Soil Structure Interaction
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Lecture 29
Beams on Elastic Foundation (Contd.,)

In the last class I determined the deflection at point A, B and D for a semi-infinite beam subjected to UDL. In this class I will continue same example problem by determining the deflection at point C, the centre point of the loaded region.

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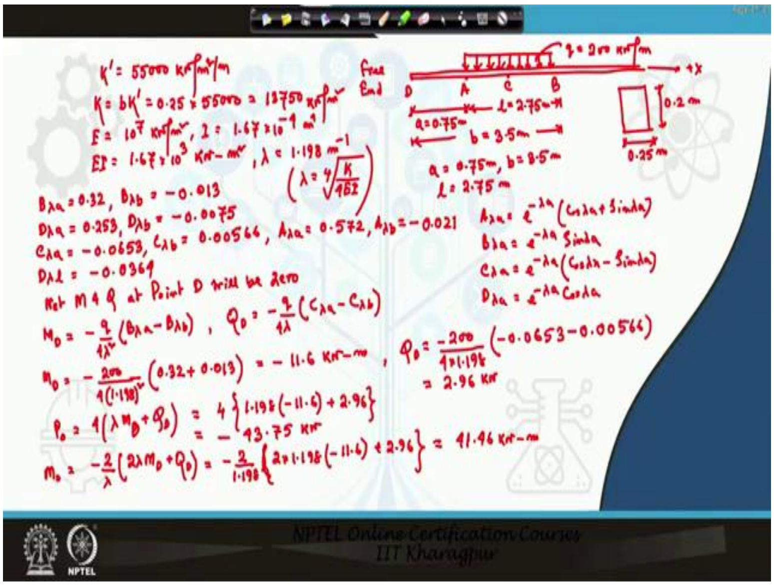

This was the problem for which the deflections at point A, B and D were calculated. Let us determine the deflection at C point first. The deflection at point D is -0.11 mm, deflection at point A is 7.83 mm and the deflection at point B is 7.53 mm. When considering deflection at point C, it is the centre point of load and so the distance from A to C and B to C is equal to l/2.

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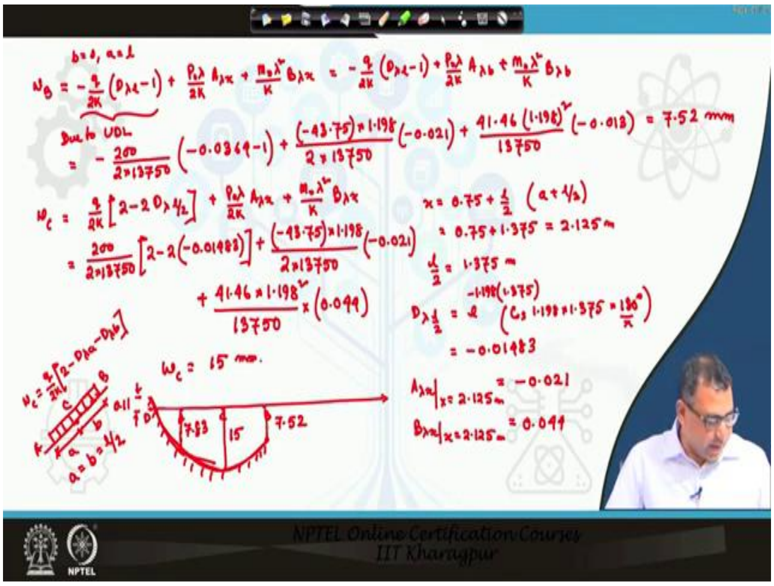

Now the point of interest is within the loaded region and so the case-1 expressions for the UDL should be considered. The deflection expression for a point within the loaded region is:

$$w_C = \frac{q}{2k}\left(2 - D_{\lambda a} - D_{\lambda b}\right)$$

Here, a & b refers to the distance from the point of interest to the ends of UDL. But, as the point C is the centre point of the UDL, a = b = l/2.

$$w_C = \frac{q}{2k}\left(2 - D_{\lambda l/2} - D_{\lambda l/2}\right)$$

$$\Rightarrow w_C = \frac{q}{2k}\left(2 - 2D_{\lambda l/2}\right)$$

Now, considering the end conditioning forces, the deflection at C will be:

$$\Rightarrow w_C = \frac{q}{2k}\left(2 - 2D_{\lambda l/2}\right) + \frac{P_o \lambda}{2k} A_{\lambda x} + \frac{M_o \lambda^2}{k} B_{\lambda x}$$

The x value is the distance between the point where the end conditioning forces are acting and the point C.

l/2 = 2.75/2 = 1.375 m

x = a + l/2 = 0.75 + (2.75/2) = 2.125 m

$$D_{\lambda l/2} = e^{-1.198 \times 1.3750}\left(\cos 1.198 \times 1.375 \times \frac{180}{\pi}\right) = -0.01483$$

$$A_{\lambda x}\Big|_{x=2.125\,m} = -0.021$$

$$B_{\lambda x}\Big|_{x=2.125\,m} = 0.044$$

$$w_C = \frac{200}{2\times 13750}\left[2-2(-0.01488)\right]+\frac{(-48.75)\times 1.198}{2\times 13750}(-0.021)+\frac{41.46\times 1.198^2}{13750}\times(0.044)$$

$$w_C = 15 \text{ mm}$$

Now we have the deflection values at 4 different points. From these, the deflection profile can be drawn as shown in the above slide. It can be seen that the deflection at the free end is found to be negative and hence it is above the original position of the beam.

