Soil Structure Interaction
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Lecture 25
Beams on Elastic Foundation (Contd.,)

In the last class I discussed about the infinite beam under multiple loading condition. In this class I will start about the semi infinite beam. The first step is to discuss what a semi infinite beam is and then, I will discuss about the different boundary conditions.

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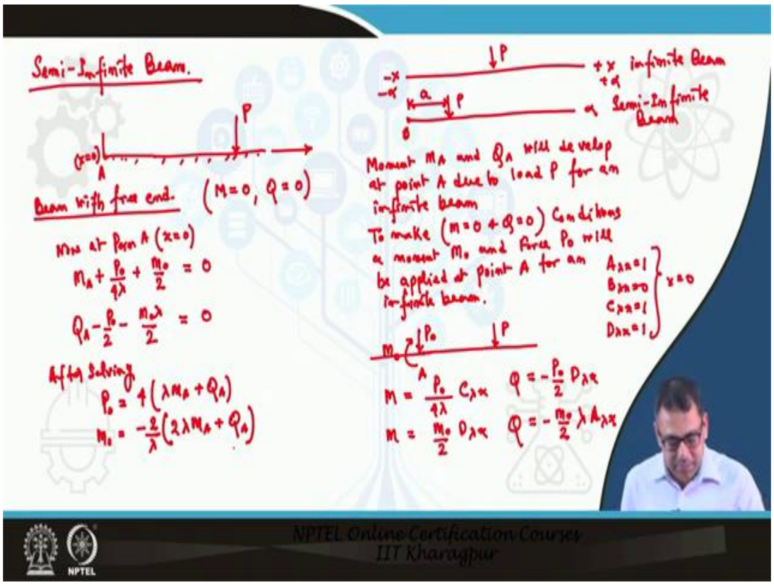


The difference between semi infinite beam and infinite been is that an infinite beam is supposed to be extended from negative infinity to positive infinity whereas for a semi-infinite beam, only one of the ends is at infinity. Suppose for a semi-infinite beam, the finite end is at point O, this point O will have some boundary condition: free or pinned or fixed etc. Boundary condition is the end condition of the beam but is not important in case of an infinite beam because the loading effect will not reach the boundary because of the infinite distance. But if the end is finite, then boundary condition is important and the type of boundary condition affects the response of the beam. This is because, when a load is applied on a semi-infinite beam, there will be some finite distance from the point O to the point of application of load.

Let us get started with a point load acting on a semi infinite beam for which the infinite end lies towards the positive infinity or to the right side. The finite end is named point A and the boundary condition is assumed to be free. It is just resting on the soil and is the most common in case of foundation or any structure on soil.

So, if a beam has a free end, the boundary conditions will be the bending moment, M and the shear force, Q of that end will be 0. But the semi infinite beam will be analysed as an infinite beam. How to do that? If the beam is infinite, there will be some bending moment and shear force acting at point A. So, by considering there is some M and Q acting at point A, we are considering the semi-infinite beam as an infinite beam.

So if it is an infinite beam, due to the application of a point load P, a moment and shear force will develop at point A. Say that the bending moment and shear force at point A are M_A and Q_A respectively which developed due to the load P. This is just a consideration and in the actual case, M_A and Q_A do not exist. So, to counteract these two, a moment M_0 and a force P_0 will be applied at the point A. Now, the forces acting at point A are: bending moment M_A & shear force Q_A due to the point load (came to picture only because the beam is considered infinite), end-conditioning forces M_0 & P_0 to be applied at point A to nullify the effect of M_A & Q_A. Ultimately the net moment and net shear at point A should be 0.

In semi-infinite beams, the formulation of 'x' and 'a' is a bit different. The distance from the finite end to the point of application of load is 'a' and the distance from the finite end to the point of interest is 'x'. So at point A, x will be 0. Now, considering this beam as infinite, the moment at any point in the beam due to the load P_0 would be:

$$M = \frac{P_0}{4\lambda} C_{\lambda x}$$

The moment at any point in the beam due to the moment M_0 would be:

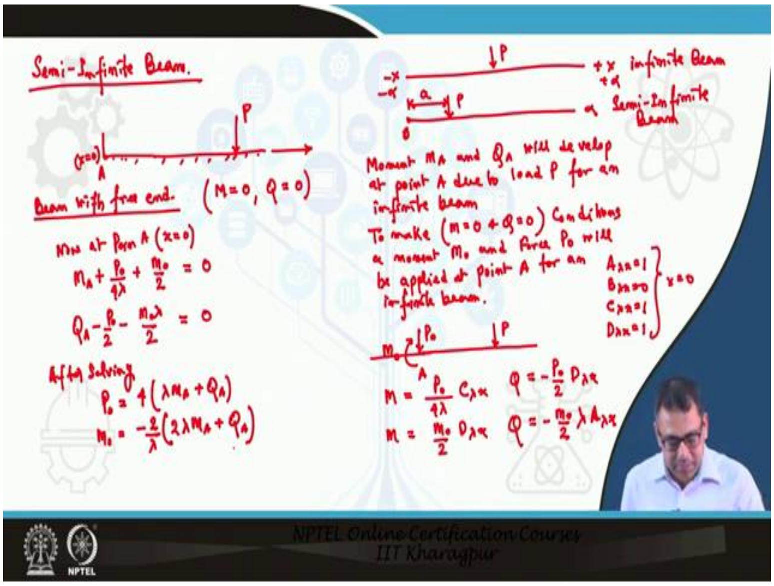
$$M = \frac{M_0}{2} D_{\lambda x}$$

The shear force at any point in the beam due to the load P_0 would be:

$$Q = -\frac{P_0}{2} B_{\lambda x}$$

The shear force at any point in the beam due to the moment M_0 would be:

$$Q = -\frac{M_0}{2} \lambda A_{\lambda x}$$



So at point A there will be a moment M_A that acts due to the application of point load P, a moment due to P_0 will act: $P_0/4\lambda$ because x will be 0 for this case and hence $C_{\lambda x}$ is unity. The moment due to the application of M0 will be: $M_0/2$ because $D_{\lambda x} = 1$. Here, P_0 and M_0 are initially considered positive, but depending upon the loading condition this may be negative also. The direction of P_0 and M_0 may differ depending upon the loading condition. So, the expression for net moment at A would be:

$$M_A + \frac{P_0}{4\lambda} + \frac{M_0}{2} = 0$$

Similarly, the shear force at A due to the point load, P will be Q_A. The shear force due to P_0 will be $-P_0/2$. One thing should be remembered here that the moment and force applied at point A is actually applied very close to point A, but slightly towards the left side. This means that there is an infinitely small distance from point A and the applied forces towards the left. So the point of interest is to the right side of the loading and all the quantities will be positive. Hence, the shear force due to P_0 will be negative. Similarly, shear force due to M_0 will be $-M_0\lambda/2$. The expression for net shear force at point A would be:

$$Q_A - \frac{P_0}{2} - \frac{M_0\lambda}{2} = 0$$

M_A and Q_A can be calculated if the distance from A to P is known. So, the unknowns in the two equations are P_0 and M_0. By solving the two equations of bending moment and shear force for P_0 and M_0, we get:

$$P_0 = 4(\lambda M_A + Q_A)$$

$$M_0 = -\frac{2}{\lambda}(2\lambda M_A + Q_A)$$

This amount of P_0 and M_0 should be applied to nullify M_A and Q_A. This is the procedure to determine P_0 and M_0 for the free end condition.

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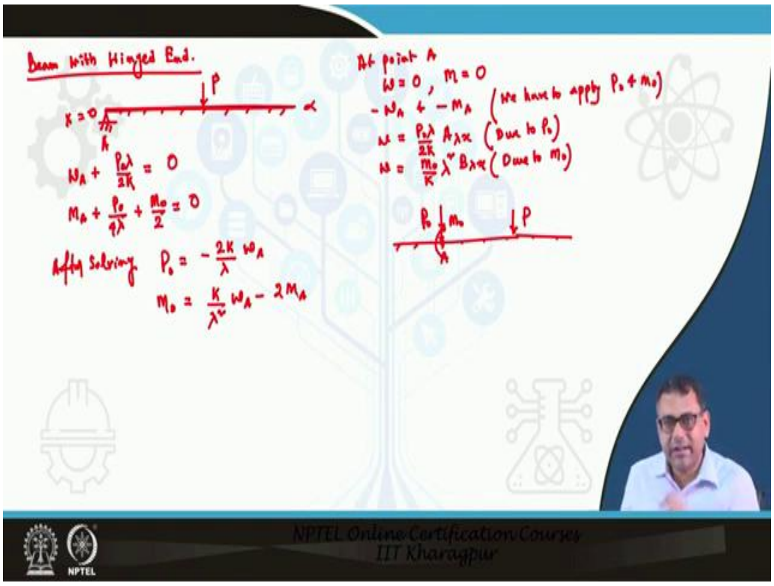

Let us see what happens if the end condition is hinge. Because of the hinged end boundary condition, the deflection and bending moment at A will be 0. Again, P_0 and M_0 should be applied such that it produces $-w_A$ and $-M_A$. Now, the boundary will be removed for consideration and the beam will be assumed as infinite. So, now the point load, P will produce a deflection (w_A) and bending moment (M_A) at point A for this infinite beam. Due to the P_0 load, the deflection will be $P_0\lambda/ 2k$. Due to the moment M_0, the deflection will be $(M_0/k\lambda^2) \times B_{\lambda x}$ which is 0 because x is 0 here. So the net deflection at A is:

$$w_A + \frac{P_0\lambda}{2k} = 0$$

Similarly the net bending moment at A will be:

$$M_A + \frac{P_0}{4\lambda} + \frac{M_0}{2} = 0$$

These are the two equations which should be solved to determine P_0 and M_0:

$$P_0 = -\frac{2k}{\lambda} w_A$$

$$M_0 = \frac{k}{\lambda^2} w_A - 2M_A$$

Till now, the free end and hinged conditions in the semi-infinite beam were discussed. In the next class I will discuss the third boundary condition when the beam has a fixed end condition. After that I will solve a numerical problem to demonstrate how to apply this principle to determine and apply these logics of infinite beam theories for a semi infinite beam to determine all these quantities. Thank you.