Soil Structure Interaction
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Lecture 22
Beams on Elastic Foundation (Contd.,)

In the last class I have discussed about an infinite beam subjected to a concentrated moment. In this class I will discuss about an infinite beam subjected to a uniformly distributed load or UDL. The method to determine all four quantities: deflection, slope, bending moment and shear force would also be discussed.

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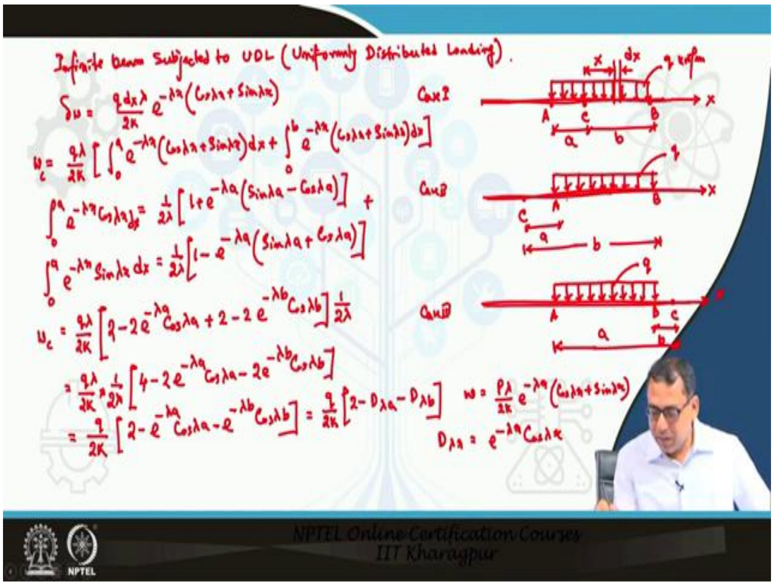

Consider an infinite beam resting on a soil and subjected to a uniformly distributed loading or UDL over a finite length, l. To solve for this condition, three different cases should be considered where: (i) the point of interest is within the UDL, (ii) the point of interest is to the left of the UDL (iii) the point of interest is to the right of the UDL. The point A is the left side end of the UDL and point B is the end of UDL to the right. C is the point of interest and in the first case, C lies within the length of the UDL. The distance between the points A and C is ‘a’ and the distance between points B and C is ‘b’ in all cases. The intensity of the UDL is q kN/m.

The value of deflection and 3 other quantities will be determined for all the 3 cases at point C. Consider the first case now where the point C is within the UDL. Take a small element of width dx at a distance of x from the point C. The load in the element dx will be treated as a concentrated load acting on the infinite beam.

The equations for all the quantities for a point load are already determined that will be used now. So, the deflection due to this load would be δw which is very small. The small deflection, δw is due to the δx portion of load and is treated as a concentrated load because of the infinitesimally small value of dx. The general expression for deflection under a point load is:

$$w = \frac{P\lambda}{2k} e^{-\lambda x}(\cos\lambda x - \sin\lambda x)$$

But here, the point load is a product of the load intensity and the very small distance over which it is acting (q × dx). So, the expression for this δw will be:

$$\delta w = \frac{qdx\lambda}{2k} e^{-\lambda x}(\cos\lambda x - \sin\lambda x)$$

Using this formulation, it is possible to develop the expression of deflection under UDL considering the entire length. This can be achieved by integrating and adding up the deflection expression from C to A with C to B. Both the parts are on two sides of the point of interest, but as this would not affect the sign of deflection, both can be added.

$$w_c = \frac{q\lambda}{2k}\left[\int_0^a e^{-\lambda x}(\cos\lambda x + \sin\lambda x)dx + \int_0^b e^{-\lambda x}(\cos\lambda x + \sin\lambda x)dx\right]$$

The integral functions would take the value as:

$$\int_0^a e^{-\lambda x}\cos\lambda x dx = \frac{1}{2\lambda}\left[1 + e^{-\lambda a}(\sin\lambda a - \cos\lambda a)\right]$$

$$\int_0^a e^{-\lambda x}\sin\lambda x dx = \frac{1}{2\lambda}\left[1 - e^{-\lambda a}(\sin\lambda a + \cos\lambda a)\right]$$

