Soil Structure Interaction

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Lecture 20
Beams on Elastic Foundation (Contd.,)

In the last class I discussed about the infinite beam subjected to a concentrated load P and derived the equations for deflection, slope, bending moment and shear force.

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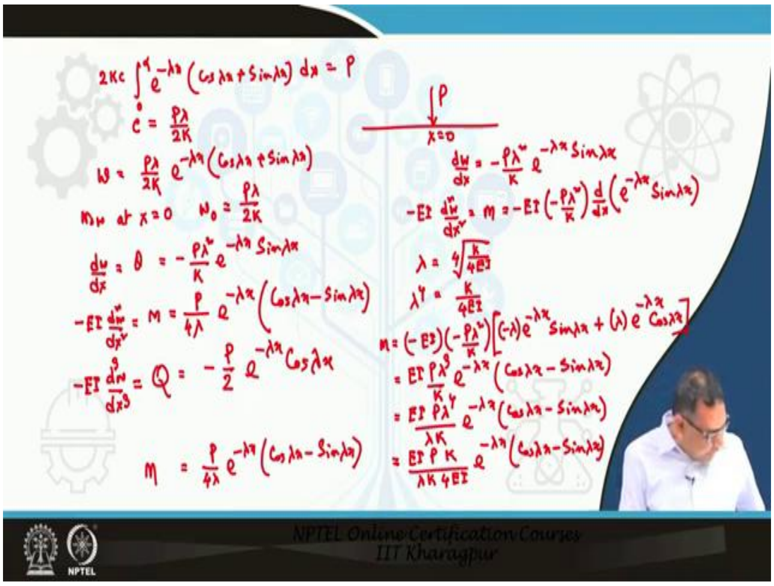

Let us start from the equations already given in the last class. The equations for all the four quantities are given, but in this class they will be derived. After that the points where these quantities will be maximum and also minimum or zero will be determined. The expressions for deflection and slope are:

$$w = \frac{P\lambda}{2k} e^{-\lambda x} [\cos \lambda x + \sin \lambda x]$$

$$\frac{dw}{dx} = \theta = -\frac{P\lambda^2}{k} e^{-\lambda x} \sin \lambda x$$

$$\Rightarrow \frac{dw}{dx} = -\frac{P\lambda^2}{k} e^{-\lambda x} \sin \lambda x$$

The bending moment expression can be obtained from the slope equation. Multiplying the above equation with -EI on both sides and differentiating it with respect to x:

$$\Rightarrow -EI \frac{d^2 w}{dx^2} = M = -EI\left(-\frac{P\lambda^2}{k}\right)\frac{d}{dx}\left(e^{-\lambda x} \sin \lambda x\right)$$

$$\Rightarrow M = -EI\left(-\frac{P\lambda^2}{k}\right)\left[(-\lambda)e^{-\lambda x}\sin\lambda x + (\lambda)e^{-\lambda x}\cos\lambda x\right]$$

$$\Rightarrow M = \left(\frac{EI \times P\lambda^3}{k}\right)e^{-\lambda x}\left[\sin\lambda x - \cos\lambda x\right]$$

$$\Rightarrow M = \left(\frac{EI \times P\lambda^3}{k}\right)e^{-\lambda x}\left[\cos\lambda x - \sin\lambda x\right]$$

$$\Rightarrow M = \left(\frac{EI \times P\lambda^4}{\lambda k}\right)e^{-\lambda x}\left[\cos\lambda x - \sin\lambda x\right]$$

We know that $\lambda = \sqrt[4]{\frac{k}{4EI}} \;\; \Rightarrow \lambda^4 = \frac{k}{4EI}$

$$\Rightarrow M = \left(\frac{EI \times Pk}{\lambda k 4EI}\right)e^{-\lambda x}\left[\cos\lambda x - \sin\lambda x\right]$$

$$\therefore M = \frac{P}{4\lambda}e^{-\lambda x}\left[\cos\lambda x - \sin\lambda x\right]$$

Similarly, by multiplying the equation of d^2w/dx^2 with -EI on both sides and differentiating it with respect to x, the equation for shear force can be determined.

$$-EI\frac{d^3w}{dx^3} = Q = -\frac{P}{2}e^{-\lambda x}\cos\lambda x$$

These are the four equations these are very important and are to be remembered.

