Soil Structure Interaction
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Lecture 26
Beams on Elastic Foundation (Contd.,)

In the last class I discussed about the two end conditions in semi-infinite beam, free end and hinged end. In today's class I will discuss about the fixed end condition and then will solve a numerical problem for the semi infinite beam.

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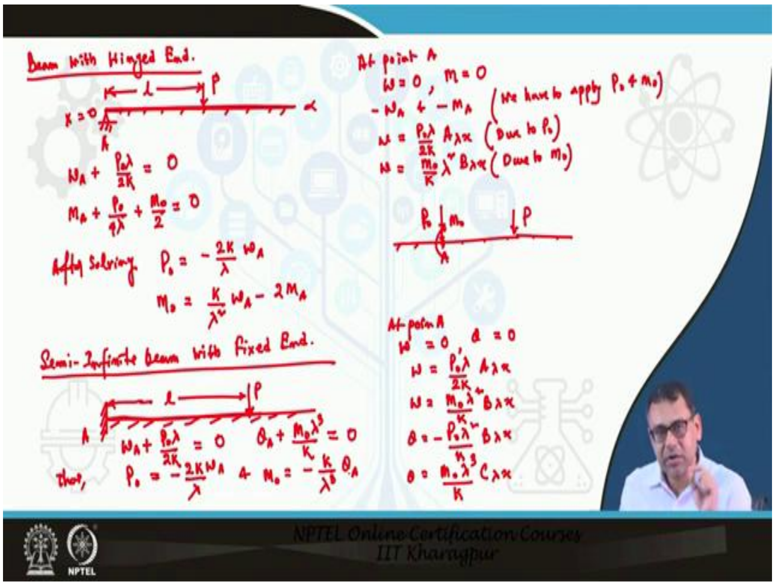

Before continuing with the semi infinite beam concept, there is a correction in the formula for triangular loading condition on infinite beam. The deflection at point A, the end of the triangular load where load intensity is zero is given below (*In lecture 24, the last term was given as* $-2\lambda l C_{\lambda l}$):

$$w_A = \frac{q_0}{4\lambda l k}\left(1 - C_{\lambda l} - 2\lambda l D_{\lambda l}\right)$$

For the semi-infinite beam with hinged end, the expressions for P_o and M_o are given above. The next end condition in the semi infinite beam is the fixed end. The fixed end is named as point A and a point load, P is considered to be acting on the beam at a distance of ‘l’ from the point A. The boundary conditions for a fixed end are that moment & shear force exists, but the deflection & slope will be 0.

The same procedure followed for the last two end conditions should be repeated here. Considering this semi-infinite beam as an infinite beam, find out the deflection and slope at A.

Then apply the end conditioning forces, P_o and M_o to make them zero. The deflection and slope due to the end conditioning forces will be:

$$w_{P_o} = \frac{P_o \lambda}{2k} A_{\lambda x}; \qquad \theta_{P_o} = -\frac{P_o \lambda^2}{k} B_{\lambda x} \ \text{\{due to the } P_o\text{\}}$$

$$w_{M_o} = \frac{M_o \lambda^2}{k} B_{\lambda x}; \qquad \theta_{M_o} = \frac{M_o \lambda^3}{k} C_{\lambda x} \ \text{\{due to } M_o\text{\}}$$

So, the net deflection at point A will be the sum of the deflection due to P (w_A), deflection due to P_o (w_{Po}) and the deflection due to M_o (w_{Mo}). But, w_{Mo} will be zero at point A because in its expression the $B_{\lambda x}$ term is present and $B_{\lambda x} = 0$ if x = 0 (i.e., the deflection at the point of action of a concentrated moment is zero). So, the net deflection at A is:

$$w_A + \frac{P_o \lambda}{2k} = 0$$

Similarly the net slope at A will be:

$$\theta_A + \frac{M_o \lambda^3}{k} = 0$$

The slope to P_o (θ_{Po}) will be zero at point A and the slope due to moment (θ_{Mo}) is considered along with the slope due to point load, P (θ_A).

The three end conditions under which the end conditioning forces P_o and M_o should be applied in a semi-infinite beam are discussed.

