Soil Structure Interaction
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Lecture 24
Beams on Elastic Foundation (Contd.,)

In my previous lecture I discussed about the response of an infinite beam subjected to triangular loading. The point of interest was considered to be within the loaded region in that case already discussed. In this class, I will first discuss how to calculate all the 4 quantities at two ends of the loading and then the other two cases when the point of interest is outside the loaded region.

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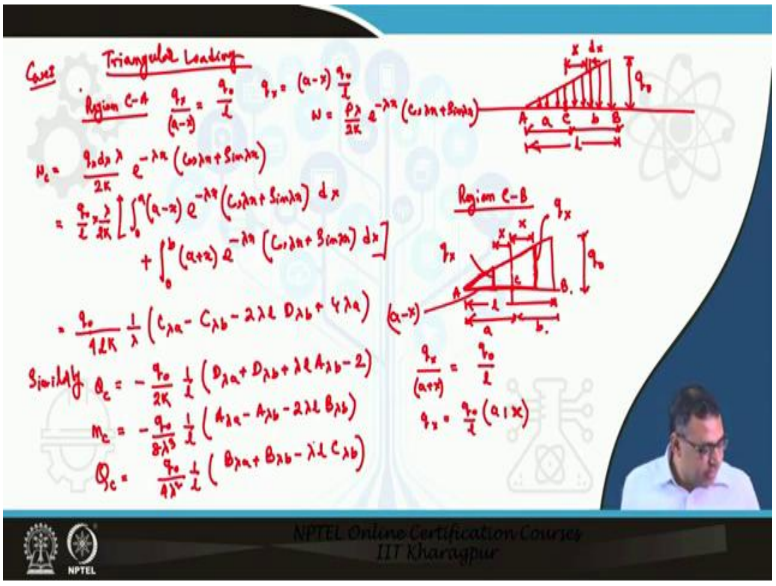

The four equations for deflection, slope, bending movement and shear force when an infinite beam is subjected to triangular loading were already discussed.

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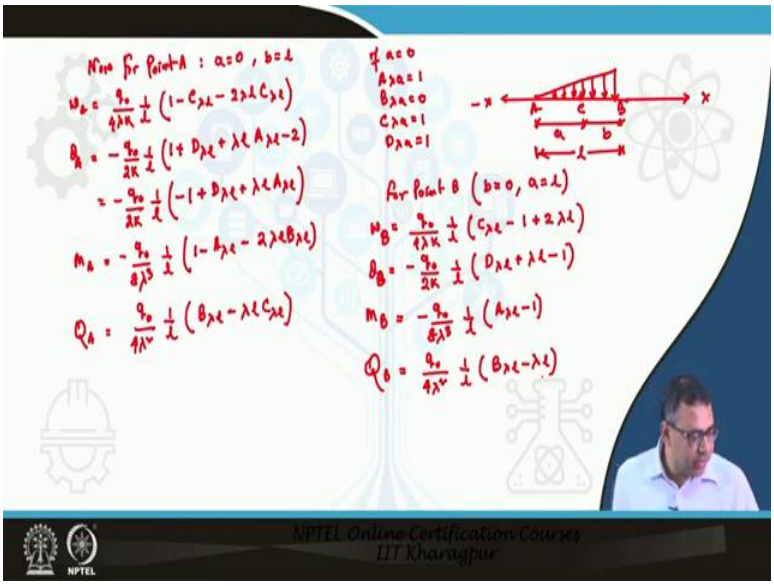

Now let us determine all these four quantities at point A, which is one of the ends of the load. For point A, ‘a’ will be zero because ‘a’ is the distance from A to the point of interest and here A is the point of interest. Similarly ‘b’ will be equal to ‘l’ because ‘b’ is the distance from B to the point of interest. Substituting these values (a = 0 & b = l) in the general expressions of the point of interest within the loaded region, we get expressions for all the quantities at point A:

$$w_A = \frac{q_0}{4\lambda lk}\left(1 - C_{\lambda l} - 2\lambda l D_{\lambda l}\right)$$ {this is $2\lambda D_{\lambda l}$, not $2\lambda C_{\lambda l}$}

$$\theta_A = -\frac{q_0}{2lk}\left(-1 + D_{\lambda l} + \lambda l A_{\lambda l}\right)$$

$$M_A = -\frac{q_0}{8l\lambda^3}\left(1 - A_{\lambda l} - 2\lambda l B_{\lambda l}\right)$$

$$Q_A = \frac{q_0}{4l\lambda^2}\left(B_{\lambda l} - \lambda l C_{\lambda l}\right)$$