O, the deflection at point C can be written as:

$$\Rightarrow w_c = \frac{q\lambda}{2k}\times\frac{1}{2\lambda}\left[2 - 2e^{-\lambda a}\cos\lambda a + 2 - 2e^{-\lambda b}\cos\lambda b\right]$$

$$\Rightarrow w_c = \frac{q\lambda}{2k}\times\frac{1}{2\lambda}\left[4 - 2e^{-\lambda a}\cos\lambda a - 2e^{-\lambda b}\cos\lambda b\right]$$

$$\Rightarrow w_c = \frac{q}{2k}\left[2 - e^{-\lambda a}\cos\lambda a - e^{-\lambda b}\cos\lambda b\right]$$

$$\Rightarrow w_c = \frac{q}{2k}\left[2 - D_{\lambda a} - D_{\lambda b}\right]$$

(Since, $D_{\lambda x} = e^{-\lambda x}\cos\lambda x$)

This is the expression for deflection at a point in an infinite beam subjected to UDL, when the point of interest is within the UDL.

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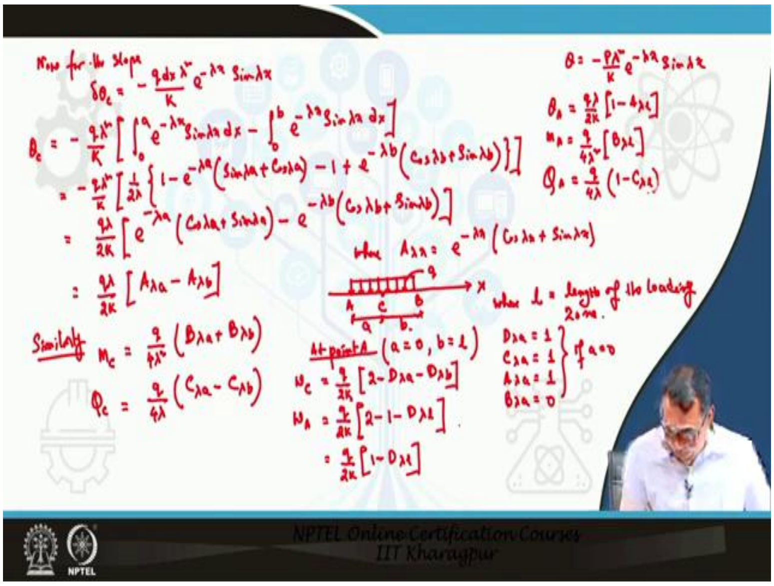

Similarly the expressions for slope, bending moment and shear force can be determined. The general expression for slope under point load is:

$$\theta = -\frac{P\lambda^2}{k} e^{-\lambda x} \sin \lambda x$$

So, the expression for slope at C, because of the load from the strip of width dx would be:

$$\delta\theta_c = -\frac{qdx\lambda^2}{k} e^{-\lambda x} \sin \lambda x$$

Similarly, the expression for slope at C, because of the entire UDL would be:

$$\theta_c = -\frac{q\lambda^2}{k}\left[\int_0^a e^{-\lambda x} \sin \lambda x dx - \int_0^b e^{-\lambda x} \sin \lambda x dx\right]$$

In the above, it should be noted that the slope because of both the sections (C to A and C to B) are with opposite signs. This is because when the load from C to A is considered, the point of interest (point C) is to the right of the load. This means that x is positive or is greater than 0 and hence the slope will be positive. But, when the load from C to B is considered, the point of interest is to the left side of the load which means that x is negative and the slope is also negative.

$$\Rightarrow \theta_c = -\frac{q\lambda^2}{k}\left[\frac{1}{2\lambda}\left\{1 - e^{-\lambda a}(\sin \lambda a + \cos \lambda a) - 1 + e^{-\lambda b}(\cos \lambda b + \sin \lambda b)\right\}\right]$$

$$\Rightarrow \theta_c = \frac{q\lambda}{2k}\left[e^{-\lambda a}(\cos \lambda a + \sin \lambda a) - e^{-\lambda b}(\cos \lambda b + \sin \lambda b)\right]$$

$$\Rightarrow \theta_c = \frac{q\lambda}{2k}\left[A_{\lambda a} - A_{\lambda b}\right]$$