The procedure to determine only the deflection was shown. The other 3 quantities can also be found out by following this procedure.

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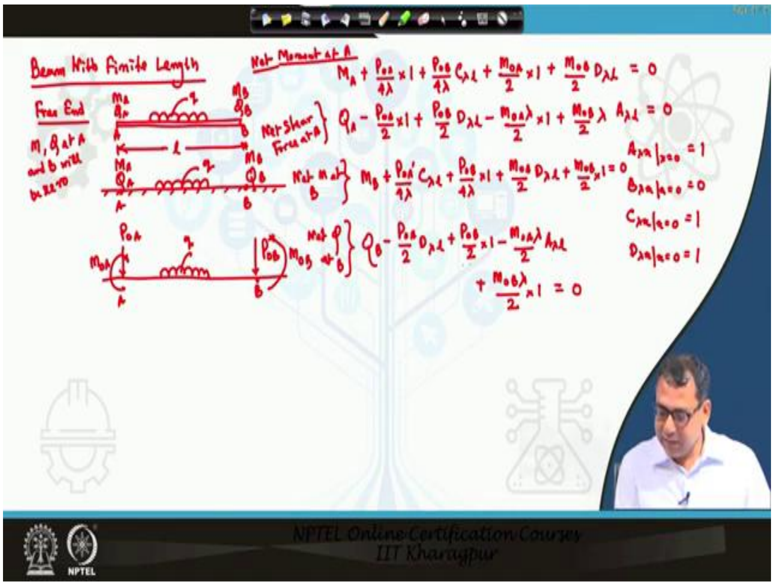

Till now the infinite beam and semi infinite beam have been discussed. The next concept is the analysis of beam with finite length. In case of infinite beam, the beam was infinite on both the sides from the loading point (right side & left side) and in a semi infinite beam, only one side was infinite and the other side was finite. But in beam with finite length both the sides of the beam will be finite and it means that the length of the beam is now finite.

In the first case, the free end condition which is very common will be discussed. A finite beam of length, l and free ends A & B is shown in the slide above. Now, consider that a UDL of intensity q is acting over the beam over a length lesser than l (not over the entire length of the beam). As the end condition is free for both the ends, the bending moment and shear force will be zero at both the ends. So, by considering the beam as infinite beam, the bending moment and shear force that would be induced at the ends due to the external load should be calculated.

Then, the value of end conditioning forces required to nullify the bending moment and shear force at the ends should be calculated. In semi-infinite beam, this procedure was applied only to one of the ends as there the other end was truly infinite, but here this procedure should be applied to both the ends. By considering the beam as infinite, the bending moment & shear force developed are termed as: M_A & Q_A at point A and M_B & Q_B at point B (shown in second figure).

But to satisfy the end conditions, the net bending moment and shear force should be 0 at points A and B. So, the end conditioning forces, P_{oA}, M_{oA}, P_{oB} and M_{oB} should be applied at both the ends to satisfy this (shown in third figure). Remember that the end conditioning forces of point A, P_{oA} & M_{oA} are at an infinitely small distance to the left side of the point A. Similarly the end conditioning forces of point B, P_{oB} & M_{oB} are at an infinitely small distance towards the right side of the point B.

Now, the expression for net bending moment at A can be written as:

$$M_A + \frac{P_{oA}}{4\lambda} \times C_{\lambda x} + \frac{M_{oA}\lambda}{2} \times D_{\lambda x} + \frac{P_{oB}}{4\lambda} \times C_{\lambda x} + \frac{M_{oA}\lambda}{2} \times D_{\lambda x}$$

In the above expression, the x value for P_{oA} & M_{oA} as they are acting at point A. The x value for P_{oB} & M_{oB} will be l because they are acting at the other end of the beam and the beam length is l. This is the value of net bending moment at point A which should be zero to satisfy the (free) end condition.

$$M_A + \frac{P_{oA}}{4\lambda} \times 1 + \frac{M_{oA}\lambda}{2} \times 1 + \frac{P_{oB}}{4\lambda} \times C_{\lambda l} + \frac{M_{oB}\lambda}{2} \times D_{\lambda l} = 0$$

$$\text{Since: } A_{\lambda x}\big|_{x=0} = 1; \quad B_{\lambda x}\big|_{x=0} = 0; \quad C_{\lambda x}\big|_{x=0} = 1; \quad D_{\lambda x}\big|_{x=0} = 1$$

