$$q_u = qN_q + 0.5\gamma B N_\gamma$$

- a) Relative density or ϕ
- b) Width of the footing
- c) Depth of the footing
- d) Unit weight of the soil
- e) Position of ground water

Now, let us look into the factors influencing bearing capacity. If the footing rest in a soil for which $c = 0$ (cohesionless soil), the first term in the bearing capacity equation, cN_c will be 0. So, the equation will be $qN_q + 1/2 (\gamma B N_\gamma)$. So, the factors affecting this equation would be: ϕ , width of footing (B), unit weight of soil (γ), depth of footing (D_f) [Since, $q = \gamma D_f$] and the position of water table.

(Refer Slide Time: 29:27)

ii) For $\phi = 0$

$$q_u = c_u N_c + q$$

- a) The bearing capacity of footing on a cohesive soil is unaffected by the width of footing
- b) The net ultimate bearing capacity ($q_{nu} = N_c c_u$) is not affected by the depth of foundation.
- c) For $\phi = 0$, $N_c = 5.14$ (smooth base) and 5.7 (rough base)

Next, for the case of cohesive soils ($\phi = 0$), where the bearing capacity equation becomes: $(c_u N_c + q)$ as $N_q = 1$ and $N_\gamma = 0$ if $\phi = 0$. So, the third term in the bearing capacity equation vanishes and as $N_q = 1$, the second term will only be q . Henceforth, the q net ultimate (q_{nu})

will just be $c_u N_c$. From this equation of net ultimate bearing capacity, it can be concluded that neither the width of footing or depth of footing have any effect on q_{nu} . But, depth of footing affects the ultimate bearing capacity. So, if $\phi = 0$, Meyerhof had given the value of $N_c = 5.14$ and 5.7 for footing with smooth base and rough base respectively.

In the next class, I will discuss about how to use these expressions to determine the bearing capacity of foundation. Thank you.