Soil Structure Interaction
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Lecture 15
Different Foundation Models (Contd.)

In this class, I will continue developing the basic differential equation of Pasternak model for nonlinear behaviour.

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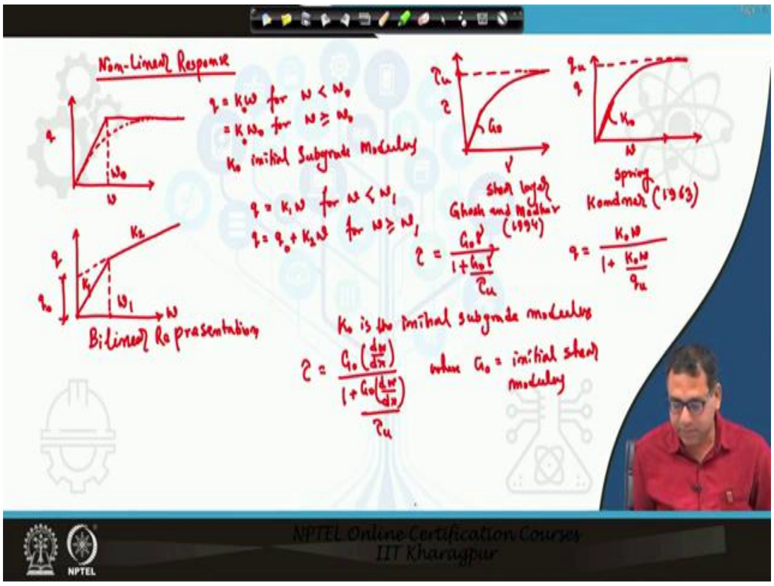

In the previous class, two different simplified approximations of nonlinear responses were discussed. One is linear and then perfectly parallel to the deflection axis and the other is bilinear representation. Now equations would be developed for two cases: one is q versus w and the other, shear stress (τ) vs shear strain (γ). The second case is needed because in Pasternak model, shear layer is considered along with the Pasternak springs. The q vs w plot is for the springs and the τ vs γ plot is for the shear layer. Kundaner (1963) proposed an equation to model non-linear response:

$$q = \frac{k_{o} w}{1 + \dfrac{k_{o} w}{q_u}}$$

The maximum value that is obtained from the load-deformation curve i.e., q value when the curve becomes almost parallel to the deflection axis is q_u. So, in the above equation, the w value (deflection) at which the q value is needed, should be substituted along with q_u and k_o. So, q value can be calculated for any deflection value with the help of the above equation. The slope of the initial portion of the curve is the initial subgrade modulus denoted by k_o.

Similarly Ghosh and Madhav gave a similar expression in 1994 for shear stress vs shear strain curve:

$$\tau = \frac{G_\circ \gamma}{1 + \frac{G_\circ \gamma}{\tau_u}}$$

$$\Rightarrow \tau = \frac{G_\circ \left(\frac{dw}{dx}\right)}{1 + \frac{G_\circ \left(\frac{dw}{dx}\right)}{\tau_u}}$$

Similar to the q ultimate, τ ultimate is the maximum value obtained from the shear stress vs shear strain (τ vs γ) curve. G_o is the initial shear modulus obtained by calculating the slope of the initial portion of the τ vs γ curve. Though different G values can be obtained along the curve, here the initial shear modulus, G_o is being used. Two expressions are given now to derive the basic differential equations for the Pasternak model where one is for the springs and the other, for the shear layer.

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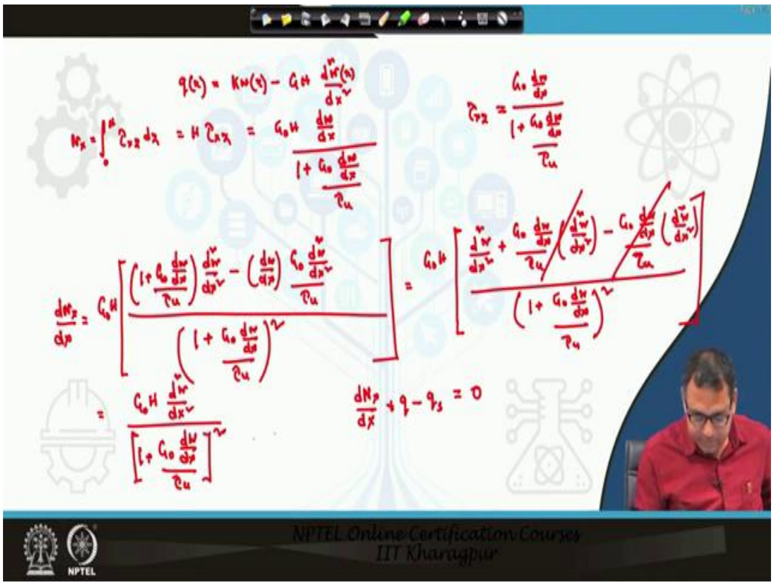

