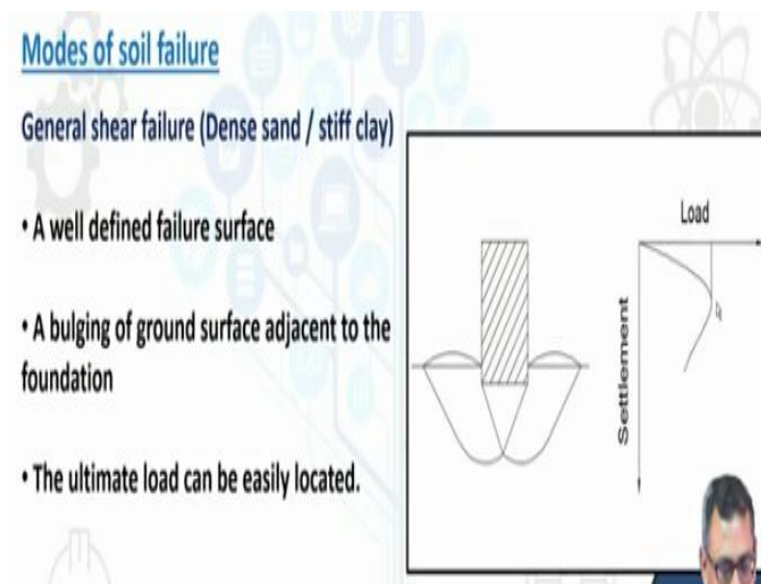


**Soil Structure Interaction**  
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**Lecture - 2**  
**Bearing Capacity of Soil**

Last class, I have discussed about various soil exploration techniques and the scope of disturbed sample and undisturbed sample usage along with the different design criteria for a shallow foundation. In this class, I will discuss about the bearing capacity and how to determine the bearing capacity of soil. So as I discussed, there are different terminology for the bearing capacity calculation listed: ultimate bearing capacity, the net ultimate bearing capacity, gross safe bearing capacity, net safe bearing capacity. The net safe bearing capacity is very important. Ultimately we will determine the net safe bearing capacity and then, then the safe bearing pressure in terms of settlement criteria. So, the smaller value of net safe bearing capacity and the safe bearing capacity in terms of settlement criteria will give the allowable bearing pressure.

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Now, we will discuss about the different modes of soil failure. When a load is applied on soil, it may fail in different modes which depend upon the type of soil. There are 3 types of failure modes as: general shear failure, local shear failure and punching shear failure. General shear failure occurs in dense sand or stiff clay. A definite failure surface develops within the soil that fails in general shear failure. There would be significant amount of bulging on both sides

of the foundation and the ultimate load can be easily determined. The load versus settlement graph gives a definite peak.

That means, the load settlement curve initially increases and then decreases giving a particular peak value. This peak value will give us the ultimate load of the foundation which is easy to locate.

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$c_u$ (kPa)	consistency	$D_r$ (%)	consistency
0 - 12.5	very soft	0-15	very loose
12.5-25	soft	15-35	loose
25-50	medium	35-65	medium
50-100	stiff	65-85	dense
100-200	very stiff	85-100	very dense
>200	hard		

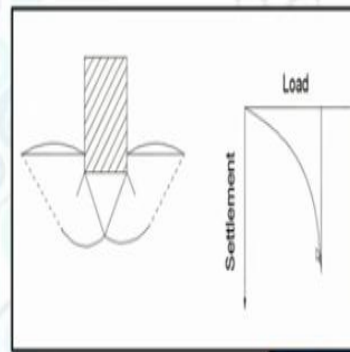
These are the different conditions to identify a soil as soft or medium or stiff as mentioned earlier, the general shear failure is applicable for stiff clay or dense sand. The undrained cohesion for a stiff soil would be between 50 and 100 kPa. If the undrained cohesion is within 0 to 12.5 kPa, then the soil is called very soft, 12.5 to 25 is for soft, 25 to 50 is for medium, 50 to 100 is for stiff, 100 to 200 is for very stiff and then if  $c_u$  is greater than 200 it is called hard.

