Soil Structure Interaction
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Lecture 14
Different Foundation Models (Contd.)

In the last class, the Pasternak model was discussed and some of the forces acting on an element of the shear layer were considered as a part of the derivation for the expression. In this class, the basic differential equation for the Pasternak model will be derived followed by the different modifications over this model.

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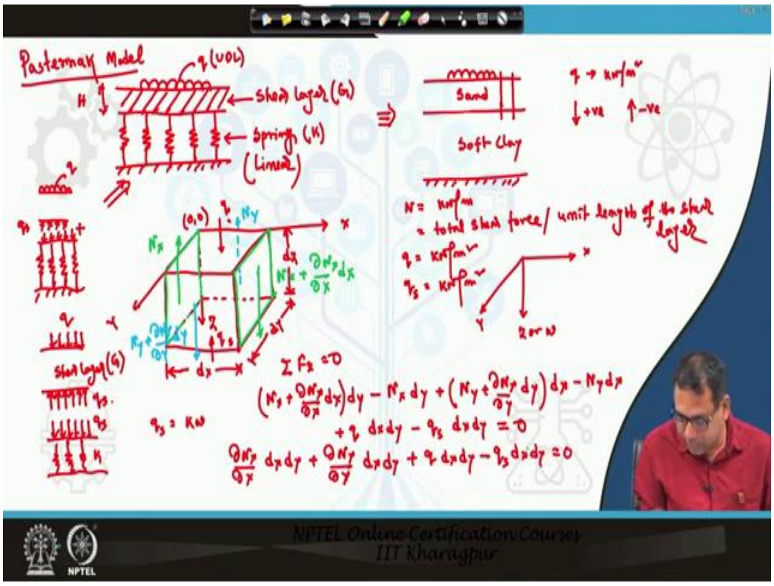

The various components and forces involved in the model were discussed in the last class and shown in the slide above. The load on the soil is applied in the form of a UDL of intensity q kN/m^2 which produces a reaction of q_s from the springs. In the present condition, the strain in the y direction will be zero for the plane strain condition to exist, which is perpendicular to x and z directions.

So far, the forces considered acting on this cube are the UDL, q at the top and the reaction q_s from the springs at the bottom. In addition to these, the side shear forces, N_x and $N_x + \frac{\partial N_x}{\partial x} dx$ act in the x direction, which are shown in green. The force $N_x + \frac{\partial N_x}{\partial x} dx$ is assumed to be acting on the right side face of the cube. The other two faces in the y direction will be subjected to shear forces, N_y and $N_y + \frac{\partial N_y}{\partial y} dy$, where the force, $N_y + \frac{\partial N_y}{\partial y} dy$ is assumed to be acting on the front side face of the cube. N is the shear force per unit length of the shear layer expressed in kN/m.

Now considering the sum of all forces acting in the z direction, the expression would be:

$$\left(N_x + \frac{\partial N_x}{\partial x} dx\right)dy - N_x dy + \left(N_y + \frac{\partial N_x}{\partial y} dy\right)dx - Nydy + qdxdy - q_s dxdy = 0$$

The forces acting downward are considered positive and the upward forces are considered to be negative. So, N_x, N_y and q_s are given negative sign and the rest are considered positive. The UDL q kN/m^2 acting on the top of the cube is acting downward (positive) and should be multiplied with the area of the top face of the cube to convert it into a force (in kN). Similarly the reaction of the springs, q_s kN/m^2 acting on the bottom face of the cube in the upward direction (negative) should also be multiplied by the same area (the top and bottom faces have the same surface area) to convert it into a force. Since the forces $N_x + \frac{\partial N_x}{\partial x} dx$ and N_x are acting along a face of width dy, these forces are multiplied with dy in the above expression. The similar formulation is followed for the other forces too. After simplifying the above expression would take the following form:

$$\frac{\partial N_x}{\partial x} dxdy + \frac{\partial N_y}{\partial y} dxdy + qdxdy + q_s dxdy = 0$$

