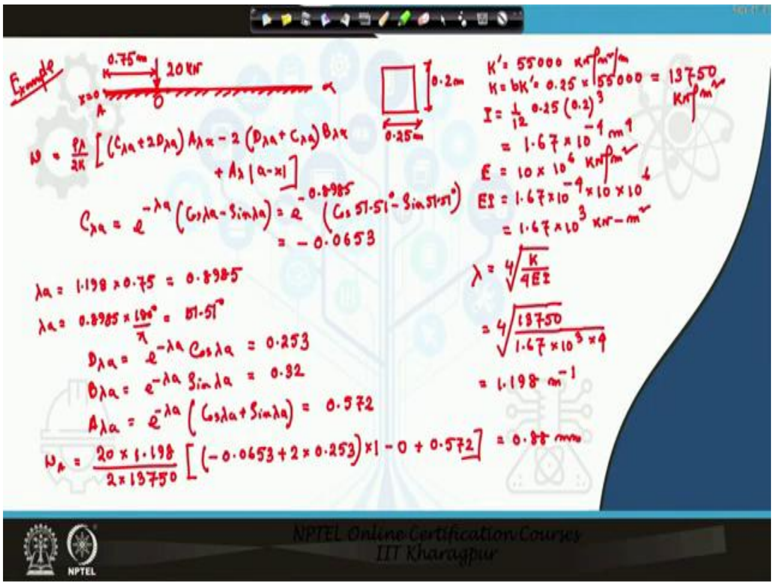
Soil Structure Interaction

Prof. Kousik Deb
Department of Civil Engineering
Indian Institute of Technology- Kharagpur

Lecture 27
Beams on Elastic Foundation (Contd.,)

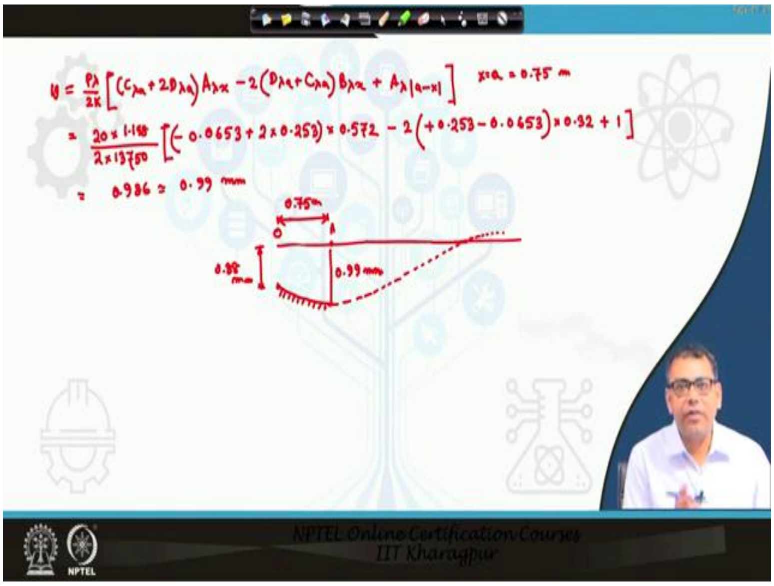
In the last class I discussed an example problem where a semi-infinite beam was subjected to a concentrated load and the end condition was free. Today I will continue the same problem.

(Refer Slide Time: 00:42)



This was the problem showing the semi infinite beam under a concentrated load of 20 kN applied at a distance of 0.75 m from the free end. The deflection of at A was calculated to be 0.88 mm. Now using the same expression the deflection at O will be determined.