Similar to consistency of different types of soil, there is a term called relative density which is usually applicable in granular soils. Relative density is the term by which one can understand the density status of the soil in the field. That means, for a soil in a very loose condition, the relative density should be between 0 and 15. But, for a dense soil where the general shear failure usually occurs, the relative density should be in between 65 and 85. The relative density values for a very dense soil would be 85 to 100 and for medium soil, 35 to 65%. So, if the relative density of a soil is determined, we can predict the type of failure the soil may undergo and for clay, this can be done by determining the consistency of it.

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Local shear failure (medium or relatively loose sand /medium and relatively soft consistency clay)

- Well defined wedge and slip surfaces only beneath the foundation
- Slight bulging of the ground surface adjacent to the foundation
- Load settlement curve does not indicate ultimate load clearly
- Significant compression of the soil directly beneath the footing

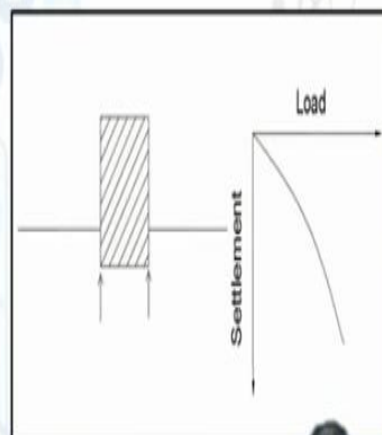


Local shear failure occurs in medium/relatively loose sand or medium/relatively soft clay. In case of a local shear failure, well defined wedges and slip surfaces will only be found beneath the foundation, and not on the sides of the foundation. Although, there would be a slight bulge in the ground surface adjacent to the foundation, it is not as significant as compared to the general shear failure. Besides, the load settlement curve does not indicate any ultimate load clearly. That means there would be no definite peak of the load settlement curve, as the curve increases initially and becomes almost parallel to the settlement axis. Another characteristic of the local shear failure is that the soil directly beneath the foundation undergoes significant compression.

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Punching shear failure (very loose sand / very soft clay)

- Poorly defined shear planes
- Soil zones beyond the loaded area being little affected
- Significant penetration of a wedge shaped soil zone beneath the foundation
- Ultimate load can not be clearly recognized



Next in the failure modes is punching shear failure which occurs in very loose sand and very soft clay. Here, there would be no definite failure surface even below the footing. The soil

zones beyond the loaded area will not be affected, which means there would be no bulging or a very little effect would be noticed. Significant penetration of the wedge shaped soil zone beneath the foundation can be expected which in turn results in significant penetration of the foundation, but within the footing zone. So, the ultimate load cannot be clearly recognized in the local shear failure.

These are the different failure modes that may occur in various types of soil.

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Terzaghi's bearing capacity theory:

The footing is a long strip or a continuous footing resting on a deep homogeneous soil having shear parameter  $c$  and  $\phi$ .

- Analysis is a 2-D condition i.e for strip footing
- The soil fails in a general shear failure mode
- The load is vertical and concentric

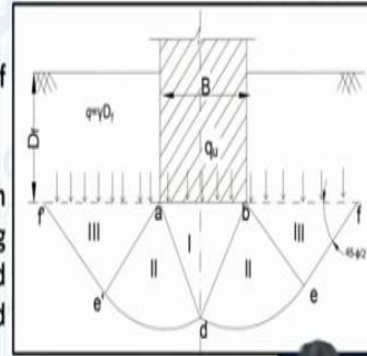
Based on the observations discussed so far, the first bearing capacity theory was proposed by Terzaghi which is very popular. This equation is generally used to determine the bearing capacity of soil. When Terzaghi derived this bearing capacity theory, he assumed or he derived this theory for strip footing or the continuous footing and it is assumed that the soil is homogeneous which fails in general shear failure. Terzaghi's bearing capacity was initially developed for dense soil and stiff clay where the general shear failure will occur, so that there will be a definite failure surface below and beyond the loaded region. The load is considered perfectly vertical and acting at the center of the footing.

