000Soil Structure Interaction
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Lecture 28
Beams on Elastic Foundation (Contd.,)

In this class I will discuss one example problem where a semi infinite beam is subjected to uniformly distributed loading. In the last class I solved one problem where the beam was subjected to one concentrated load.

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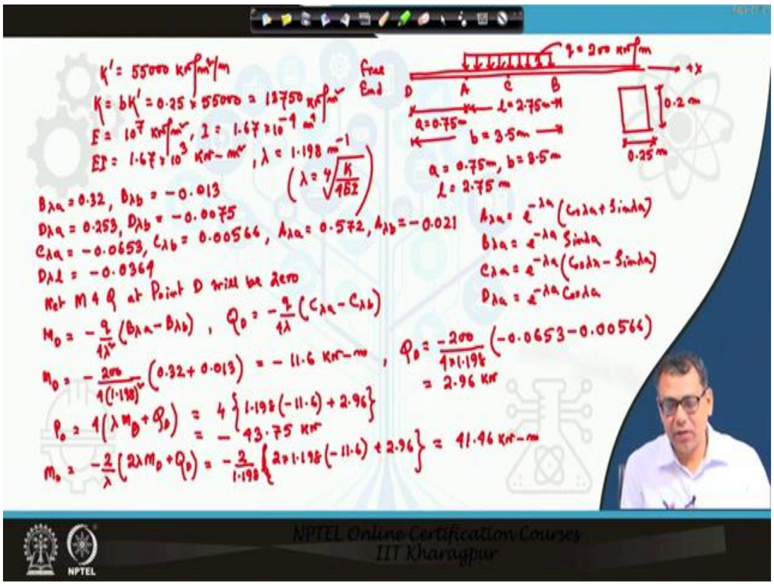

In the problem, the semi-infinite beam has its finite end towards left and its infinite end towards right which is indicated as +x direction. The finite end of the beam is free. The UDL of intensity, q = 200 kN/m is placed on the beam and the left side end of the UDL is at point A and the right side end is at point B. The distance between the points: D & A is a = 0.75 m, D & B is b = 3.5 m. The length of the UDL or the distance between A & B is l = 2.75 m and the centre of the UDL is considered as point C. First, the deflection will be determined at all these four points, A, B, C and D. After that the other quantities will be determined.

The k' value is same as that of the previous problem 55000 $kN/m^2/m$, b = 0.25 and h = 0.2 m. So, the value of k is 13750 kN/ m^2 (0.25 × 55000). EI = 1.67×10^3 kN-m^2. As all these values are same, the λ value will also be the same = 1.198 m^{-1}. $\left\{\lambda = \sqrt[4]{\frac{k}{4EI}}\right\}$

The values of all the coefficients should be determined for all possible combinations. The coefficients are:

$$e^{-\lambda a}\left(\cos\lambda a+\sin\lambda a\right)=A_{\lambda a}$$

$$e^{-\lambda a}\left(\sin\lambda a\right)=B_{\lambda a}$$

$$e^{-\lambda a}\left(\cos\lambda a-\sin\lambda a\right)=C_{\lambda a}$$

$$e^{-\lambda a}\left(\cos\lambda a\right)=D_{\lambda a}$$

As the values of a, b and l are already defined, the appropriate values can be substituted in the above expressions to find out all the four quantities. The values will be given directly as they are calculated many times in the previous lectures:

$A_{\lambda a}$ = 0.572; $A_{\lambda b}$ = -0.021

$B_{\lambda a}$ = 0.32; $B_{\lambda b}$ = -0.013

$D_{\lambda a}$ = 0.253; $D_{\lambda b}$ = -0.0075

$C_{\lambda a}$ = -0.0653; $C_{\lambda b}$ = 0.00566

$D_{\lambda l}$ = -0.0364

As the end condition is free here, the moment and shear force at that point will be zero. So, considering this as an infinite beam, calculate the moment and shear force at D due to the UDL. For an infinite beam under UDL, the expressions for all the four quantities for 3 different cases were given already.

