Soil Structure Interaction

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Lecture 16
Different Foundation Models (Contd.)

In this class I will discuss the governing differential equation of a two parameter model under the axi-symmetrical circular loading condition. Then I will discuss about the time dependent effect on the settlement response.

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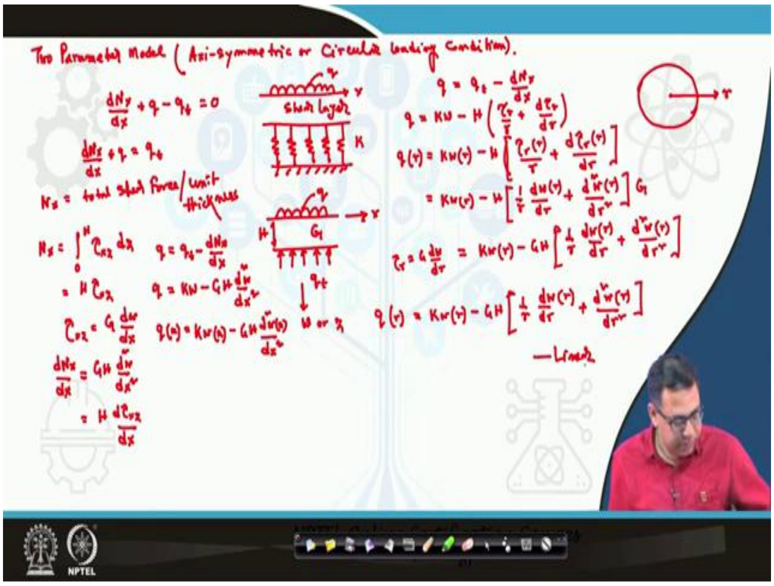

Let us start with the governing differential equation of a plate resting on two parameter model under axi-symmetric or circular loading condition. In the two parameter model, the differential equation for a beam under UDL was defined as:

$$\frac{dN_x}{dx} + q - q_t = 0$$

$$\Rightarrow \frac{dN_x}{dx} + q = q_t$$

where, N_x is the total shear force per unit thickness, q is the intensity of the load or UDL and q_t is the uniform reaction from the springs beneath the shear layer. The thickness of the shear layer is H, the value of shear modulus is G and the spring constant is k (for the springs representing soft soil). This was under 1-D condition where the right side horizontal direction is the x direction and the vertical direction is represented as the direction of w (settlement).

Now, the N_x value can be determined by:

$$N_x = \int_0^H \tau_{xz} dz$$

$$\Rightarrow N_x = H\tau_{xz}$$

Let us first start with the 1-D case and then can formulate equations for the axi-symmetric case. It is known that:

$$\tau_{xz} = G\frac{dw}{dx}$$

By substituting the above value in the expression for N_x and differentiating it with respect to x, we get:

$$\frac{dN_x}{dx} = GH\frac{d^2w}{dx^2}$$

$$\Rightarrow \frac{dN_x}{dx} = H\frac{d\tau_{xz}}{dx}$$

Now from the basic equation for the two parameter model,

$$\Rightarrow q = q_t - \frac{dN_x}{dx}$$

$$\Rightarrow q = kw - GH\frac{d^2w}{dx^2}$$

$$\Rightarrow q(x) = kw(x) - GH\frac{d^2w(x)}{dx^2}$$

This is the expression that is derived for the 1D case. Usually q and w are functions of x as the loading and the deflection may change when x changes.

Now let us get started with the axi-symmetric condition or the circular condition where the radial direction is represented by r and the downward direction, with w.

So the basic equation, $q = q_t - \frac{dN_x}{dx}$ for the 1D case, can be written as:

$$q = kw - H\left(\frac{\tau_r}{r} + \frac{d\tau_r}{dr}\right)$$

$$\Rightarrow q(r) = kw(r) - H\left[\frac{\tau_r(r)}{r} + \frac{d\tau_r(r)}{dr}\right]$$

$$\because \left\{\tau_r = G\frac{dw}{dr}\right\}$$

$$\Rightarrow q(r) = kw(r) - GH\left[\frac{1}{r}\frac{dw(r)}{dr} + \frac{d^2w(r)}{dr^2}\right]$$

This equation is for the model under linear case. If the non-linearity should be introduced in this model, the formulation is explained below.