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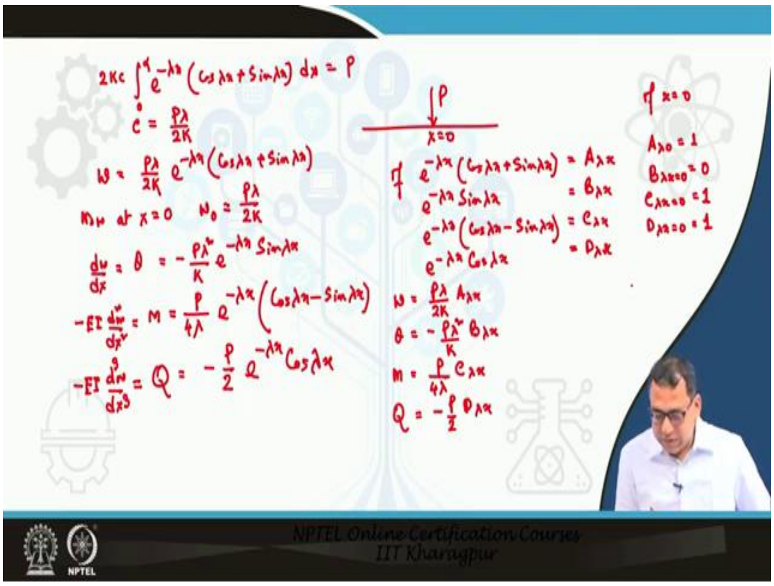

Next part in the infinite beam concept is to determine the points where each of the quantity will be 0 or minimum. Before that, let us define coefficients which will be used in this concept frequently.

$$e^{-\lambda x}(\cos \lambda x + \sin \lambda x) = A_{\lambda x}$$

$$e^{-\lambda x}(\sin \lambda x) = B_{\lambda x}$$

$$e^{-\lambda x}(\cos \lambda x - \sin \lambda x) = C_{\lambda x}$$

$$e^{-\lambda x}(\cos \lambda x) = D_{\lambda x}$$

By substituting the above values of coefficients in the formulae derived, we get:

$$w = \frac{P\lambda}{2k} A_{\lambda x}$$

$$\theta = -\frac{P\lambda^2}{k} B_{\lambda x}$$

$$M = \frac{P}{4\lambda} C_{\lambda x}$$

$$Q = -\frac{P}{2} D_{\lambda x}$$

If x = 0: $A_{\lambda x} = 1$; $B_{\lambda x} = 0$; $C_{\lambda x} = 1$; $D_{\lambda x} = 1$;

These 4 equations will be required later on to solve problems of beam under different types of loading conditions.

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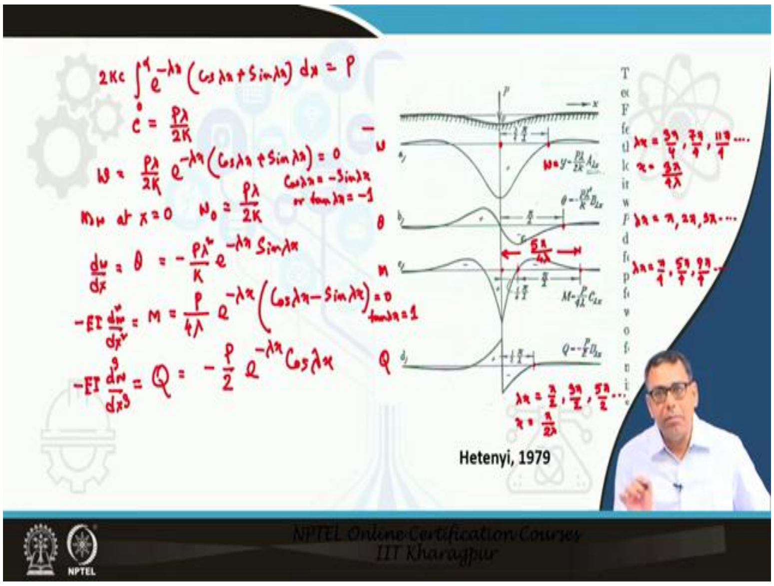

In the above slide, the profiles of deflection, slope, bending moment and shear force are given in the same order mentioned. These profiles and the equations mentioned next to the profile are valid only for a point load acting on an infinite beam.

Now, what are the points where the deflection will be 0? From the equation of deflection it is evident that is w should be zero, (cosλx + sinλx) should be zero:

$$\Rightarrow \cos\lambda x + \sin\lambda x = 0 \;\; \Rightarrow \cos\lambda x = -\sin\lambda x$$

$$\Rightarrow \tan\lambda x = -1$$

So, if tanλx value is equal to -1, the deflection will be 0. But tanλx will be -1 if: $\lambda x = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}$..... So, the nearest point of zero deflection from the load would be at a distance of (3π/4λ) units.

Similarly if slope has to be 0, sinλx should be 0. This is possible when: λx = 0, π, 2π.....

Similarly for bending moment to be 0: cosλx – sinλx = 0

$$\Rightarrow \cos\lambda x - \sin\lambda x = 0 \;\; \Rightarrow \cos\lambda x = \sin\lambda x$$

$$\Rightarrow \tan\lambda x = 1$$

So, if tanλx value is equal to 1, the bending moment will be 0. But tanλx will be 1 if: $\lambda x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}$....

Similarly for shear force to be 0 cosλx should be 0. This is possible when: $\lambda x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$....