These are the conditions or assumption in Terzaghi's bearing capacity theory.

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Zone – I (zone abd)

- The soil in this zone remains in a state of elastic equilibrium
- The soil wedge abd immediately beneath the footing is prevented from undergoing any lateral movement by the friction and adhesion between the base of footing and soil.



Zone II (bed and ae'd) : Zone of radial shear

Zone III (bef and ae'f) : Rankine passive zone

Terzaghi assumed 3 different zones: a triangular portion, zone-1 and a curved portion, zone-2 and again there would be a straight line portion, zone-3. The zone-1 is called the state of static equilibrium. The zone-2 is the zone of radial shear and zone-3 is called the Rankine passive zone. Here, the effect of soil on the bearing capacity is taken as a surcharge effect.

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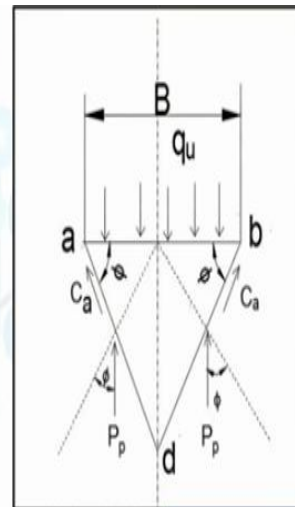
The equation developed for the ultimate bearing capacity is

$$q_u = cN_c + \gamma D_f N_q + \frac{1}{2} \gamma B N_\gamma$$

$$N_c = \cot \phi \left[ \frac{a^2}{2 \cos^2 \left( 45^\circ + \frac{\phi}{2} \right)} - 1 \right] \quad N_q = \left[ \frac{a^2}{2 \cos^2 \left( 45^\circ + \frac{\phi}{2} \right)} \right]$$

$$N_\gamma = \frac{1}{2} \left[ \frac{K_p}{\cos^2 \phi} - 1 \right] \tan(\phi)$$

where  $a = e^{\left( \frac{3\pi}{4} - \frac{\phi}{2} \right) \tan \phi}$



The different forces acting on the wedge and then finally these expressions are proposed. There are 3 terms in this equation: the first term is  $cN_c$ ; the second term is  $\gamma D_f N_q$  and the third term is  $\frac{1}{2} \gamma B N_\gamma$ .  $N_c$ ,  $N_q$ ,  $N_\gamma$  are the bearing capacity factors. The expressions for the bearing capacity factors are given in the slide and as it is evident, they are a function of  $\phi$ , friction angle. The factor, 'a' is also a function of friction angle,  $\phi$ . Note that this equation was developed and is valid only for a strip footing.

As a summary of defining all the terms of the equation:-

$N_c$ ,  $N_q$ ,  $N_\gamma$  are the bearing capacity factors

$c$  is the cohesion of the soil

$\gamma$  is the unit weight of the soil

$D_f$  is the depth of foundation

$B$  is the width of foundation

The first term in the bearing capacity equation considers the contribution due to cohesion. The second term is  $\gamma \times D_f$  which is nothing but the surcharge of the soil above the foundation level. The third term is the contribution due to the unit weight of the soil below the foundation.

There are two unit weights ( $\gamma$ ) in the equation which means that for a homogeneous soil, the same  $\gamma$  can be used for both the terms. But ideally, the second term  $\gamma$  is the unit weight of the soil above foundation level (from the ground level to the foundation level) and the third term  $\gamma$  is the unit weight of the soil below the foundation level.