Similarly for Point B (b = 0 & a = l),

$$w_B = \frac{q_0}{4lk\lambda}\left(C_{\lambda l} - 1 + 2\lambda l\right)$$

$$\theta_B = -\frac{q_0}{2lk}\left(D_{\lambda l} + \lambda l - 1\right)$$

$$M_B = -\frac{q_0}{8l\lambda^3}\left(A_{\lambda l} - 1\right)$$

$$Q_B = \frac{q_0}{4l\lambda^2}\left(B_{\lambda l} - \lambda l\right)$$

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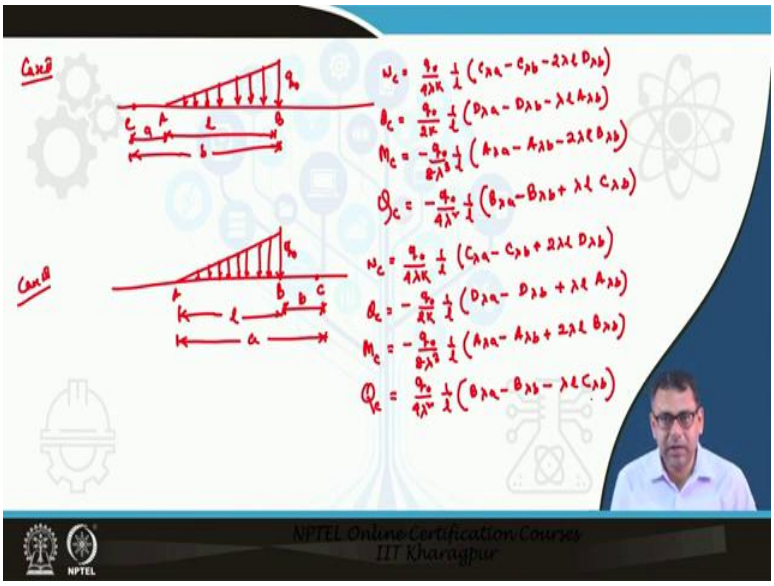

Let us discuss about case 2 and case 3 now. In the case 2, the point of interest, C is to the left of the loading 'a' units away from point A and 'b' units away from point B. The expressions for this case would be:

$$w_C = \frac{q_0}{4lk\lambda}\left(C_{\lambda a} - C_{\lambda b} - 2\lambda l D_{\lambda b}\right)$$

$$\theta_C = \frac{q_0}{2lk}\left(D_{\lambda a} - D_{\lambda b} - \lambda l A_{\lambda b}\right)$$

$$M_C = -\frac{q_0}{8l\lambda^3}\left(A_{\lambda a} - A_{\lambda b} - 2\lambda l B_{\lambda b}\right)$$

$$Q_C = \frac{q_0}{4l\lambda^2}\left(B_{\lambda a} - B_{\lambda b} + \lambda l C_{\lambda b}\right)$$

Similarly, the expressions for case-3 when the point of interest it to the right of the loading and outside the loaded region will be:

$$w_C = \frac{q_0}{4lk\lambda}\left(C_{\lambda a} - C_{\lambda b} + 2\lambda l D_{\lambda b}\right)$$

$$\theta_C = -\frac{q_0}{2lk}\left(D_{\lambda a} - D_{\lambda b} + \lambda l A_{\lambda b}\right)$$

$$M_C = -\frac{q_0}{8l\lambda^3}\left(A_{\lambda a} - A_{\lambda b} + 2\lambda l B_{\lambda b}\right)$$

$$Q_C = \frac{q_0}{4l\lambda^2}\left(B_{\lambda a} - B_{\lambda b} - \lambda l C_{\lambda b}\right)$$

Till now, four types of loading are discussed: concentrated load, concentrated movement, UDL and triangular loading. The next loading condition is the case of multiple loading where multiple concentrated loads are applied on an infinite beam.

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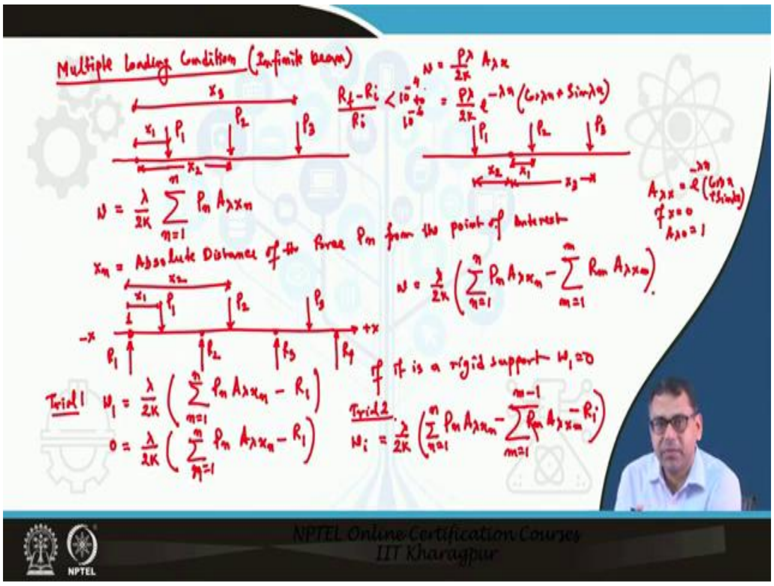