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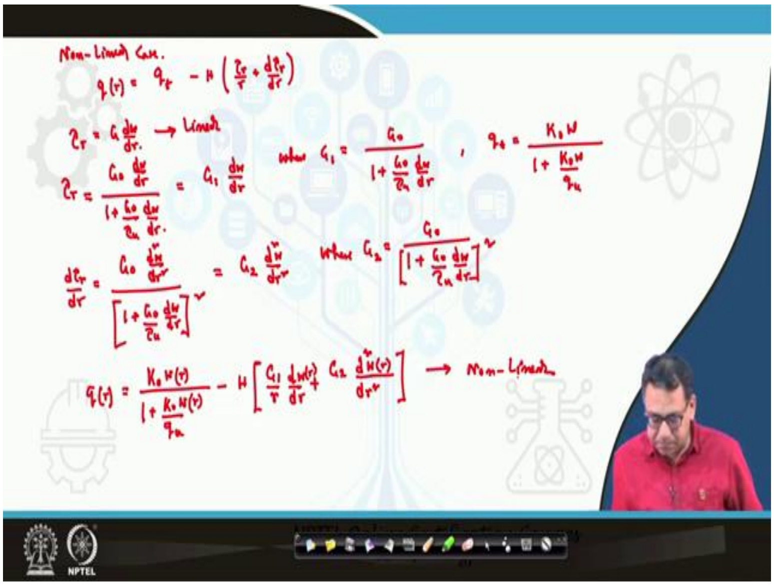

The expression for the load intensity is already known:

$$q(r) = q_t - H\left(\frac{\tau_r}{r} + \frac{d\tau_r}{dr}\right)$$

In case of the linear model, the value of τ_r is: $\tau_r = G\frac{dw}{dr}$

Now for the nonlinear the τ_r expression will be:

$$\tau_r = \frac{G_\circ \frac{dw}{dr}}{1 + \frac{G_\circ}{\tau_u}\frac{dw}{dr}} = G_1\frac{dw}{dr}$$

$$\text{where, } G_1 = \frac{G_\circ}{1 + \frac{G_\circ}{\tau_u}\frac{dw}{dr}} \text{ and } q_t = \frac{k_\circ w}{1 + \frac{k_\circ w}{q_u}}$$

The terms q_u, τ_u, k_o and G_o are already explained and mean the same here. Now considering the derivate of τ_r:

$$\frac{d\tau_r}{dr} = \frac{G_\circ \frac{d^2w}{dr^2}}{\left[1 + \frac{G_\circ}{\tau_u}\frac{dw}{dr}\right]^2} = G_2\frac{d^2w}{dr^2}$$

$$\text{where, } G_2 = \frac{G_\circ}{\left[1 + \frac{G_\circ}{\tau_u}\frac{dw}{dr}\right]^2}$$

In terms of nonlinearity of soil the expression of q_r will be:

$$q(r) = \frac{k_\circ w(r)}{1 + \frac{k_\circ w(r)}{q_u}} - H\left[\frac{G_1}{r}\frac{dw(r)}{dr} + G_2\frac{d^2 w(r)}{dr^2}\right]$$

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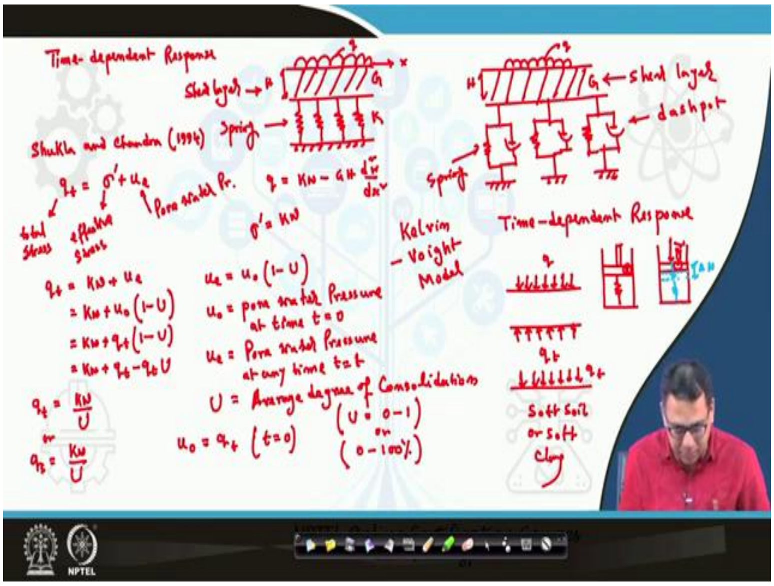

The next concept is the time dependent response of soil. In the normal case, under 1D case when UDL is applied:

$$q = kw - GH\frac{d^2 w}{dx^2}$$

The above expression simulates a condition where the shear layer is underlain by softsoil represented by a series of springs. But if the time-dependent response is to be considered, the springs should be replaced with a spring and dashpot system (*shown in the figure on the right side top corner above*). This model is called the Kelvin-Voight model. This dashpot will take care of the time dependent response. But in this model, the problem is to determine the model parameters which are important here.