Similarly, the net shear force at A will be:

$$Q_A - \frac{P_{oA}}{2} \times 1 - \frac{M_{oA}\lambda}{2} \times 1 + \frac{P_{oB}}{2} \times D_{\lambda l} + \frac{M_{oB}\lambda}{2} \times A_{\lambda l} = 0$$

The formulae for shear force due to point load and concentrated moment have a negative sign. But in the above expression, the terms with P_{oB} & M_{oB} are positive because the point of interest is to the left of the point where these forces are acting. So, negative of the original formula should be considered and hence the last two terms are positive. This problem was not there for the bending moment expression as irrespective of the side the point of interest is on, no negative sign need to be applied to the formula in case of bending moment and deflection.

Similarly, the net bending moment at point B:

$$M_B + \frac{P_{oA}}{4\lambda} \times C_{\lambda l} + \frac{M_{oA}\lambda}{2} \times D_{\lambda l} + \frac{P_{oB}}{4\lambda} \times 1 + \frac{M_{oB}\lambda}{2} \times 1 = 0$$

Similarly, the net shear force at point B:

$$Q_B - \frac{P_{oA}}{2} \times D_{\lambda l} - \frac{M_{oA}\lambda}{2} \times A_{\lambda l} + \frac{P_{oB}}{2} \times 1 + \frac{M_{oB}\lambda}{2} \times 1 = 0$$

These are the 4 cases on net bending moment and shear force for the two different edges. If these equations are solved, the different quantities can be obtained, but it is very complicated to solve these equations. This process was easy in case of semi-infinite beam, but here, using the same process is very difficult. So a simplified procedure will be adopted to solve for the finite beam.

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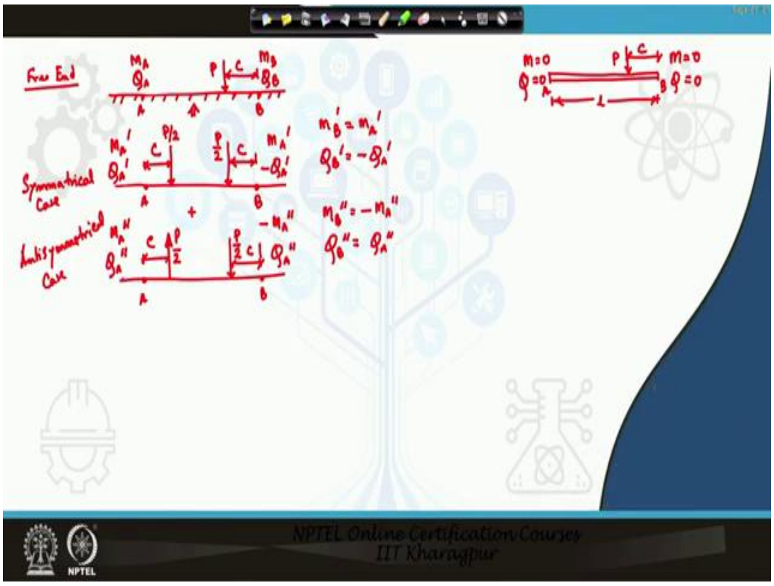

The finite beam with point load acting on it will be considered as the infinite beam as usual. Then the simplified procedure starts. Now, A and B are the ends of the finite beam and a point load of P acts at a distance of ‘c’ from the point B as shown in the first figure above. As this is considered infinite, bending moment and shear force will develop at both the ends and are termed as: M_A & Q_A at point A and M_B & Q_B at point B. Now, this condition is split into two cases: Symmetric case and Anti-symmetric case.

In the symmetric case, a load of P/2 is assumed to act from a distance ‘c’ from the point B (same point where P is acting) along with another P/2 load at a distance ‘c’ from the point A (second figure). In the anti-symmetric case, the loading condition is almost similar to that of the symmetric case, but the P/2 load near to point A acts in the opposite direction or upwards. This upward force is considered because, there is no force acting at that point and so the P/2 force considered in the symmetric case should be nullified.

In the symmetric case, the loading condition is also symmetric as both the loads of equal magnitude are acting at equal distances from the ends and are in the same direction. In this condition, the bending moment and shear force developed at the ends are termed as: M_A' & Q_A' at point A and M_B' & Q_B' at point B.

Where: $M_B' = M_A'$ and $Q_B' = -Q_A'$

In the anti-symmetric case, both the loads are of equal magnitude and are acting at equal distances from the ends, but are in the opposite direction. In this condition, the bending moment and shear force developed at the ends are termed as: M_A'' & Q_A'' at point A and M_B'' & Q_B'' at point B.

Where: $M_B'' = -M_A''$ and $Q_B'' = Q_A''$

In the next class I will discuss how to determine the other quantities using these two loading conditions for a finite beam subjected to a concentrated load. Thank you.