Soil Structure Interaction
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Lecture 23
Beams on Elastic Foundation (Contd.,)

In the last class, I have discussed about an infinite beam subjected to UDL and also the three cases considered to evaluate that loading condition. The three cases differ in the relative position of the point of interest where in case-1, it lies within the loaded region, in case-2, it lies to the left of the UDL and in case-3, it lies to the right of the UDL. The equations were derived for the first case to determine the deflection and other quantities at the two end points of the UDL along with a point within. Today, I will solve one numerical problem and then will discuss about the other two cases.

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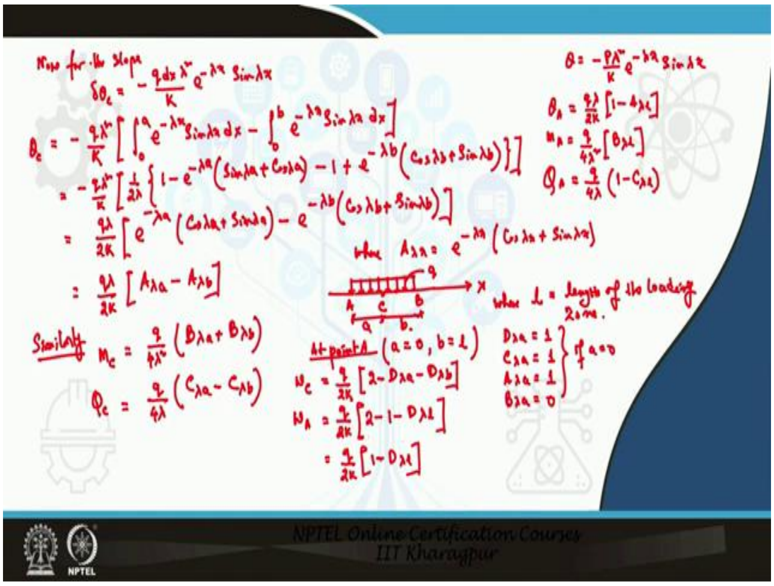

The equations derived in the last class are shown in the slide above and below. Points A and B are the end points or extremes of the UDL, where point C was considered to be somewhere within the UDL.

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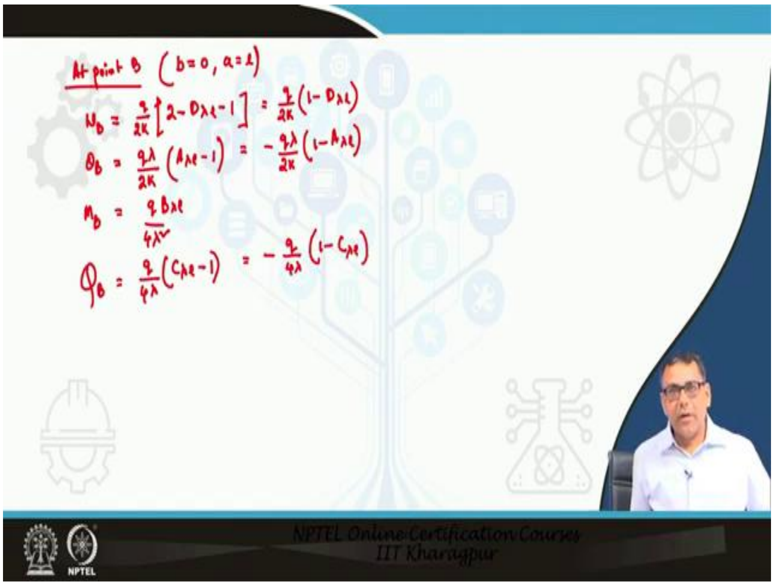

The above slide shows the equations for deflection, slope, bending moment and shear force at point B.

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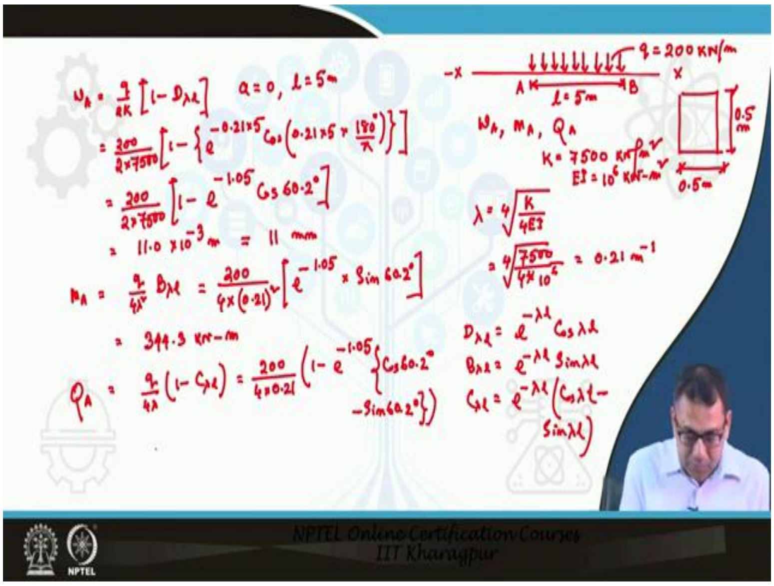