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$\phi$	Terzaghi's Bearing Capacity Factor		
	$N_c$	$N_q$	$N_\gamma$
0	5.7	1.0	0.0
5	7.3	1.6	0.5
10	9.6	2.7	1.2
15	12.9	4.4	2.5
20	17.7	7.4	5
25	25.1	12.7	9.7
30	37.2	22.5	19.7
35	57.8	41.4	42.4
40	95.7	81.3	100.4
45	172.3	173.3	297.5
50	347.5	415.1	1153.2

Terzaghi had also given in tabular form, the different bearing capacity factors ( $N_c$ ,  $N_q$ ,  $N_\gamma$ ). Values are given for different  $\phi$  values ranging from 0 to 50° of soil. For a  $\phi$  value of 0° (purely cohesive soil), the  $N_c$  value is 5.7,  $N_q$  is 1 and  $N_\gamma$  is 0.

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### Ultimate bearing capacity for local shear failure

$$\text{Mobilized cohesion: } c_m = \frac{2}{3}c$$

$$\text{Mobilized angle of shearing resistance: } \phi_m = \tan^{-1}\left(\frac{2}{3}\tan\phi\right)$$

$$q_u = \frac{2}{3}cN'_c + \gamma D_f N'_q + \frac{1}{2}\gamma B N'_\gamma$$

Terzaghi's bearing capacity expression was initially applicable only in case of general shear failure, but it can also be used for local shear failure condition by modifying the  $c$  and  $\phi$  values. This modification includes reducing the  $c$  value to two-thirds of the original value  $\left(c_m = \frac{2}{3}c\right)$ . The mobilized angle of shearing resistance ( $\phi_m$ ) is given by:

$\phi_m = \tan^{-1}\left(\frac{2}{3}\tan\phi\right)$  So, instead of using  $\phi$  and  $c$ , the mobilized values,  $c_m$  and  $\phi_m$  should be used. So, for the equation in case of local shear failure, the bearing capacity factors would be chosen based on the new  $\phi$  value. The ultimate bearing capacity in this case will be:

$\frac{2}{3}cN'_c + \gamma D_f N'_q + \frac{1}{2}\gamma B N'_\gamma$ . So,  $N'_c$ ,  $N'_q$  and  $N'_\gamma$  means that it is for local shear failure and the bearing capacity factors are determined based on  $\phi_m$  but not  $\phi$ .

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For sandy soil ( $c' = 0$ )

- $\phi \geq 36^\circ$  - Purely general shear failure,  $\phi \leq 29^\circ$  - Purely local shear failure
- $\phi$  between this range represents the mixed state of general and local shear failure

For c- $\phi$  soil

- Failure of soil specimen occur at a relatively small strain (less than 5%) - General shear failure
- If stress – strain curve does not show peak and has a continuously rising pattern upto a strain of 10- 20% - Local shear failure

The next question is that when to consider general shear failure and when, local shear failure. There is a guideline given for this purpose. If soil has its friction angle,  $\phi \geq 36^\circ$  then the soil is expected to fail by general shear failure, and if  $\phi \leq 29^\circ$ , local shear failure is likely to occur. If the  $\phi$  value lies in between  $29^\circ$  and  $36^\circ$ , a mixed type of failure may be encountered. So, if  $\phi \geq 36^\circ$ , directly we can use the  $N_c$ ,  $N_q$ ,  $N_\gamma$ , the original Terzaghi's bearing capacity factors as well as equation. If  $\phi \leq 29^\circ$ , the  $\phi$  and  $c$  values should be modified.

For example: if  $\phi = 29^\circ$ , convert it to  $\phi_m$  which would be equal to:  $\tan^{-1}\left(\frac{2}{3} \tan 29^\circ\right)$ .