Consider an infinite beam subjected to three point loads, P_1, P_2, P_3. This is just an example to show the process to solve when there are multiple loads acting on an infinite beam. So, any type of multiple load can be considered like 2 concentrated loads or 2 UDLs + 1 concentrated load or 1 UDL + 1 triangular load + one moment. The procedure to calculate deflection in case of multiple loads will be explained and this can be applied to determine the other quantities also. The usual expression for deflection due to a point load is:

$$w = \frac{P\lambda}{2k} A_{\lambda x}$$

Similarly, for a beam subjected to multiple point loads, the expression for deflection would be:

$$w = \frac{\lambda}{2k} \sum_{n=1}^{n} P_n A_{\lambda x_n}$$

The distance x_n is the absolute distance of the force P_n from the point of interest where the deflection should be determined. Since the interested quantity now is deflection, x_n cannot have negative value because the sign of deflection is not affected by whether the point of interest is to the left or right of the load. But, this is not the case while determining slope and shear force. These rules are already discussed.

Suppose the point of interest, C is to the left of all the loads and the distances between C and P_1, P_2, P_3 are x_1, x_2, x_3 respectively. If the point of interest is between P_1 & P_2 and close to P_2, then the distance between: C & P_2 will be x_1, C & P_1 will be x_2 and C & P_3 will be x_3.

The loading condition considered is without any reaction. But if there is some rigid support, there would be some support reaction. Suppose there are four supports offering a reaction of R_1, R_2, R_3 and R_4. Deflection should be determined for this case of three point loads and four support reactions.

$$w = \frac{\lambda}{2k}\left(\sum_{n=1}^{n} P_n A_{\lambda x_n} - \sum_{n=1}^{n} R_n A_{\lambda x_n}\right)$$

This way the deflection at any point considering the support reactions can be determined. But the question is how to determine the support reaction. All the P values are known, but the R values are unknown. To determine the deflection, both are necessary and so to calculate the R_n, it is better to follow trial and error process. Let us say, that the deflection of the first support (with R_1 reaction) should be found out. Neglect the affect of all other support reactions other than R_1 and consider all the loads as the first trial. The equation for this condition would be:

$$w_1 = \frac{\lambda}{2k}\left(\sum_{n=1}^{n} P_n A_{\lambda x_n} - R_1\right)$$

As the deflection is being determined at the same point where R_1 is acting, the x_n or x value for R_1 will be 0 and hence the $A_{\lambda x}$ coefficient will be unity. The support is rigid and hence there will be no deflection at this support ($w_1 = 0$):

$$0 = \frac{\lambda}{2k}\left(\sum_{n=1}^{n} P_n A_{\lambda x_n} - R_1\right)$$

In the above expression, the only unknown is R_1 and hence that can be calculated. Basically, the deformation due to P_1 at point 1, due to P_2 at point 1, due to P_3 at point 1 are summed up from which R_1 is subtracted. Remember that while determining R_1, all other support reactions are not been considered. Similarly, all the support reactions can be determined by neglecting the other support reactions when calculating one particular support reaction.

But these values are not correct because some existing forces are being neglected which is not the actual case. In the next trail (trial-2), all the R values obtained from trial-1, though not correct, should be used in the following formula:

$$w_i = \frac{\lambda}{2k}\left(\sum_{n=1}^{n} P_n A_{\lambda x_n} - \sum_{m=1}^{m-1} R_m A_{\lambda x_m} - R_i\right)$$

The summation of the load (P_n) effects should be subtracted with all the support reactions multiplied with the $A_{\lambda x}$ coefficient except for the support reaction at which the deflection is being calculated. If the deflection is to be calculated at 2^{nd} support, R_2 should be subtracted separately because x_2 will be 0 and $A_{\lambda x_2}$ will be 1. Using this method, the support reaction for all other points can also be obtained.

After determining all the support reactions in a particular trial, compare those values with that of the previous trial values. If the difference is considerable, repeat the procedure while checking the difference between consecutive trial results. Unless the difference between the present reaction values and the previous reaction values is very small, this process should be repeated. The tolerance limit may be fixed using the ratio $\left(R_{\text{final}} - R_{\text{initial}}\right) / R_{\text{initial}}$ for which the value should ideally be between 10^{-4} and 10^{-6}. Unless this tolerance value is achieved, the trial should be continued. This is the process to determine the support reaction values in case of a multiple loading condition.

Till now I have discussed only about the infinite beam and in the next class I will start discussing about the second type of beam that is semi-infinite beam under different boundary conditions. Thank you.