Now, the basic differential equation for Pasternak model will be derived for the plane strain condition in the xy plane. The basic equation for this plane is:

$$q(x) = k \times w(x) - GH \frac{d^2 w(x)}{dx^2}$$

The expression for shear force, N_x is:

$$N_x = \int_0^H \tau_{xz} dz = \tau_{xz} H = \frac{G_\circ H\left(\frac{dw}{dx}\right)}{1+\frac{G_\circ\left(\frac{dw}{dx}\right)}{\tau_u}}\text{; As, } \tau_{xz} = \frac{G_\circ\left(\frac{dw}{dx}\right)}{1+\frac{G_\circ\left(\frac{dw}{dx}\right)}{\tau_u}}$$

Differentiating N_x, we get:

$$\frac{dN_x}{dx} = G_\circ H\left[\frac{\left(1+\frac{G_\circ\left(dw/dx\right)}{\tau_u}\right)\frac{d^2w}{dx^2} - \left(\frac{G_\circ\left(d^2w/dx^2\right)}{\tau_u}\right)\frac{dw}{dx}}{\left(1+\frac{G_\circ H\left(dw/dx\right)}{\tau_u}\right)^2}\right]$$

$$\Rightarrow \frac{dN_x}{dx} = G_\circ H\left[\frac{\frac{d^2w}{dx^2} + \frac{G_\circ\left(dw/dx\right)}{\tau_u}\left(\frac{d^2w}{dx^2}\right) - \frac{G_\circ\left(dw/dx\right)}{\tau_u}\left(\frac{d^2w}{dx^2}\right)}{\left(1+\frac{G_\circ\left(dw/dx\right)}{\tau_u}\right)^2}\right]$$

$$\Rightarrow \frac{dN_x}{dx} = \frac{G_\circ H\left(d^2w/dx^2\right)}{\left[1+\frac{G_\circ\left(dw/dx\right)}{\tau_u}\right]^2}$$

Now, consider the basic equation of Pasternak model under plane strain condition:

$$\frac{dN_x}{dx} + q - q_s = 0$$

$$\Rightarrow q = q_s - \frac{dN_x}{dx}$$

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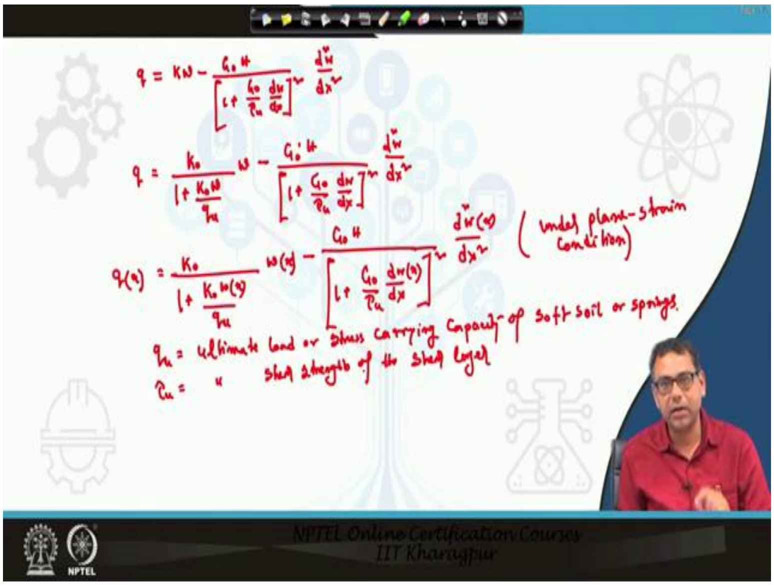

Substituting the already known values of q_s and (dN_x/dx) in the above equation, we get:

$$q = kw - \frac{G_\circ H}{\left[1 + \frac{G_\circ}{\tau_u}\left(\frac{dw}{dx}\right)\right]^2} \frac{d^2 w}{dx^2}$$