Then based on that  $\phi_m$ , determine the bearing capacity factors, which will be  $N_c'$ ,  $N_q'$  and  $N_\gamma'$ . If the  $\phi$  value lies between  $29^\circ$  and  $36^\circ$ , linear interpolation should be done to determine the bearing capacity factors. In case of a c- $\phi$  soil, the mode of failure cannot be decided either by the  $\phi$  value or  $c$  value alone in which case, an analysis of the load settlement curve of the soil would be required.

If the stress-strain curve or the load settlement curve does not show any peak and has a continuously rising pattern up to a strain of 10 to 20%, then we will go for the local shear failure. If the curve indicates failure of soil specimen at a relatively small strain, less than 5%, then we will go for general shear failure.

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## Ultimate bearing capacity of strip, square, circular and rectangular footing

$$q_u = \alpha_1 c N_c + \gamma D_f N_q + \alpha_2 \gamma B N_\gamma$$

For  $\phi = 0$  (saturated clay),  $q_{nu} = 5.7 c_u$

For strip footing:  $\alpha_1 = 1.0, \alpha_2 = 0.5$

The effect of submergence is to reduce the undrained shearing strength  $c_u$  due

to a softening effect. The shear strength parameter should be determined

the laboratory under saturated condition.

For circular footing:  $\alpha_1 = 1.3, \alpha_2 = 0.3$

For Rectangular Footing:  $\alpha_1 = \left(1 + 0.3 \frac{B}{L}\right)$   $\alpha_2 = 0.5 \left(1 - 0.2 \frac{B}{L}\right)$

*L = Length of the footing*

Originally Terzaghi's equation was applicable only for strip footing, but later on it has been modified and now it can be used for square footing, circular footing or rectangular footing but only with the application of certain correction factors. Two correction factors are introduced:  $\alpha_1$  and  $\alpha_2$  where,  $\alpha_1$  is applied for the first term ( $\alpha_1 \times c N_c$ ) and  $\alpha_2$  is applied for the third term.

Now, for a strip footing:  $\alpha_1$  and  $\alpha_2$  will be half because in the original expression, it is  $(1/2 \times \gamma B N_q)$ . For a square footing,  $\alpha_1$  and  $\alpha_2$  will be 1.3 and 0.4 respectively. For a circular footing,  $\alpha_1$  and  $\alpha_2$  will be 1.3 and 0.3. For a rectangular footing,  $\alpha_1$  will be  $\left(1 + 0.3 \frac{B}{L}\right)$  and  $\alpha_2$  will be

$0.5 \left(1 - 0.2 \frac{B}{L}\right)$  [B is the width of the footing and L is the length of the footing]

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### Effect of water table :

$q_u = c N_c + q N_q + 0.5 \gamma B N_\gamma$  reduced as the effective weight below the water table is equal to the submerged unit weight.

For  $\phi = 0$  (saturated clay),  $q_{nu} = 5.7 c_u$

The effect of submergence is to reduce the undrained shearing strength  $c_u$  due to a softening effect. The shear strength parameter should be determined in the laboratory under saturated condition.

The next aspect to be considered is the water table effect because in Terzaghi's original equation, the water table effect was not introduced. But when a foundation is being designed, the water table should be taken into consideration. Now how we will incorporate the water table effect?

When bearing capacity is being worked out for a saturated clay ( $\phi = 0$ ), the  $N_\gamma$  would be zero and  $N_q = 1$ . So, the third term in Terzaghi's equation would be zero and  $q_u$  would be equal to  $(cN_c + q)$ . But the net ultimate bearing capacity is:  $q_{nu} = q_u - q$  (where,  $q = \gamma \times D_f$ ). So,  $q_{nu}$  would be equal to  $c_u N_c$ . But if  $\phi=0$ ,  $N_c$  will be 5.7 which makes  $q_{nu} = 5.7N_c$ .

So for a saturated clay ( $\phi = 0$ ), net ultimate,  $q_{nu}$  will be  $5.7c_u$ .