The example problem considers a case where a UDL of length 5 m and intensity, 200 kN/m is applied on an infinite beam resting on soil. The positive x direction (+x) is to the right and the negative x direction (-x) is to the left. The beam is same as the beam for which the solution was already given when a point load is acted upon it. The problem here is to determine the deflection, bending moment and shear force at point A. The k value given is the same as 7500 kN/m^2 and if it is in kN/m^3, it will be 1500 kN/m^3 because the dimension of the beam is point 0.5×0.5 m.

Firstly, calculate the λ value using the formula:

$$\lambda = \sqrt[4]{\frac{k}{4EI}} = \sqrt[4]{\frac{7500}{4\times10^{6}}} = 0.21m^{-1}$$

Now, the deflection at point A should be calculated using the formula:

$$\Rightarrow w_A = \frac{q}{2k}\left[1 - D_{\lambda l}\right]$$

$$\Rightarrow w_A = \frac{200}{2\times7500}\left[1-\left\{e^{-0.21\times5}\cos\left(0.21\times5\times\frac{180}{\pi}\right)\right\}\right]$$

$$\Rightarrow w_A = \frac{200}{2\times7500}\left[1 - e^{-1.05}\cos(60.2^{\circ})\right]$$

$$\therefore w_A = 11.0\times10^{-3}m = 11mm$$

Similarly the bending moment can be calculated:

$$M_A = \frac{q}{4\lambda^2}\left[B_{\lambda l}\right] = \frac{200}{4\times(0.21)^2}\left[e^{-1.05}\times\sin(60.2^{\circ})\right]$$

$$\therefore M_A = 344.3kN - m$$

To calculate the shear force at A:

$$Q_A = \frac{q}{4\lambda}\left(1 - C_{\lambda l}\right)$$

$$\Rightarrow Q_A = \frac{200}{4\times(0.21)^2}\left[1 - e^{-1.05}\left\{\cos(60.2^{\circ}) - \sin(60.2^{\circ})\right\}\right]$$

$$\therefore Q_A = 269kN$$

These are the values of deflection, bending moment and shear force at point A due to the application of this UDL. The deflection, bending moment and shear force at point B or any point within the UDL can also be calculated using the expressions derived and discussed so far.

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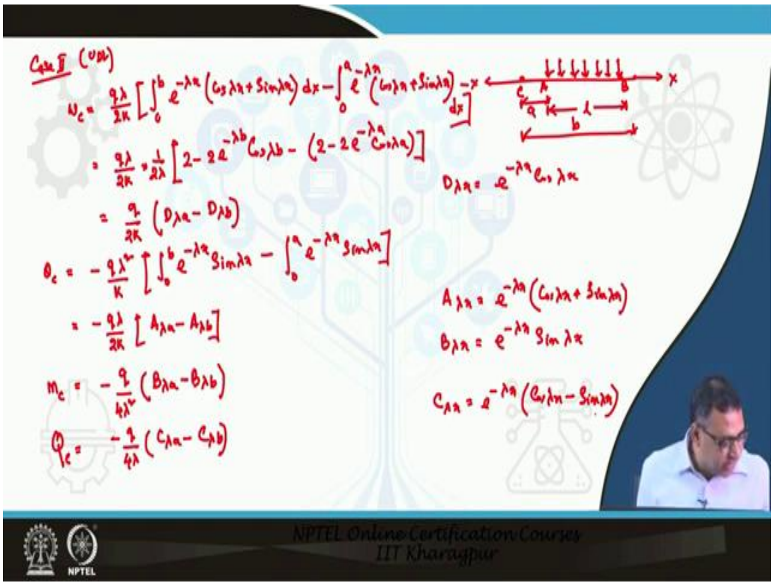

Now let us move on to the second and third cases where the point of interest lies to the left and right of the UDL respectively. The length of the UDL is l, A and B are the end points of the UDL and C is the point of interest that lies to the left of the UDL for this particular case. The distance between A and C is ‘a’ and the distance between C and B is ‘b’. In this case, b is greater than l and l is greater than a (b > l > a).

