**Soil Structure Interaction**
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**Lecture 17**
**Different Foundation Models (Contd.)**

In the previous lecture we have discussed about the time-dependent response of the soil and how to incorporate the time dependent response in the available models. Today I will first summarise all the models that I have discussed till now. And then I will discuss about the beam foundation or the elastic foundation.

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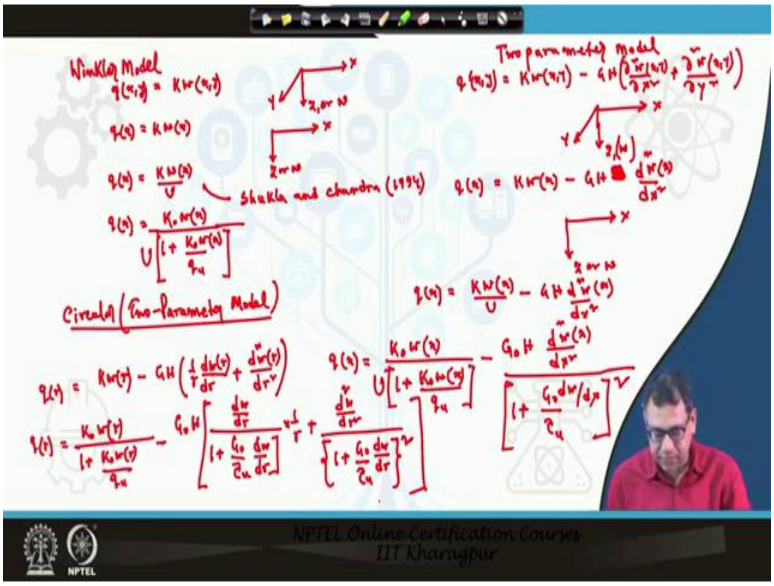


If the directions of x, y and z or w are as shown in the figure above, let us see how the equations for various models will be. Firstly, for Winkler model:

$$q(x, y) = kw(x, y)$$

For 1D case: $q(x) = kw(x)$

Time-dependent response: $q(x,t) = \dfrac{k}{U} w(x,t)$

Time-dependent and non-linear response: $q(x,t) = \dfrac{k_{\circ} w(x,t)}{U\left[1 + \dfrac{k_{\circ} w(x,t)}{q_u}\right]}$

The first equation is for the 2D condition in the directions x and y whereas the second equation captures only the variations in x direction. The time-dependent response is given by Shukla and Chandra (1994) which is already mentioned.

Now for the two parameter model:

$$q(x,y) = kw(x,y) - GH\left(\frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2}\right)$$

Equation for the 1D case:

$$q(x) = kw(x) - GH\left(\frac{d^2 w(x)}{dx^2}\right)$$

Time-dependent and linear response:

$$q(x,t) = \frac{kw(x,t)}{U} - GH\frac{d^2 w(x,t)}{dx^2}$$

Time-dependent and non-linear response:

$$q(x,t) = \frac{k_\circ w(x,t)}{U\left[1 + \frac{k_\circ w(x,t)}{q_u}\right]} - \frac{G_\circ H \frac{d^2 w(x,t)}{dx^2}}{\left[1 + \frac{G_\circ}{\tau_u}\frac{dw(x,t)}{dx}\right]^2}$$

For the circular case two parameter model, the expression is:

$$q(r) = kw(r) - GH\left[\frac{1}{r}\frac{dw(r)}{dr} + \frac{d^2 w(r)}{dr^2}\right]$$

After incorporating non-linearity:

$$q(r) = \frac{k_\circ w(r)}{1 + \frac{k_\circ w(r)}{q_u}} - H\left[\frac{G_1}{r}\frac{dw(r)}{dr} + G_2\frac{d^2 w(r)}{dr^2}\right]$$

This is the summary of all the expressions that are developed till now. These expressions will be used later on, for different models.

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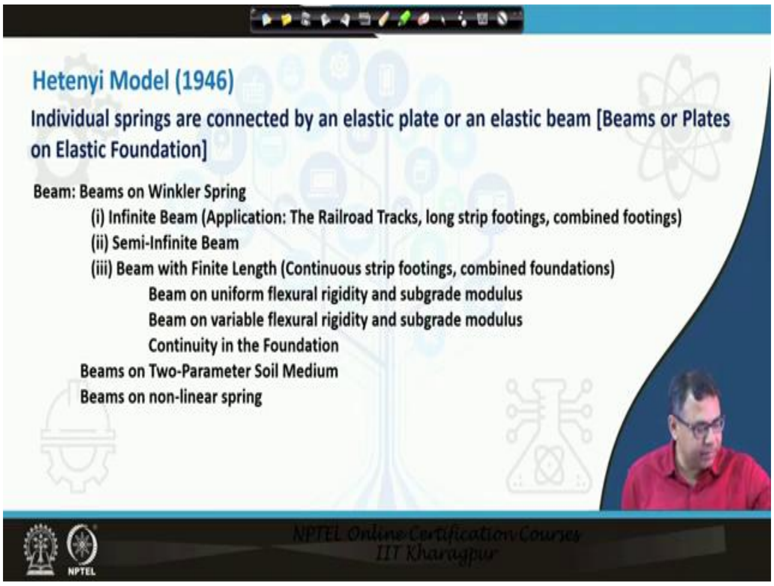


The next model in the discussion is the Hetenyi model. In the two parameter model or Pasternak model, the springs are connected with elastic plate or by an elastic beam. So this section can be named as beams and plates on elastic foundation. The things that will be covered under this topic is about the various types of beams, various types of plates and the application area of these different types of beams or plates.