To avoid this problem, an indirect method is proposed by which we can determine the time dependent response of the soil but indirectly. This is first proposed by Shukla and Chandra in 1994 and later on number of researchers have used this process or the methodology. This basically involves splitting the model and considering the forces acting on each part. The figure below the spring and dashpot system shows this clearly. The shear layer is subjected to a uniform pressure of q in the top because of the UDL and a uniform reaction of q_t from the soft soil beneath it. This reaction also acts on the bottom of the shear layer downwards as the springs in the spring & dashpot system exert the reaction towards both sides.

The soil will show the time dependent response as of in clay or particularly, soft clay. The time dependent response is very important in clay soil because in sandy soil, the settlement is immediate settlement but in clayey soil, most of the settlement is consolidation settlement which is the time dependent.

It is known that when a stress is applied on a saturated soil, some portion of it will be taken by the soil skeleton (represented by springs here) and the remaining will be taken by the water. So, the total stress, q_t is equal to the effective stress (σ') plus pore water pressure (u_e).

$$q_t = \sigma' + u_e$$

The effective stress is directly taken by the springs. The remaining pressure is taken by the water which is nothing but pore water pressure. The dashpot in the model represents the water which simulates the water behaviour appropriately. At the instance (time, t = 0), stress is applied on a saturated soil all the stress will be taken up by the water and it slowly transfers it to the soil skeleton. First only pore pressure develops and slowly it gets transferred to the effective stress and finally pore pressure becomes 0. The dashpot also exhibits similar behaviour which is why this is appropriate to model softsoil.

As the effective stress can only act on the springs, we can write:

$$\sigma' = kw$$

So, the expression for the total stress can be re-written as:

$$q_t = kw + u_e$$

This u_e can be written as:

$$u_e = u_\circ(1 - U)$$

where, u_o = Pore water pressure at time t = 0

u_e = Pore water pressure at any time t = t

U = Average degree of consolidation

U varied from 0 % – 100 % or 0 - 1

As already mentioned, $u_o = q_t$ at time t = 0 i.e., at the instant of loading the entire stress is carried by the water or entire stress will be equal to the pore pressure. Using these relations, the total stress equation can be written as:

$$\Rightarrow q_t = kw + u_o(1-U)$$

$$\Rightarrow q_t = kw + q_t(1-U)$$

$$\Rightarrow q_t = kw + q_t - q_t U$$

$$q_t = \frac{kw}{U} \text{ or } q_s = \frac{kw}{U}$$

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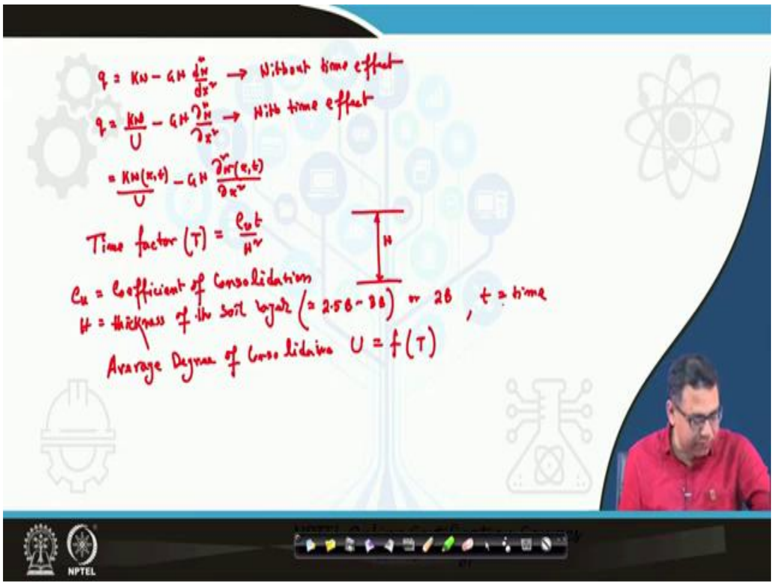