(Refer Slide Time: 01:21)



Using the same expression of deflection:

$$w = \frac{P\lambda}{2k}\left[A_{\lambda|a-x|} + (C_{\lambda a} + 2D_{\lambda a})A_{\lambda x} - 2(C_{\lambda a} + D_{\lambda a})B_{\lambda x}\right]$$

At point O, x = a = 0.75 m

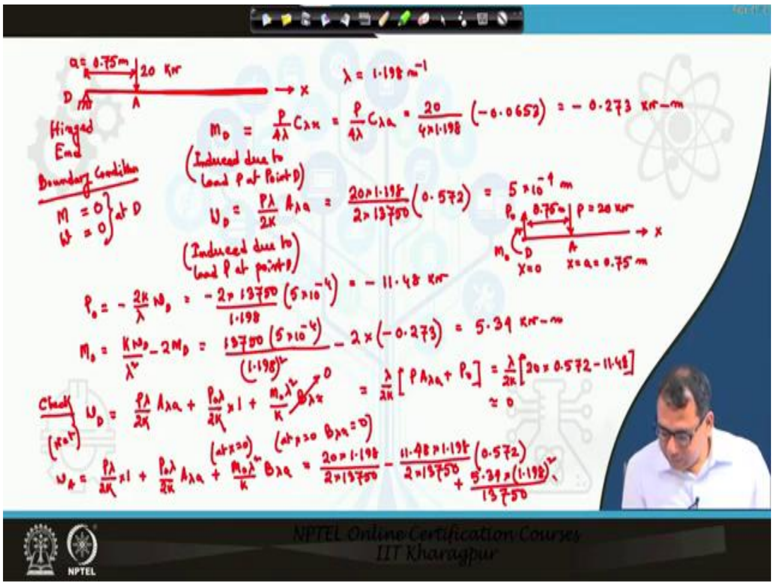
$$w_O = \frac{20 \times 1.198}{2 \times 13750}[(-0.0653 + 2 \times 0.253) \times 0.572 - 2(0.253 - 0.0653) \times 0.32 + 1]$$

$$w_O = 0.986 \approx 0.99mm$$

Here, deflection at two points is calculated. If the deflection profile should be drawn, the deflection should be calculated at more points. From that, a pattern similar to the one shown in the above slide can be developed accurately.

Next let us solve the same problem under different boundary condition.

(Refer Slide Time: 08:01)



The boundary condition for the previous example was free end which will now be changed to hinged end. There is a small change in naming the points here. Consider the hinged end as point D and the point where the 20 kN load is acting as A. The distance between these two points is 0.75 m. As all conditions other than end condition are same, λ and EI values will also be same as that of calculated in the previous problem i.e., EI = 1.67 × 10^3 kN-m^2 & λ = 1.198 m^{-1}.

For a hinged end, the boundary conditions are that the bending moment and deflection are zero. So imagining this beam as an infinite beam, first calculate the bending moment and deflection induced due to the point load and then apply end conditioning forces to make them zero.

The bending moment induced at D due to the point load is:

$$M_D = \frac{P}{4\lambda} C_{\lambda x} = \frac{P}{4\lambda} C_{\lambda a}$$

$$\Rightarrow M_D = \frac{20}{4\times 1.198}(-0.0653) = -0.273 kN-m$$

The deflection at D due to the point load is:

$$w_D = \frac{P}{2k} A_{\lambda a} = \frac{20}{2\times 13750}(0.572) = 5\times 10^{-4} m$$

To make these M_D and w_D values zero, two end conditioning forces, P_o and M_o should be applied at the point D. The expressions for these end conditioning forces were already derived.

$$P_{\circ} = -\frac{2k}{\lambda} w_D = -\frac{2\times 13750}{1.198}(5\times 10^{-4}) = -11.48 kN$$

$$M_{\circ} = \frac{k w_D}{\lambda^2} - 2 w_D = \frac{13750\times(5\times 10^{-4})}{1.198^2} - [2\times(-0.273)] = 5.34 kN-m$$

Here the value of P_o is negative which shows that the assumed direction of P_o is wrong and it acts upwards. As the M_o value is positive, the direction of it is correct.