First, the discussion will be mainly about the beams on Winkler springs in which three types of beams will be dealt with. They are the infinite beam, the Semi-infinite beam and the Beam with Finite Length. Now the question is how to define each beam and what is the application of each beam. Let us start with the application of the infinite beam. The infinite beam application is the railroad track or the long strip of footings or combined footing.

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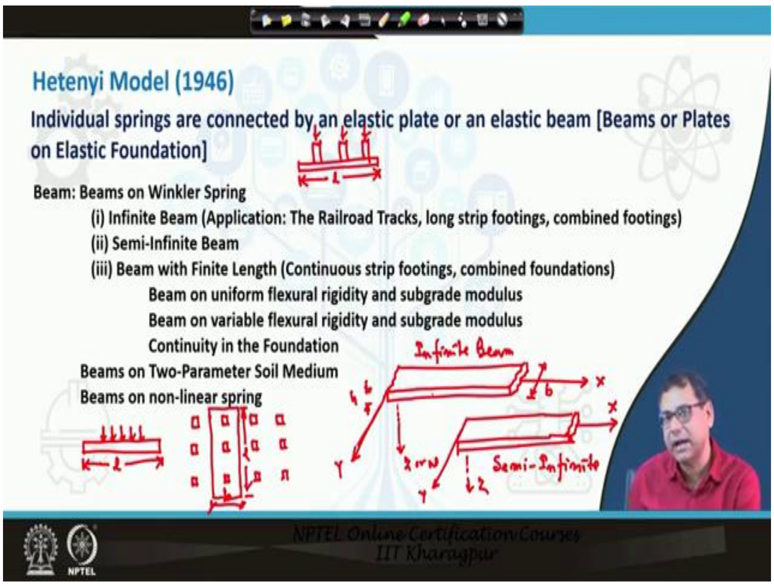

The infinite beam may be shown in the way represented in the above slide. Given the directions of x, y and z or w, the width of the beam may be represented by b and the thickness of the beam by h. The x direction is the direction of the beam length and it is infinite. But in the width and thickness directions, the beam has finite values. As mentioned already, the analysis of infinite beam is required to design a railroad track or a combined footing or a long strip footing.

The next one is about the semi-infinite beam for which the applications are similar to that of infinite beam. But, what is a semi-infinite beam and what is the difference between a semi-infinite beam and an infinite beam? In an infinite beam, if a load is applied at a particular point on the beam, the distance to the left side and right side of the load will be infinite. But in the semi-infinite beam one side of the load in the beam will be finite and the other side, infinite. If the above slide is referred to for the semi-infinite beam figure, it can be noted that one side is finite and the other is infinite.

The next type of beam is the finite beam pr the beam with finite length. The strip footing and combined footing can be analyzed as an infinite beam as well as finite beam. If a load is applied on a finite beam the distance from the load and both ends of the beam in length direction is finite. Suppose, for a structure a number of columns are to be laid in a square or rectangular pattern. The columns in one row or column may be provided with a combined footing which can be analysed as a finite beam. The beam with finite length can be under plane strain condition also, but that will be discussed later on.

So the types of beams are: infinite beam, where the length on both sides of the loading is infinite, the semi-infinite beam, where the length on one side of the loading is finite (and the other side of the loading, infinite) and the beam with finite length with both the sides of the loading, the beam has a finite length. Then we can analyse for the beams with uniform flexural rigidity resting on soil of uniform subgrade modulus and beams with variable flexural rigidity resting on soil of variable subgrade modulus. After that we will discuss about the continuity in the foundation. Later, after discussing about the beams on two parameter model and beams on non-linear springs, the discussion about plates will be taken up.

The subgrade modulus of the soil may be constant and may be varying along the entire length of the beam. If it is constant, it means that it is not a function of x and if it is varying along the beam length, it will be a function of x. In the previous model it was discussed that the deformation will be uniform, but it is true only if the subgrade modulus is uniform throughout x or throughout the beam length. If the subgrade modulus under a certain footing is non-uniform, then the deformation will also be non-uniform even if uniform load is applied. So, uniform deformation can be obtained only if both subgrade modulus and the load are uniform.