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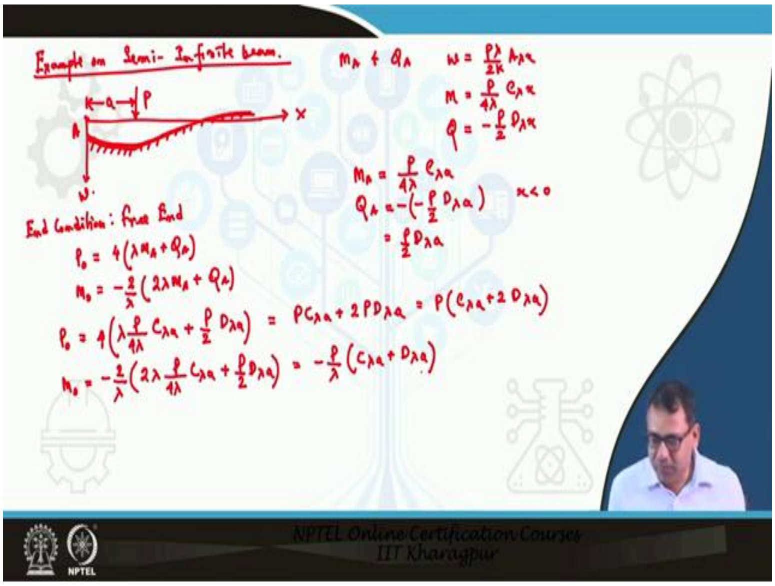

Let us solve a numerical problem by first deriving the expressions for a similar condition and then substituting the values of the example in them. Consider a semi-infinite beam with a free end and is subjected to a point load, P at a distance of ‘a’ from the finite end (which is free). The beam is resting on soil and the deflection, w is in the downward direction.

Only the deflection part will be derived and discussed in detail. The similar procedure can be applied to determine the other three quantities. The probable deflection profile is drawn in the above slide. First we should calculate the values of moment M_A and shear force Q_A at A due to the point load, P considering the beam as an infinite beam. Remember that moment and shear force will be zero at A truly because it is a free end.

In this explanation, the deflection in a semi-infinite beam with free end condition under point load will be dealt with. Using this, slope, bending moment and shear force can be found out. If there is any other type of load like concentrated moment or UDL or triangular loading, this procedure can be applied to it. Also if there is another end condition like hinge or fix or roller, this procedure can be applied to solve.

Now, as the beam is considered as an infinite beam, the expressions for an infinite beam under point load will be written down first:

$$w = \frac{P\lambda}{2k} A_{\lambda x} \qquad \theta = -\frac{P\lambda^2}{k} B_{\lambda x}$$

$$M = \frac{P}{4\lambda} C_{\lambda x} \qquad Q = -\frac{P}{2} D_{\lambda x}$$

The moment at point A, M_A due to point load, P will be:

$$M_A = \frac{P}{4\lambda} C_{\lambda a}$$

As the distance between point A and the point load is ‘a’, x = a.

Similarly, shear force at point A, Q_A due to point load, P will be:

$$Q_A = -\frac{P}{2} D_{\lambda a} \Rightarrow Q_A = -\left(-\frac{P}{2} D_{\lambda a}\right) \Rightarrow Q_A = +\frac{P}{2} D_{\lambda a}$$

But here, the point of interest is to the left side of the loading which means x < 0. There would be no effect on bending moment in case of a point load whether the point of interest is to the left or right of the loading. But the shear force will be negative if the point of interest is to the left of the point of loading. (***Under point load:*** *shear force and slope will be negative if point of interest is to the left of load, but bending moment and deflection will be positive on both sides*)

The expressions for the end conditioning forces, P_o and M_o are:

$$P_o = 4(\lambda M_A + Q_A)$$

$$M_o = -\frac{2}{\lambda}(2\lambda M_A + Q_A)$$

Substituting the M_A and Q_A values in the expression for P_o:

$$P_o = 4\left(\lambda \frac{P}{4\lambda} C_{\lambda a} + \frac{P}{2} D_{\lambda a}\right)$$

$$\Rightarrow P_o = PC_{\lambda a} + 2PD_{\lambda a}$$

$$\Rightarrow P_o = P(C_{\lambda a} + 2D_{\lambda a})$$

Substituting the M_A and Q_A values in the expression for M_o:

$$M_o = -\frac{2}{\lambda}\left(2\lambda \frac{P}{4\lambda} C_{\lambda x} + \frac{P}{2} D_{\lambda x}\right) = -\frac{P}{\lambda}(C_{\lambda a} + D_{\lambda a})$$