The case-1 was when the point of interest is within the loaded region, case-2 was when the point of interest is to the left side of loaded region and the case-3 was when point of interest is to the right side of the loaded region. Here the bending moment and shear force are to be determined at point D, which is to the left side of the loaded region. So, now the expressions given under case-2 (for infinite beam) should be considered. The expression for bending moment is:

$$M_D=-\frac{q}{4\lambda^2}\left(B_{\lambda a}-B_{\lambda b}\right)$$

$$\Rightarrow M_D=-\frac{200}{4\times1.198^2}(0.32+0.013)=-11.6kN-m$$

The expression for shear force is:

$$Q_D=-\frac{q}{4\lambda}\left(C_{\lambda a}-C_{\lambda b}\right)$$

$$Q_D = -\frac{200}{4 \times 1.198}\left(-0.0653 - 0.00566\right) = 2.96kN$$

Now, the end conditioning forces should be determined and the expressions for free end are:

$$P_o = 4\left(\lambda M_D + Q_D\right) = 4\left[1.198\left(-11.6\right) + 2.96\right] = -43.75kN$$

$$M_o = -\frac{2}{\lambda}\left(2\lambda M_D + Q_D\right)$$

$$\Rightarrow M_o = -\frac{2}{1.198}\left[2 \times 1.198\left(-11.6\right) + 2.96\right] = 41.46kN - m$$

Now, these values can be checked by calculating the net bending moment and net shear force at point D.

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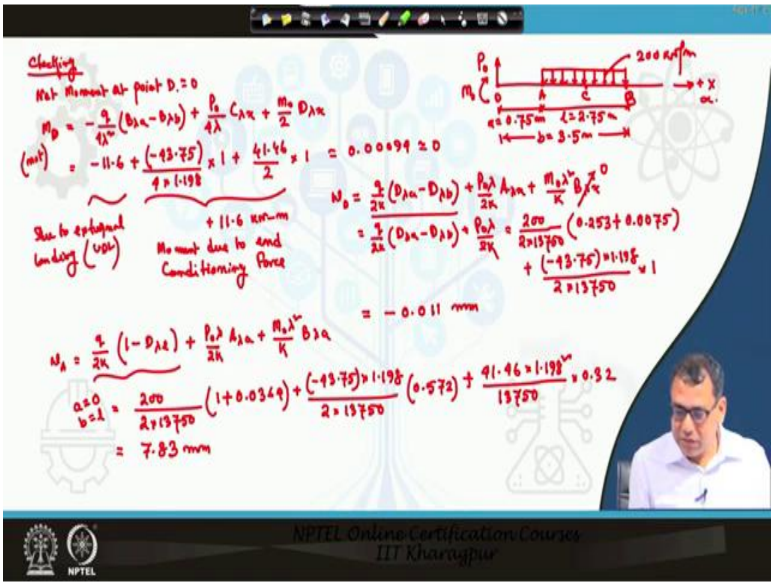

First, the net moment at point D will be checked considering all the forces acting on the beam:

$$M_D = -\frac{q}{4\lambda^2}\left(B_{\lambda a} - B_{\lambda b}\right) + \frac{P_\circ}{4\lambda}C_{\lambda x} + \frac{M_\circ}{2}D_{\lambda x}$$

$$M_D = \underbrace{-11.6}_{\text{Due to external load (UDL)}} + \underbrace{\frac{-43.75}{4 \times 1.198} \times 1 + \frac{41.46}{2} \times 1}_{\substack{+11.6 \\ \textit{Due to end conditioning forces}}} = 0.00094 \approx 0$$

As the point of interest is D, the x value for the end conditioning forces, P_o & M_o will be 0. Another way to check the end conditioning forces is to calculate the moment due to the end conditioning forces (+11.6) alone and see if that value is equal in magnitude and opposite sign to the moment (-11.6) induced due to the external load. Similarly, the net shear force can also be checked.