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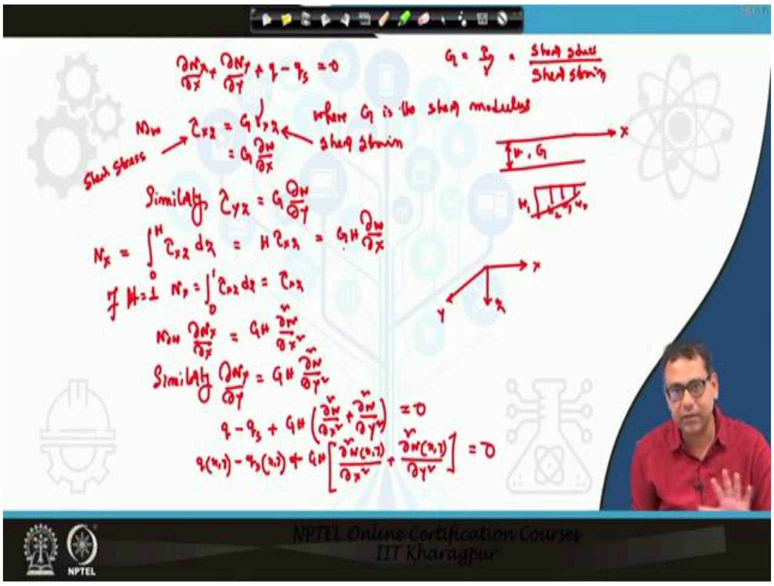

As the dxdy term is common in all the terms, it can be vomited by cancelling, further simplifying the equation to:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_y}{\partial y} + q - q_s = 0$$

The above sum is basically the shear stress in the xz plane which is equal to 0. By definition, shear modulus, *G* is a ratio of shear stress over shear strain. So, shear stress in a plane will be equal to *G* multiplied by the shear strain, γ_{xz} in that particular plane (here, the xz plane).

$$\tau_{xz} = G\gamma_{xz}$$

But the shear strain can be written as $\partial w/\partial x$ in xz plane.

$$\tau_{xz} = G\frac{\partial w}{\partial x} \text{ in the xz plane}$$

$$\tau_{yz} = G\frac{\partial w}{\partial y} \text{ in the yz plane}$$

The shear force, N_x is the total of unit force acting per unit thickness of the shear layer. This force can be obtained when an integral over 0 to H is applied for the product of shear stress for a particular unit thickness and the corresponding thickness (dz), H being the total thickness of shear layer. To evaluate the total shear force acting on the shear layer, the thickness of it is assumed to be divided into unit thicknesses of dz which, when integrated over the full thickness, gives H again. So, the shear force acting on that small element is ($\tau_{xz}\times dz$) which is integrated from 0 to H, to get the total shear force.

$$N_x = \int_0^H \tau_{xz} dz \quad \Rightarrow N_x = H\tau_{xz} \quad \Rightarrow N_x = GH\frac{\partial w}{\partial x}$$

If H =1: $$\Rightarrow N_x = \int_0^1 \tau_{xz} dz \qquad \Rightarrow N_x = \tau_{xz}$$

Now consider the expression for N_x and:

$$\frac{\partial N_x}{\partial x} = GH\frac{\partial^2 w}{\partial x^2}$$

$$\text{Similarly, } \frac{\partial N_y}{\partial y} = GH\frac{\partial^2 w}{\partial y^2}$$

Substituting the above values in the expression for total shear stress in xz plane ($q - q_s + \frac{\partial N_x}{\partial x} + \frac{\partial N_y}{\partial y} = 0$), we get:

$$q - q_s + GH\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = 0$$

$$\Rightarrow q(x,y) - q_s(x,y) + GH\left(\frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2}\right) = 0$$

In the expression above, all the parameters vary with x and y except for the parameters G and H.

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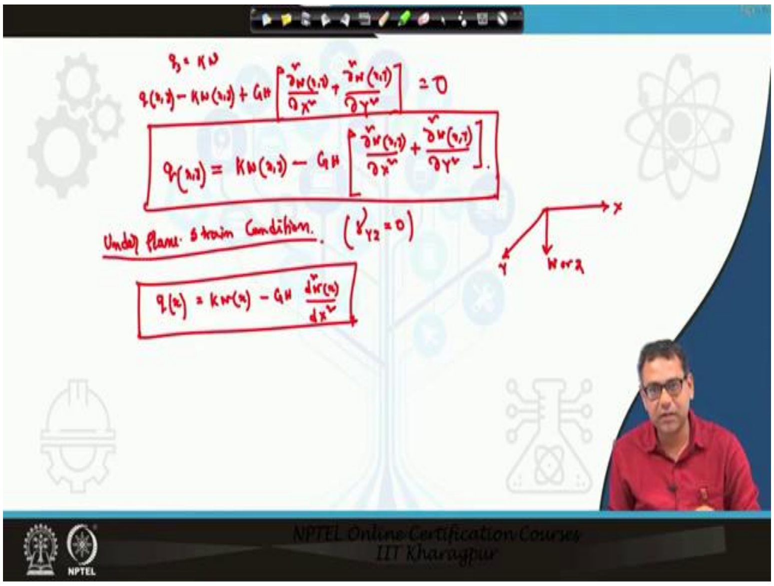

As per Winkler model, q_s=k×w.