All the required quantities should be determined for this condition in the infinite beam. The deflection at the point C can be calculated by considering that the loading is applied between A & C and also between B & C. In the next step, the loading between C & A will be subtracted resulting in the presence of net loading only between A & B.

$$w_C = \frac{q\lambda}{2k}\left[\int_0^b e^{-\lambda x}(\cos\lambda x + \sin\lambda x)dx - \int_0^b e^{-\lambda x}(\cos\lambda x + \sin\lambda x)dx\right]$$

$$\Rightarrow w_C = \frac{q\lambda}{2k} \times \frac{1}{2\lambda}\left[2 - 2e^{-\lambda b}\cos\lambda b - \left(2 - 2e^{-\lambda a}\cos\lambda a\right)\right]$$

$$\therefore w_C = \frac{q}{2k}\left(D_{\lambda a} - D_{\lambda b}\right)$$

The deflection at C can also be determined using the same procedure:

$$\theta_C = -\frac{q\lambda^2}{k}\left[\int_0^b e^{-\lambda x}\sin\lambda x dx - \int_0^a e^{-\lambda x}\sin\lambda x dx\right]$$

$$\therefore \theta_C = -\frac{q\lambda}{2k}\left[A_{\lambda a} - A_{\lambda b}\right]$$

Similarly, the bending moment and shear force can also be determined:

$$M_C = -\frac{q}{4\lambda^2}\left(B_{\lambda a} - B_{\lambda b}\right)$$

$$Q_C = -\frac{q}{4\lambda}\left(C_{\lambda a} - C_{\lambda b}\right)$$

The UDL is actually present only between A & B and so the effect of load within C & A was subtracted to determine the actual deflection.

These are the expressions for all the quantities when the point of interest is to the left of loading.

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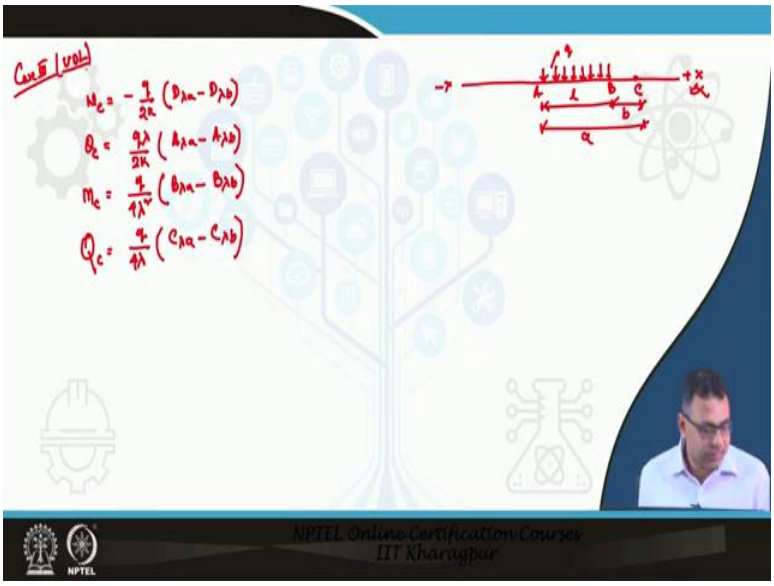

Now consider the third case where the point of interest is considered to the right of the loading. In the previous case, the loading was assumed to be present from B to C and then the part from A to C was subtracted. But here the opposite process should be followed. First consider the load to be acting from A to C and then subtract the load effect from B to C. This will be the only difference from the previous case because of which the equations will have the opposite sign.

$$w_C = -\frac{q}{2k}\left(D_{\lambda a} - D_{\lambda b}\right)$$

$$\theta_C = \frac{q\lambda}{2k}\left[A_{\lambda a} - A_{\lambda b}\right]$$

$$M_C = \frac{q}{4\lambda^2}\left(B_{\lambda a} - B_{\lambda b}\right)$$

$$Q_C = \frac{q}{4\lambda}\left(C_{\lambda a} - C_{\lambda b}\right)$$

These are the expressions for all the quantities at a point which is to the right side of the UDL. The required quantities can be found out at A or B by substituting the appropriate ‘a’ & ‘b’ values in the equations of any one of the three cases.