The deflection at D due to all the three forces acting on the beam is:

$$w_D = \frac{q}{2k}\left(D_{\lambda a} - D_{\lambda b}\right) + \frac{P_o\lambda}{2k}A_{\lambda x} + \frac{M_o\lambda^2}{k}B_{\lambda x}$$

$$\Rightarrow w_D = \frac{200}{2\times 13750}\left(0.253 + 0.0075\right) + \frac{(-43.75)\times 1.198}{2\times 13750}\times 1 = -0.011mm$$

Now the deflection at point A which is the left side end of the UDL, will be calculated.

$$w_A = \frac{q}{2k}\left(\underbrace{1}_{a=0} - \underbrace{D_{\lambda l}}_{b=l}\right) + \frac{P_o\lambda}{2k}A_{\lambda a} + \frac{M_o\lambda^2}{k}B_{\lambda a}$$

$$\Rightarrow w_A = \frac{200}{2\times 13750}\left(1 + 0.0364\right) + \frac{\left(-43.75\right)\times 1.198}{2\times 13750}(0.572) + \frac{41.46\times 1.198^2}{13750}\times 0.32 = 7.83mm$$

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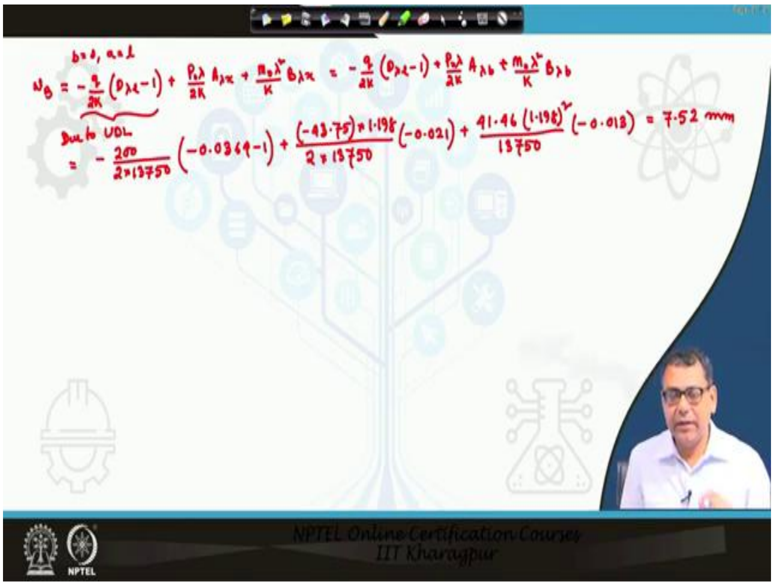

Next, the deflection at point B, the right side edge of the UDL, will be calculated. In this case, the $D_{\lambda b}$ term in the deflection due to load will be 1 as b = 0 and $D_{\lambda a}$ will be $D_{\lambda l}$ as a = l.

$$w_B = \underbrace{-\frac{q}{2k}(D_{\lambda l} - 1)}_{\text{Due to UDL}} + \frac{P_o\lambda}{k}A_{\lambda b} + \frac{M_o\lambda^2}{k}B_{\lambda b}$$

$$w_B = -\frac{200}{2\times 13750}\left(-0.0364 - 1\right) + \frac{(-43.75)\times 1.198}{2\times 13750}(-0.021) + \frac{41.46\times 1.198^2}{13750}\left(-0.018\right) = 7.52mm$$

The first term is due to the UDL and the next two terms are due to the end conditioning forces.

Till now the deflections at point A, point B and point D are determined. In the next class I will first determine the deflection at point C which is the centre point of the loaded region and then I will draw the deflection profile for this particular beam under this loading and the hinge condition. Thank you.