The expression for a time-independent response was derived to be:

$$q = kw - GH\frac{d^2w}{dx^2}$$

But now, the expression changes to:

$$q(x,t) = \frac{kw(x,t)}{U} - GH\frac{\partial^2 w(x,t)}{\partial x^2}$$

This expression shows the time dependent response in case of a linear model, but it can be incorporated in the non-linear model also. Now the displacement is being indirectly related to the degree of consolidation. Now the aim is to correlate it with the real time. In the above expression the displacement is written as a function of t. Let us see how this is true. The time factor is:

$$T = \frac{c_v t}{H^2}$$

where, cv is the coefficient of consolidation, H is the thickness of the soil layer (generally taken as 2.5 – 3B or 2B) and t is the time. The average degree of consolidation U is a function of time factor, T. So, indirectly that average degree of consolidation is a function of T also.

If the thickness of a soil layer and coefficient of consolidation of the soil layer are known then for a particular time at which the settlement needs to be determined, the time factor can be calculated. For the corresponding time factor, the degree of consolidation can be calculated. This degree of consolidation value can be substituted in the equation for time-dependent response to get the settlement at that time.

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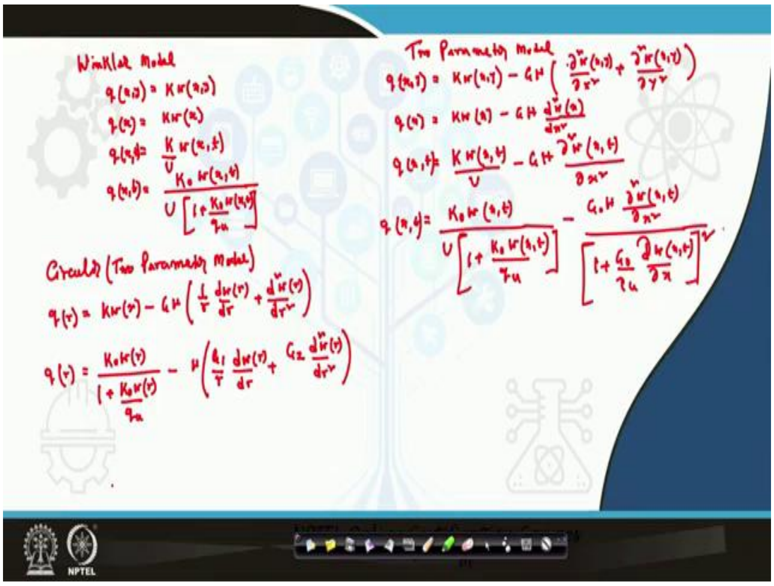

Quickly let us see all the equations developed till now. For Winkler model:

$$q(x,y) = kw(x,y)$$

For 1D case: $q(x) = kw(x)$

Time-dependent response: $q(x,t) = \frac{k}{U} w(x,t)$

Time-dependent and non-linear response: $q(x,t) = \frac{k_{\circ} w(x,t)}{U\left[1 + \frac{k_{\circ} w(x,t)}{q_u}\right]}$

For the circular case two parameter model, the expression is:

$$q(r) = kw(r) - GH\left[\frac{1}{r}\frac{dw(r)}{dr} + \frac{d^2 w(r)}{dr^2}\right]$$

After incorporating non-linearity:

$$q(r)=\frac{k_{\circ}w(r)}{1+\dfrac{k_{\circ}w(r)}{q_u}}-H\left[\frac{G_1}{r}\frac{dw(r)}{dr}+G_2\frac{d^2w(r)}{dr^2}\right]$$

Equation for the two parameter model in xy directions:

$$q(x,y)=kw(x,y)-GH\left(\frac{\partial^2 w(x,y)}{\partial x^2}+\frac{\partial^2 w(x,y)}{\partial y^2}\right)$$

Equation for the 1D case:

$$q(x)=kw(x)-GH\left(\frac{d^2w(x)}{dx^2}\right)$$

Time-dependent and linear response:

$$q(x,t)=\frac{kw(x,t)}{U}-GH\frac{\partial^2 w(x,t)}{\partial x^2}$$

Time-dependent and non-linear response:

$$q(x,t)=\frac{k_{\circ}w(x,t)}{U\left[1+\dfrac{k_{\circ}w(x,t)}{q_u}\right]}-\frac{G_{\circ}H\dfrac{\partial^2 w(x,t)}{\partial x^2}}{\left[1+\dfrac{G_{\circ}}{\tau_u}\dfrac{\partial w(x,t)}{\partial x}\right]^2}$$

These are all the expressions that we have developed till now. In next class I will start another topic. Thank you.