So, the above equation can be further written as:

$$q(x,y) - k \times w(x,y) + GH\left(\frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2}\right) = 0$$

$$\Rightarrow q(x,y) = k \times w(x,y) - GH\left(\frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2}\right)$$

This is the basic equation of the Pasternak model for the xy plane. The plane strain condition is a special case which can be derived with the help of the general case shown above. For the plane strain condition, the shear strain in yz direction will be 0 (γ_{yz}=0). It is known that, $\tau_{yz} = G\gamma_{yz}$ and $N_y = H\tau_{yz}$. So, if the shear strain in yz direction is zero (γ_{yz}=0), the value τ_{yz} would be zero and eventually N_y term would vanish from the above equation.

$$q(x) = k \times w(x) - GH\frac{d^2 w(x)}{dx^2}$$

This is the basic equation of Pasternak model under plane strain condition. The equation for plane strain condition is in terms of x alone because of its independency on the y direction.

There are still two issues with this model: the time dependent settlement is not considered and the response is linear.

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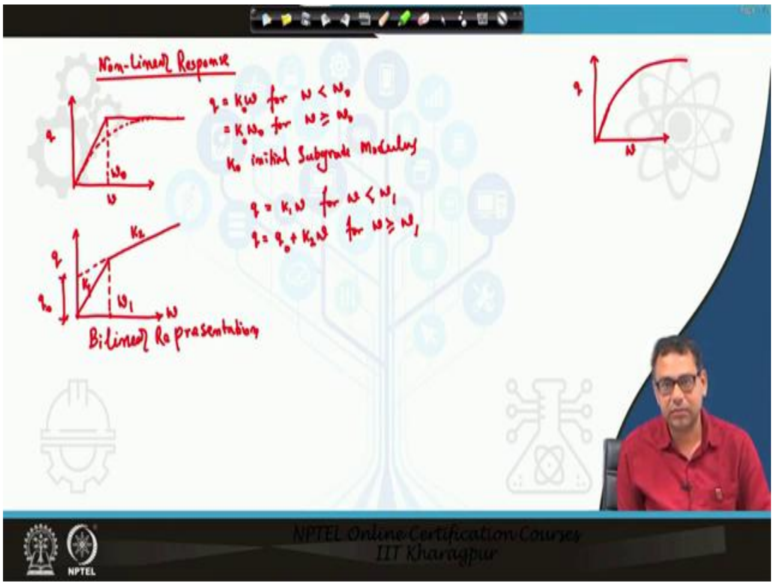

Now, let us try to include non-linearity in this equation. Consider a load versus deformation plot or a q by w plot which is non-linear as shown in the top right figure of the above slide. This non-linear behaviour can be idealised into two parts for simplification, in which one changes linearly up to a certain value of deformation (w_0) and the other is parallel to the X-axis beyond w_0. So, beyond w_0, the stress q is constant and only the deformation varies. As long as the deformation, w is less than w_0, the q value would be k_0w and beyond that, it would be k_0w_0:

$$q = k_0 w \text{ for } w < w_0;$$

$$q = k_0 w_0 \text{ for } w > w_0;$$

where, k_0 is the initial subgrade modulus

The value of k_0 would be determined from the initial portion of the curve because the subgrade modulus, in this case is varying due to the nonlinear behaviour. So, as the k value keeps on changing, the subgrade modulus is calculated from the initial portion of the curve and so it is called the initial subgrade modulus.

Another way of representing the above non-linear behaviour is the bilinear response. A bilinear representation is in which, a non-linear curve is represented with two straight lines of different slopes like the figure shown above. In the slide above, the graph with bilinear representation shows a linear behaviour of slope k_1 up to a deflection value of w_1 and then the slope changes to k_2 beyond w_1.

$$q = k_1 w \text{ for } w < w_1$$

$$q = q_0 + k_2 w \text{ for } w > w_1$$

where, q_0 is the projection of the line with slope k_2 on the Y-axis.

The non-linear response discussed now is just a simplified approach where the actual non-linear response is idealised by few linear responses approximately. In the next class, I will discuss about the actual nonlinear response and the procedure to model it which will be introduced in the basic governing differential equation developed for the Pasternak model and then will try to develop basic differential equation of the Pasternak model under nonlinear response, thank you.