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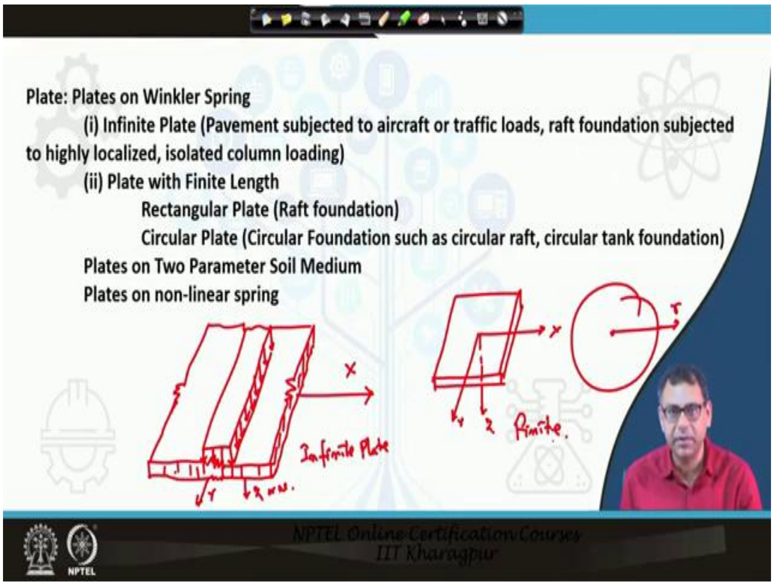


Within the discussion about plates on Winkler springs, again infinite plate and finite plate will be covered. Infinite plate has its applications where the pavement is subjected to aircraft or traffic loads, raft foundation subject to highly localised or isolated column loading. Then the finite plate can be a rectangular plate (raft foundation) or a circular plate (circular raft and circular tank foundation). Then the plates resting on two parameter soil medium and on the nonlinear springs will be discussed.

An infinite plate may be defined as a plate which is infinite in all four directions (+x, -x, +y & -y). The directions of x, y and z or w are shown in the slide above. The finite plate, as already discussed can be rectangular or circular, but is finite in all the four directions. If the plate is circular, the directions will be r and z or w. The application areas of these are already mentioned.

Now, let us derive the basic differential equation for a beam which can be used for any type of beam. Then we will go specifically for infinite beam, semi-infinite beam and then for the beam with finite length.

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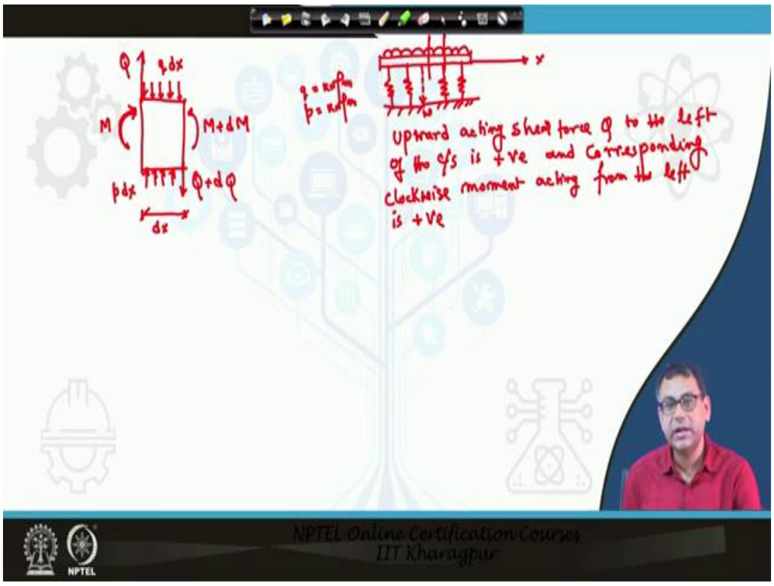


Consider a beam with its length in the horizontal direction along the x-axis. The vertical direction, along which the settlement of beam occurs, is the w axis or z axis. Now, this beam rests on soil, which is idealized as springs. Imagine that a UDL of intensity q is applied over the beam. Remember that the beam is an elastic beam resting on springs. If a small segment of length ‘dx’ of the beam is considered, what will be the forces acting on that segment?

First the q UDL will act on the top of it, shear force, Q will act on one of the sides of the element (say, left side) and a bending moment, M. The Q and M are considered to be acting on the left side of the element. Now, on the right side of the element, there would be a very small difference in the shear force & bending moment and hence are considered to be (Q + dQ) & (M + dM) respectively. There will be a reaction force acting on the base of the footing from the springs which can be considered as p.

The units of q will be kN/m as it is the intensity of UDL and so, the total load acting on the element due to the UDL is q × dx. Similarly the units of p will be kN/m and hence the total load or force due to the spring reaction will be p × dx. These are the forces that act on a small segment of the beam.

Now, the sign convention when designing or analysing a beam is very important. The upward acting shear force, Q to the left of the cross section of the beam is considered positive. That means shear force acting upward and is to the left of the beam (or observation point) should be considered positive. Similarly the clockwise moment acting on the left is again positive i.e., M is positive. That means a clockwise moment acting on the left side of the beam (or observation point) should be considered positive. This is the sign convention to be followed.

In the next class I will derive the basic differential equation of this beam considering all these forces and then I will give you the idea how to solve the differential equation for a finite as well as an infinite beam. Thank you.