So, these are the three cases for a UDL acting on an infinite beam when the point of interest is within and outside the loaded region.

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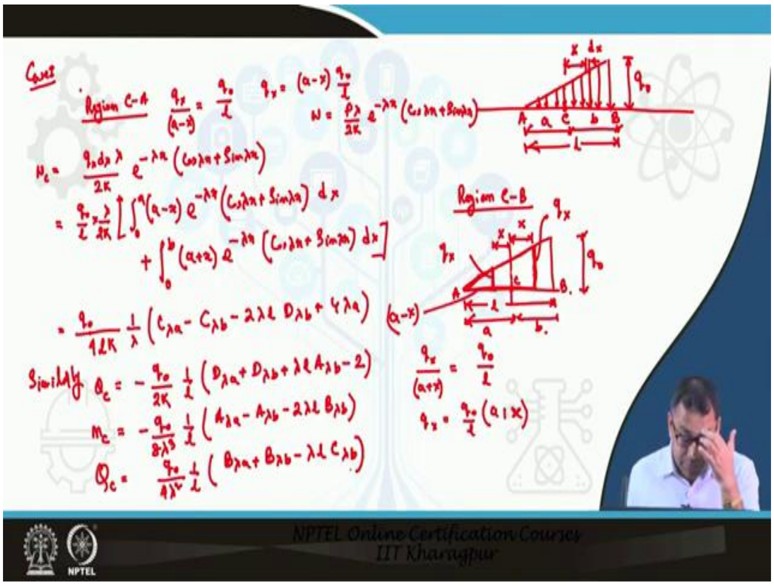

Till now point load, concentrated moment and UDL have been discussed. The next problem or the loading condition is the triangular loading acting on an infinite beam. In the triangular loading also there are three cases, when the point of interest is within and outside the loaded region.

All the three cases will be discussed now starting with the first case where the point of interest is within the loaded region. The highest load intensity of this triangular load is considered to be q_0. The ends of the load are at points A & B where the load is of zero intensity at A and of q_0 intensity at B. The distance from A to C is ‘a’, from C to B is ‘b’ and from A to B is ‘l’.

Now, there is load on both sides of the point of interest. First consider the load in the region C to B which is to the right of the point of interest, C and consider a small segment in it of width dx. This segment is at a distance of x units from C. The load intensity in the dx segment should be determined now using the available information. When the dx segment is in C-B region, we can write:

$$\frac{q_x}{a+x} = \frac{q_0}{l}$$

$$\Rightarrow q_x = \frac{q_0}{l}(a+x)$$

Similarly, consider the segment of dx width to be present in the C-A region again x units from C, but this time the segment is to the left of the point of interest.

$$\frac{q_x}{a-x} = \frac{q_0}{l}$$

$$\Rightarrow q_x = \frac{q_0}{l}(a-x)$$

Now, the expression for w_c considering the load from the segment can be written as:

$$w_C = \frac{q_x dx\lambda}{2k} e^{-\lambda x}(\cos\lambda x + \sin\lambda x)$$

The above formulation can be used to calculate the total deflection at C due to the entire triangular load and by integrating the entire areas from C to A and C to B:

$$\Rightarrow w_C = \frac{q_x}{l} \times \frac{\lambda}{2k}\left[\int_0^a (a-x)e^{-\lambda x}(\cos\lambda x + \sin\lambda x)dx + \int_0^b (a+x)e^{-\lambda x}(\cos\lambda x + \sin\lambda x)dx\right]$$

$$\therefore w_C = \frac{q_0}{4lk\lambda}\left(C_{\lambda a} - C_{\lambda b} - 2\lambda l D_{\lambda b} + 4\lambda_a\right)$$

Similarly,

$$\theta_C = -\frac{q_0}{2lk}\left(D_{\lambda a} + D_{\lambda b} + \lambda l A_{\lambda b} - 2\right)$$

$$M_C = -\frac{q_0}{8l\lambda^3}\left(A_{\lambda a} - A_{\lambda b} - 2\lambda l B_{\lambda b}\right)$$

$$Q_C = \frac{q_0}{4l\lambda^2}\left(B_{\lambda a} + B_{\lambda b} - \lambda l C_{\lambda b}\right)$$

The only difference between the triangular load and UDL cases is the expressions, but the procedure involved in deriving these expressions is same for both the cases. The expressions for all the different quantities are given.

In the next class, I will show how to determine the required quantities at points A and B. After that I will discuss the other two cases when the point of interest is outside the loaded region. Thank you.