Now it has to be checked whether these end conditioning forces, P_o & M_o are serving the purpose in making the deflection and moment zero at the hinge. The check can be made by calculating the net deflection and net moment at D.

The forces acting on the beam are: P = 20 kN, P_o = 11.48 kN and M_o = 5.34 kN-m. The net deflection due to all these forces should be zero.

$$w_D = \frac{P\lambda}{2k} A_{\lambda a} + \frac{P_{\circ}\lambda}{2k}\times 1 + \underbrace{\frac{M_{\circ}\lambda^2}{k} B_{\lambda a}}_{0}$$

The last term in the above expression is 0 as $B_{\lambda x}$ at x = 0 is 0.

$$\Rightarrow w_D = \frac{\lambda}{2k}(P A_{\lambda a} + P_{\circ})$$

$$\Rightarrow w_D = \frac{\lambda}{2k}(20\times 0.572 + 11.48) \approx 0$$

As the net deflection is zero, the end conditioning forces are satisfying. Similarly the net moment at D should also be zero that can be checked.

In the last problem, as the end condition was free, the deflection was determined at the end. But here, as the end is hinged, the deflection will be zero at the end. So, only the deflection under the point load (point A) will be found out:

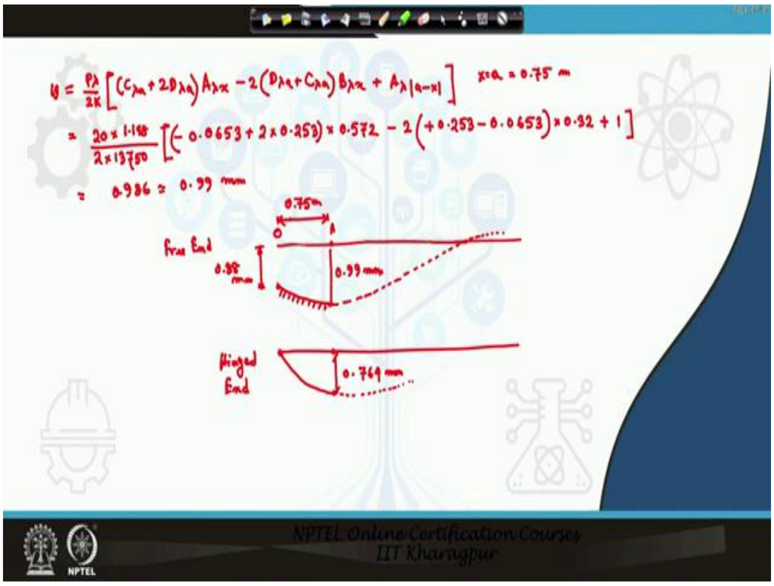
$$w_A = \frac{P\lambda}{2k}\times 1 + \frac{P_o\lambda}{2k}A_{\lambda a} + \frac{M_o\lambda^2}{k}B_{\lambda a}$$

$$\Rightarrow w_A = \frac{20\times 1.198}{2\times 13750} - \frac{11.48\times 1.198}{2\times 13750}(0.572) + \frac{5.34\times 1.198^2}{13750}\times 0.32 = 0.764mm$$

As the deflection is calculated below the point load, the x value for the point load, P will be 0 and the x value for the end conditioning forces, P_o & M_o will be equal to 'a'.

So, keeping all other conditions same, if the end condition is changed to hinge from a free end condition, the deflection under the point load reduced from 0.99 mm to 0.764 mm.

(Refer Slide Time: 25:00)



If this deflection profile is drawn for a fixed end beam keeping all other conditions same, the deflection below the point load will be lesser than 0.746 mm. These are the two different end conditions for the same beam under same loading conditions. Similarly, the deflection for a beam with fixed end condition can also be determined. So, next class I will solve another problem where I will consider one UDL.

Here I solved the problem considering a concentrated load, but in the next class I will solve a problem considering UDL on the beam. I will determine the deflection at different points under that UDL. Thank you.