The effect of submergence on soil is to reduce the undrained shearing strength,  $c_u$  due to softening effect. The shear strength parameters should be determined in the laboratory under saturated condition. Remember that whenever soil is under submerged condition, its cohesion or the strength will reduce. So when it is needed to determine the properties of clay or any soil, it should be done under submerged condition or the saturated condition.

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Water table located above the base of footing:

The effective surcharge is reduced as the effective weight below the water table is equal to the submerged unit weight.

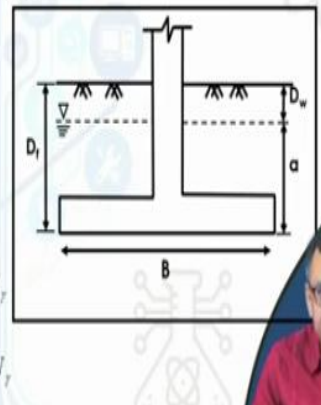
$$q = D_w \gamma + a \gamma'$$

$$\text{As, } a = D_f - D_w \quad q = \gamma' D_f + (\gamma - \gamma') D_w$$

$$q_u = c_u N_c + [\gamma' D_f + (\gamma - \gamma') D_w] N_q + \frac{1}{2} \gamma' B N_\gamma$$

$$\text{If } D_w = 0 \text{ (i.e., } a = D_f) \quad q_u = c_u N_c + \gamma' D_f N_q + \frac{1}{2} \gamma' B N_\gamma$$

$$\text{If } a = 0 \text{ (i.e., } D_f = D_w) \quad q_u = c_u N_c + \gamma D_f N_q + \frac{1}{2} \gamma' B N_\gamma$$



The effect of water table is actually introduced in Terzaghi's equation by using the effective surcharge in the calculations. This implies using a reduced surcharge as the effective weight below the water table is equal to the submerged unit weight.

As we are using the effective stress of soil we should know of total stress and effective stress. When a load is applied on soil, the soil skeleton takes some portion of the stress and the water takes some portion of the stress. Due to this, pore pressure will be generated in the soil. The

effective stress is the stress taken by the soil skeleton only. That means, we can express effective stress as: total stress minus pore water pressure. ( $\sigma' = \sigma - u$ )

From the figure:  $D_w$  is the position of water table below the ground level;

$D_f$  is the depth of foundation and

$B$  is the width of foundation.

The soil above the ground level is not in submerged condition, but the soil below the ground level is submerged condition. Initially when there is no water,  $q = \gamma \times D_f$ . Now with the water table,  $q$  will be  $(D_w \times \gamma + a \times \gamma')$ . Now what is  $\gamma'$ ?  $\gamma'$  is basically the effective unit weight of soil.

At a point at a depth of 'h' from ground level with water table at the ground level, effective stress will be: total stress,  $(\gamma \times h) - (\gamma_{\text{water}} \times h)$ . ( $\sigma' = \gamma h - \gamma_w h$ )

where,  $\gamma$  is the unit weight of soil (here, as the soil is in submerged condition,  $\gamma$  is  $\gamma_{\text{saturated}}$  or  $\gamma_{\text{sat}}$ ) and  $\gamma_{\text{water}}$  is the unit weight of water.

Now, effective stress,  $(\sigma' = (\gamma - \gamma_w)h) \Rightarrow (\sigma' = \gamma'h)$

$\gamma'$  is the submerged unit weight of soil (may be referred to as  $\gamma_{\text{sub}}$  sometimes).

The same concept is used here, that here  $(D_w \times \gamma + a \times \gamma')$ . As the soil below water table will be submerged condition,  $\gamma'$  is used here and 'a' is the height, so:  $(a \times \gamma')$ . Although initially, both were  $\gamma$  and this may be different  $\gamma_s$  according to the condition. It may be  $\gamma_{\text{bulk}}$  above the water table or  $\gamma_{\text{sat}}$ , (saturated unit weight).  $\gamma_w$  is the unit weight of the water, generally taken as  $10 \text{ kN/m}^3$ . Sometimes, you may find in some problems that  $\gamma_{\text{bulk}}$  will be equal to  $\gamma_{\text{sat}}$  and in some cases, these two can be different. You can refer to any soil mechanics book for the detailed derivation of this equation. Also, there would be detailed explanation about  $\gamma_{\text{bulk}}$ ,  $\gamma_{\text{sat}}$ ,  $\gamma_w$  and  $\gamma_{\text{sub}}$  or  $\gamma'$ .