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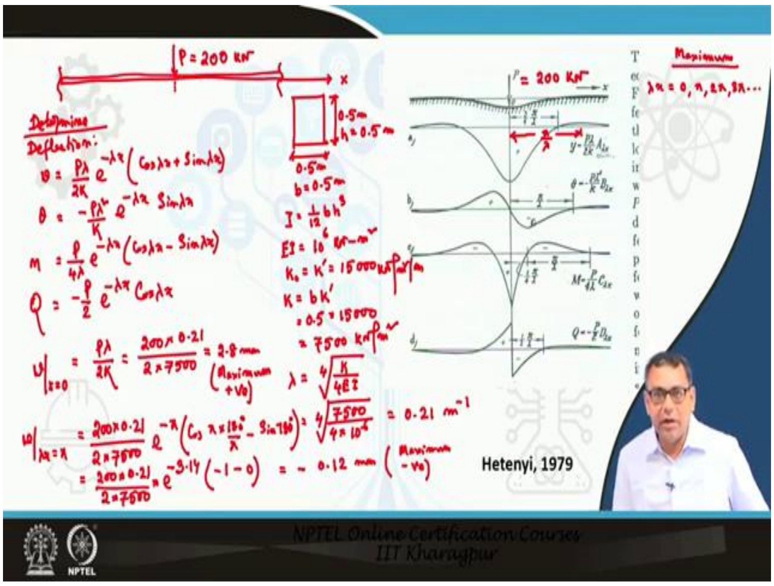

Let us now solve a numerical problem where an infinite beam is subjected to a point load, P = 200 kN. After finding the solution for this problem, we will try to find out the points at which maximum deflection, bending moment, shear force and slope occur. The point where the load is acting upon is treated as the 0 point (x = 0). The cross section of the beam is 0.5 m × 0.5 m i.e., b = h = 0.5 m. Usually the moment of inertia (I value) is calculated separately $bh^3/12$ and will be multiplied with the modulus of elasticity of the material to get the EI value. But here the EI value is directly given as 10^6 kN-m^2. The $k_o = k' = 15000$ kN/m^2/m.

So the k value can be found out by: $k = b \times k_\circ = 0.5 \times 15000 = 7500 kN/m^2$

The λ value can be determined by:

$$\lambda = \sqrt[4]{\frac{k}{4EI}} = \sqrt[4]{\frac{7500}{4 \times 10^6}} = 0.21\,\mathrm{m}^{-1}$$

The expressions for the four quantities are:

$$w = \frac{P\lambda}{2k} e^{-\lambda x} [\cos \lambda x + \sin \lambda x]$$

$$\theta = -\frac{P\lambda^2}{k} e^{-\lambda x} \sin \lambda x$$

$$M = \frac{P}{4\lambda} e^{-\lambda x} [\cos \lambda x - \sin \lambda x]$$

$$Q = -\frac{P}{2} e^{-\lambda x} \cos \lambda x$$

The points at which these quantities will be 0 are already discussed. The next question is what are the points at which these quantities will be maximum? The maximum value of these quantities should be calculated for any design purpose and they may be positive or negative. So, both maximum positive and maximum negative values should be determined.

If at a point, the deflection is maximum, then the slope or the first order derivative of the deflection, (dw/dx) should be 0. If slope has to be 0, the value of sinλx has to be 0. For this condition to be possible, the requirement is: λx = 0, π, 2π, 3π.....

So, at λx = 0, the deflection will be maximum and the next maximum will be at λx = π. If the deflection profile is observed, it can be understood that at λx = 0, the deflection is positive and at λx = π, the deflection is negative. So, at x = 0, the deflection is maximum positive and at x = π/λ, the deflection is negative.

If the x value increases, the value of deflection also will reduce as the distance from the load is increasing. In the expression the negative power to the distance term ($e^{-\lambda x}$) captures this effect properly. So as the x value increases, the w value decreases. This is why it is better to concentrate on the closer points when finding out the maximum values either negative or positive.

In a problem if it is mentioned to determine any quantity for a particular λx then substitute that λx value in the expressions or else if it is mentioned to determine the maximum value then both the maximum positive as well as negative value should be determined.

The deflection at x = 0 will be:

$$w\big|_{x=0} = \frac{P\lambda}{2k} = \frac{200\times 0.21}{2\times 7500} = 2.8mm \quad \text{(Maximum +ve)}$$

$$w\big|_{\lambda x=\pi} = \frac{200\times 0.21}{2\times 7500} e^{-\pi}\left[\cos\left(\pi\times\frac{180^\circ}{\pi}\right) - \sin 180^\circ\right]$$

$$w\big|_{\lambda x=\pi} = \frac{200\times 0.21}{2\times 7500} e^{-3.14}\left[-1-0\right] = -0.12mm \text{ (Maximum -ve)}$$

If the deflection is determined at x = 2π, the value will be definitely less than the above values. In the next class I will show the maximum points for the slope, bending moment and shear force. Then I will continue the problem of infinite beam for different loading condition. Thank you.