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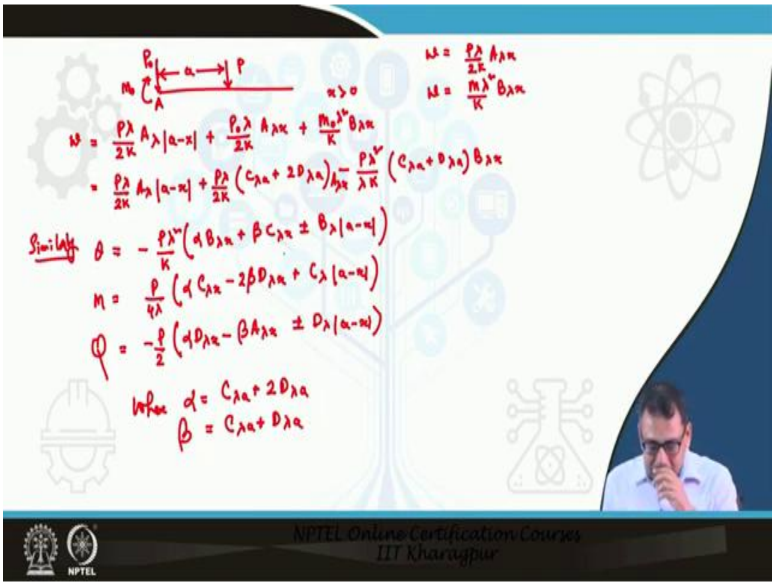

So, now there are three forces acting on the beam. Point load, P at a distance of ‘a’ from the point A or the free ends. The end conditioning forces, P_o and M_o acting at A but at an infinitely small distance towards the left side of the load. So, as far as the load P is concerned, the point of interest (A) is to the left side of the load but considering P_o and M_o, the point of interest is to the right side of the loads.

The deflection due to the all forces: two loads and one moment should be calculated. For the given conditions, the final deflection expression at any point in the beam can be given by:

$$w = \frac{P\lambda}{2k} A_{\lambda|a-x|} + \frac{P_o\lambda}{2k} A_{\lambda x} + \frac{M_\circ\lambda^2}{k} B_{\lambda x} \rightarrow (1)$$

If deflection should be calculated at some point in the beam other than A, the above expression can be used. The x value is the distance between that point and point A. So, the first term captures the effect of the point load, P in which the absolute difference between x and a is used. This is because if the point of interest is between A and the load, the distance from the load to the point of interest would be (a - x). But if the point of interest is to the right of the load, the distance between the load and the point of interest would be (x - a). The next two terms in the above expression are for P_o and M_o. Wherever the point of interest in the beam, it will always be to the right side of the load considering these two end conditioning forces and for this condition, all quantities are positive.

Though deflection is positive irrespective of which side the load is, this explanation is being given for clarity in the later expressions for slope and shear force which are negative if the point of interest is to the left of the load.

Substituting the values of P_o and M_o in the above expression:

$$w = \frac{P\lambda}{2k} A_{\lambda|a-x|} + \frac{P\lambda}{2k}\left(C_{\lambda a} + 2D_{\lambda a}\right)A_{\lambda x} - \frac{P\lambda^2}{\lambda k}\left(C_{\lambda a} + D_{\lambda a}\right)B_{\lambda x}$$

$$w = \frac{P\lambda}{2k}\left[A_{\lambda|a-x|} + \left(C_{\lambda a} + 2D_{\lambda a}\right)A_{\lambda x} - 2\left(C_{\lambda a} + D_{\lambda a}\right)B_{\lambda x}\right]$$

Similarly:

$$\theta = -\frac{P\lambda^2}{k}\left(\alpha B_{\lambda x} + \beta C_{\lambda x} \pm B_{\lambda|a-x|}\right)$$

$$M = \frac{P}{\lambda}\left(\alpha C_{\lambda x} - 2\beta D_{\lambda x} \pm C_{\lambda|a-x|}\right)$$

$$Q = -\frac{P}{2}\left(\alpha D_{\lambda x} - \beta A_{\lambda x} \pm D_{\lambda|a-x|}\right)$$

$$\text{where,}\quad \alpha = C_{\lambda a} + 2D_{\lambda a}$$

$$\beta = C_{\lambda a} + D_{\lambda a}$$

The first term in equation (1) is written last in the last three expressions (for θ, M and Q). Out of those, for the expressions of slope and shear force, the ± sign is given because the point of interest may be left side or right side. This term will be positive if the point of interest is to the right of the load and will be negative if point of interest is to the left of the load. Note that this is only for slope and shear force.

These are the expressions to determine w, θ, M and Q at any point in a semi-infinite beam with a free end subjected to a point load, P.