If  $\gamma_{\text{sat}}$  and  $\gamma_{\text{bulk}}$  are different, then two different values will be given for: unit weight of the soil above water table and the saturated unit weight of the soil below water table. Finally, the equation for  $q$  will be:  $(\gamma \times D_w) + (a \times \gamma_{\text{sub}})$ .

If I substitute  $a = (D_f - D_w)$ , the expression changes to:  $q = \gamma'D_f + (\gamma - \gamma')D_w$  ( $D_w$  is the height of the water table from the ground). Now, substitute the value of  $q$  in the bearing capacity

$$\text{equation: } q_u = c_u N_c + [\gamma'D_f + (\gamma - \gamma')D_w] N_q + \frac{1}{2} \gamma' B N_\gamma.$$

Here, as the water table is above the footing base, the third term will be definitely  $\gamma_{\text{sub}}$  or  $\gamma'$ . If the water table is at the ground level, the second term would also be  $\gamma'$  along with the third term. As, here it is not at ground level, we have to modify it in this way.

The next case we will discuss is if  $a = 0$ , that means water table is at the base of the foundation and  $D_w$  will be  $D_f$ . Now the second term  $\gamma$  will be  $\gamma_{\text{bulk}}$  because there water table is not present, but the third term  $\gamma$  will be  $\gamma_{\text{sub}}$  because third term  $\gamma$  represents the soil below the footing base which is in totally submerged condition. So, third term will be  $\gamma_{\text{sub}}$  or  $\gamma'$ .

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Water table located at a depth  $b$  below the base of footing

In this case, the surcharge term is not affected. However, the unit weight in the third term of bearing capacity equation is modified as

$$\bar{\gamma} = \gamma' + \frac{b}{B}(\gamma - \gamma')$$

$$q_u = c_u N_c + \gamma D_f N_q + \frac{1}{2} B \left[ \gamma' + \frac{b}{B}(\gamma - \gamma') \right] N_\gamma$$

*Handwritten notes:  $\gamma = \gamma_{\text{bulk}}$ ,  $\gamma' = \gamma_{\text{sub}}$*

The next case is when the water table lies below the footing base. If the water table position is below the footing level at a depth which is greater than or equal to the width of foundation from the footing level, then the water table effect need not be considered. But, if the water table is located within a distance  $B$  from the base of the foundation, then our expression will be:  $q_u = c_u N_c + \gamma D_f N_q + \frac{1}{2} \left[ \gamma' + \frac{b}{B}(\gamma - \gamma') D_w \right] B N_\gamma$ . The  $\gamma$  in second term would be  $\gamma_{\text{bulk}}$  or simply  $\gamma$  as the water table is not affecting the soil above the foundation.

So instead of using only  $\gamma$ , we should be using  $\left[ \gamma' + \frac{b}{B}(\gamma - \gamma') D_w \right]$ . Usually if only  $\gamma$  is given, it means that it is the bulk density,  $\gamma_{\text{bulk}}$  and sometimes  $\gamma'$  may be written as  $\gamma_{\text{sub}}$ . In the last equation, if  $b=B$  ( $b$  is the depth of water table from base of the footing), this equation turns to the original bearing capacity equation indicating that the water table effect need not be incorporated.

In the next class, I will discuss about another condition where load is inclined and not acting at the center. In that case, how to determine the bearing capacity expression, bearing capacity of the foundation will be discussed in the next class. Thank you.