Similarly, the expressions for bending moment and shear force can be derived:

$$M_c = \frac{q}{4\lambda^2}\left(B_{\lambda a} + B_{\lambda b}\right)$$

$$Q_c = \frac{q}{4\lambda}\left(C_{\lambda a} - C_{\lambda b}\right)$$

These two equations can be derived using the same methodology used for deflection and slope. Till now, equations were derived for a point which is a distance 'a' from point A and 'b' from point B. Now, let us determine all the four quantities at point A which is at a distance 'b' from point B. As the point of interest is A now, the value of a is 0 and that of b is l (length of the UDL). Substituting these values in the expression for deflection, we get:

$$\Rightarrow w_C = \frac{q}{2k}\left[2 - D_{\lambda a} - D_{\lambda b}\right]$$

{a = 0 and b = l}

$$\Rightarrow w_A = \frac{q}{2k}\left[2 - 1 - D_{\lambda l}\right]$$

$$\Rightarrow w_A = \frac{q}{2k}\left[1 - D_{\lambda l}\right]$$

Similarly,

$$\theta_c = \frac{q\lambda}{2k}\left[A_{\lambda a} - A_{\lambda b}\right]$$

$$\Rightarrow \theta_A = \frac{q\lambda}{2k}\left[1 - A_{\lambda l}\right]$$

$$M_A = \frac{q}{4\lambda^2}\left[B_{\lambda l}\right]$$

$$Q_A = \frac{q}{4\lambda}\left(1 - C_{\lambda l}\right)$$

{If x = 0: $A_{\lambda x} = C_{\lambda x} = D_{\lambda x} = 1$ and $B_{\lambda x} = 0$}

These are the expressions of deflection, slope, bending moment and shear force at point A. Similarly these can be determined for point B.

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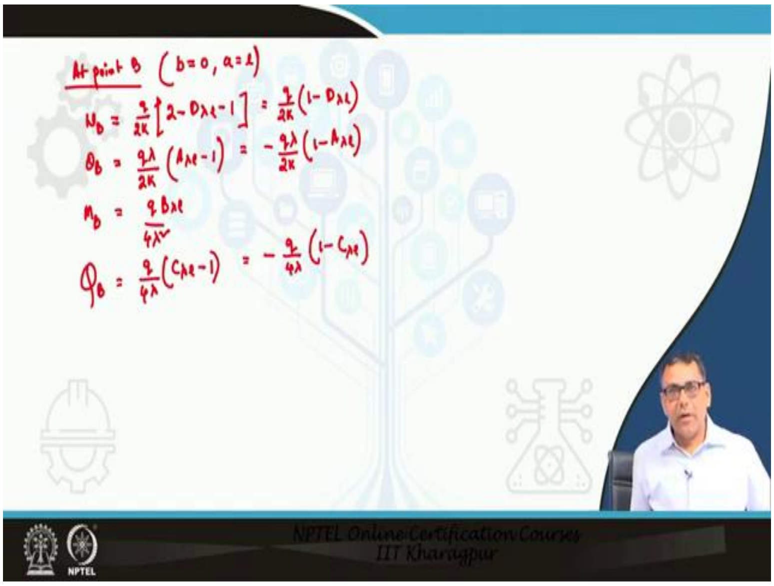

When the point of interest is B, b will be equal to 0 and a will be equal to l because here, point C and point B are same. Substituting these values in the equations of deflection, slope, bending moment and shear force, we get:

$$w_D = \frac{q}{2k}\left[2 - D_{\lambda l} - 1\right] = \frac{q}{2k}\left(1 - D_{\lambda l}\right)$$

$$\theta_B = \frac{q}{2k}\left(A_{\lambda l} - 1\right) = -\frac{q}{2k}\left(1 - A_{\lambda l}\right)$$

$$M_B = \frac{q}{4\lambda^2} B_{\lambda l}$$

$$Q_B = -\frac{q}{4\lambda}\left(1 - C_{\lambda l}\right)$$

In this way all the quantities can be calculated at the two edges of the load and at any point within the UDL.

In the next class I will solve one numerical example of this UDL case and then I will discuss two other cases where the point C is at left side of the loading and right side of the loading. After that, I will explain the case of UVL with triangular loading and I will try to derive the other equation for different loading condition. Thank you.