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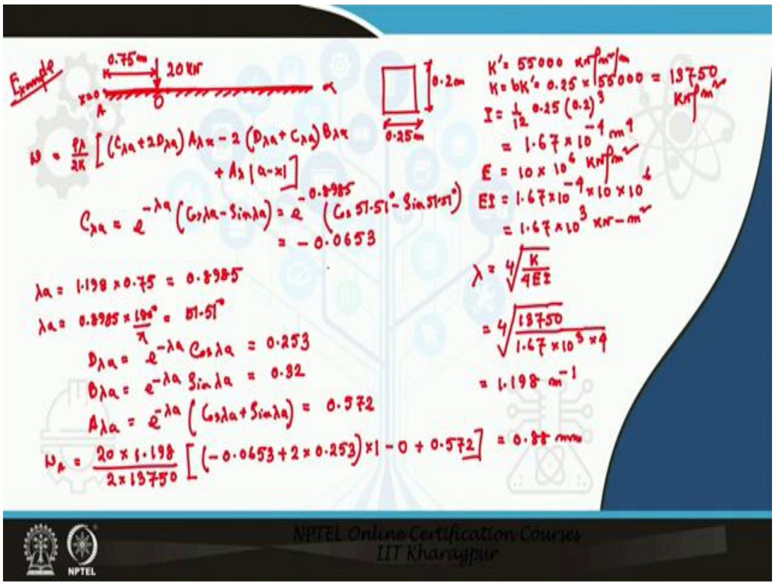

Let us solve a numerical problem where a semi infinite beam with free end is subjected to a point load. Consider a semi-infinite beam with its free, finite end at point A. A point load, P = 20 kN acts on the beam at point O which is 0.75 m to the right of point A. The cross section of the beam is given as: b = 0.25m and h = 0.2 m. $k_o = k' = 55000\ kN/m^2$, $E = 10 \times 10^6\ kN/m^2$.

The k value will be:

$$k = bk' = 0.25 \times 55000 = 13750 kN/m^2$$

The moment of inertia, I:

$$I = \frac{bh^3}{12} = \frac{0.25 \times 0.2^3}{12} = 1.67 \times 10^{-4} m^4$$

$$EI = 10 \times 10^6 \times 1.67 \times 10^{-4} = 1.67 \times 10^3 kN - m^2$$

The λ value can be calculated by:

$$\lambda = \sqrt[4]{\frac{k}{4EI}} = \sqrt[4]{\frac{13750}{4 \times 1.67 \times 10^3}} = 1.198 m^{-1}$$

To calculate the deflection and other quantities, firstly the λa value and the coefficient values should be calculated.

$$\lambda a = 1.198 \times 0.75 = 0.8985$$

$$\Rightarrow \lambda a = 0.8985 \times \frac{\pi}{180} = 51.51^\circ$$

$$A_{\lambda a} = e^{-\lambda a}(\cos \lambda a + \sin \lambda a) = e^{-0.8985}\left[\cos(51.51^\circ) + \sin(51.51^\circ)\right] = 0.572$$

$$B_{\lambda a} = e^{-\lambda a}(\sin \lambda a) = 0.32$$

$$C_{\lambda a} = e^{-\lambda a}(\cos \lambda a - \sin \lambda a) = -0.0653$$

$$D_{\lambda a} = e^{-\lambda a}(\cos \lambda a) = 0.253$$

Now, the expression for deflection at any point in this beam is:

$$w = \frac{P\lambda}{2k}\left[A_{\lambda|a-x|} + (C_{\lambda a} + 2D_{\lambda a})A_{\lambda x} - 2(C_{\lambda a} + D_{\lambda a})B_{\lambda x}\right]$$

Deflection at point A will be:

$$w_A = \frac{20 \times 1.198}{2 \times 13750}[(-0.0653 + 2 \times 0.253) \times 1 - 0 + 0.572] = 0.88mm$$

As the deflection is taken at point A, x = 0 and so in the term $(C_{\lambda a} + 2D_{\lambda a})A_{\lambda x}$, $A_{\lambda x}$ will be 1. Similarly the term $2(C_{\lambda a} + D_{\lambda a})B_{\lambda x}$, will be zero as at x = 0, $B_{\lambda x}$ is also zero.

If the deflection at point O should be calculated, then x = a because the distance of point O from the free end, point A is ‘a’. This will be calculated along with the deflection at few other points.

In the next class I will first calculate the deflection at point O where x is equal to 0.75 m and then I will discuss about the other conditions of the beam. After that I will start the concept of beam with finite length. Thank you.