Till now, the non-linearity has been applied to the N_x term which represents the shear layer. So, in the above equation, the non-linear response is incorporated only in the shear layer. Since the term $k \times w$ is nothing but the reaction from springs (q_s), non-linearity should be introduced in this term or precisely, in k. So the above equation is rewritten as:

$$q(x) = \frac{k_\circ}{1 + \frac{k_\circ w(x)}{q_u}} w(x) - \frac{G_\circ H}{\left[1 + \frac{G_\circ}{\tau_u}\left(\frac{dw(x)}{dx}\right)\right]^2} \frac{d^2 w(x)}{dx^2}$$

This is the basic differential equation of Pasternak model under plane strain condition. The non-linear behaviour of both shear layer and springs was incorporated in the above equation. q_u is the ultimate load or stress carrying capacity of the soft soil or weak soil. τ_u is the ultimate shear strength of the shear layer.

The equation derived now overcomes all the four limitations of the Winkler model. There is connectivity between the springs and so the deflections are not confined only to the loaded region. Non-uniform settlement can also be modelled along with the non-linear response of the soil. Still, the time dependent response of the soil has not been incorporated in this equation.

This equation should be solved to make use of its application, but that part will be discussed in the later classes of the course. Ultimately, the deflection, slope, bending moment and shear force are to be calculated. So, deriving an equation is not merely sufficient, but it should be solved to determine the required values.

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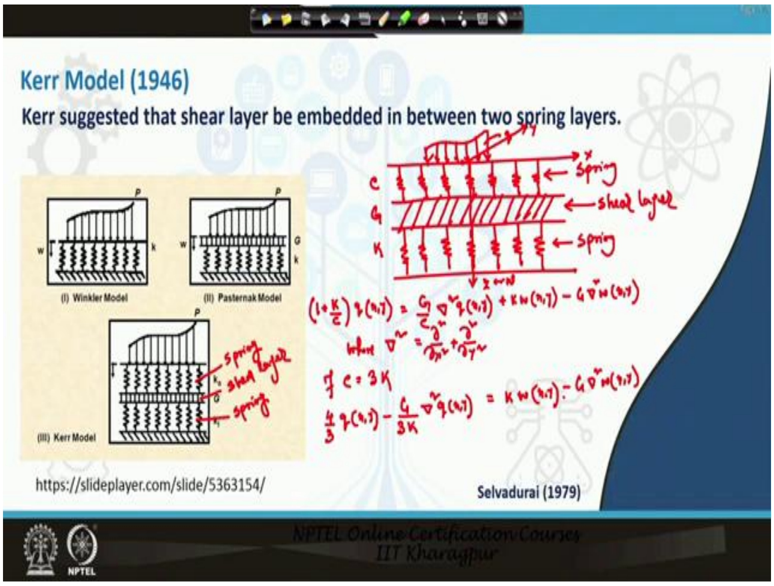

The next model in the discussion is called the Kerr model. In Pasternak model, the approximation was that the soft/weak soil layer is overlain by a shear layer whereas in this model, a shear layer would be sandwiched between two layers of soft/weak soil (idealised by springs). The loading is applied on the upper layer of springs with spring constant, C. G is the shear modulus of the shear layer and k is the spring constant of the lower layer of springs. Here, 3 parameters are involved whereas in the previous model (Pasternak model), 2 parameters were involved. The basic governing differential equation will not be derived for this case as it was derived for the Pasternak model.

$$\left(1+\frac{k}{C}\right)q(x,y)=\frac{G}{C}\nabla^2 q(x,y)+kw(x,y)-G\nabla^2 w(x,y)$$

$$\text{where, } \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$$

If C=3k i.e., the upper springs are stiffer than the lower springs, the equation can be written as:

$$\frac{4}{3}q(x,y)-\frac{G}{3k}\nabla^2 q(x,y)=kw(x,y)-G\nabla^2 w(x,y)$$

In this class, the Pasternak model and the method to incorporate non-linear response in the governing differential equation followed by the Kerr model was discussed. In the next class, I will discuss the Hetenyi model and then I will introduce the structural elements (beams and plates) in that model.

So in the next class I will introduce the structural element, beam first followed by plate resting on springs and the two parameter soil media. Later on, I will try to determine the deflection, slope, bending moment and shear force of that structure element. Thank you.