Soil Structure Interaction
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Lecture 13
Different Foundation Models

In this class, the different foundation models that are used to idealize soil for the soil structure interaction problem will be discussed.

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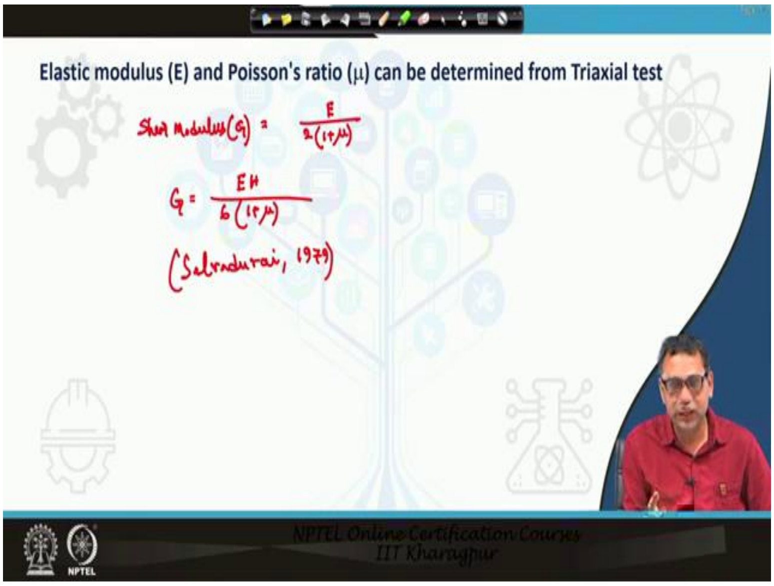

Before discussing about the foundation models, there is one thing to be discussed about determining the k value from tests other than plate load test. As already mentioned, the k value can be determined from the triaxial test, the consolidation test, the California bearing ratio or CBR test. But there are few other properties that are required to be determined for the soil structure interaction problem. Elastic modulus and Poisson's ratio are such properties which can be determined from the triaxial test. But the actual property that is very important is the shear modulus and this can be calculated from the elastic modulus. If the elastic modulus and Poisson's ratio are known, the shear modulus can be determined easily by the expression:

$$G = \frac{E}{2(1+\mu)}$$

where, G is the shear modulus, E is the elastic modulus and μ is the Poisson's ratio of the soil. There is another expression to determine the shear modulus from E and μ proposed by Selvadurai (1979):

$$G = \frac{EH}{6(1+\mu)}$$

where, H is the influence zone depth can be taken as 2.5 to 3 times of B (B is the width of the foundation), G is the shear modulus, E is the elastic modulus and μ is the Poisson's ratio of the soil.

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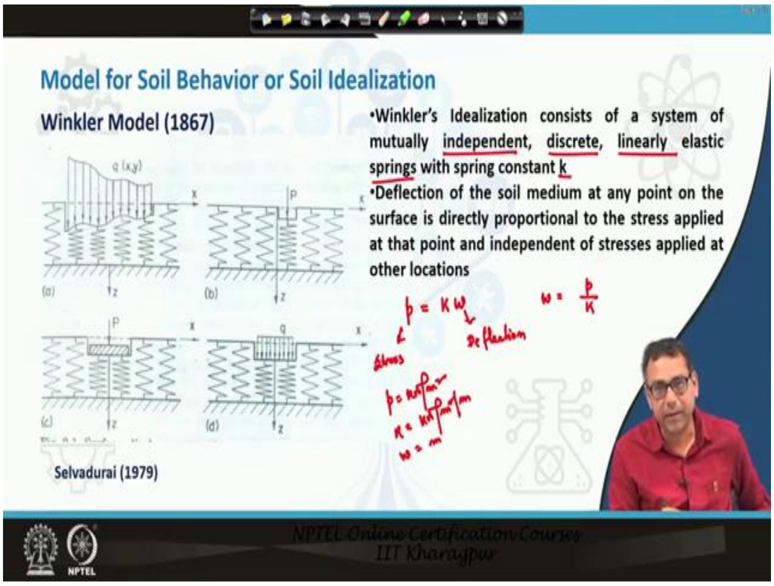

Let us begin with the modelling of soil for the soil structure interaction problem. The Winkler model (1867) has already been discussed where soil is replaced by linear springs. These springs are independent, discrete and linearly elastic with spring constant k and that k is termed as modulus of subgrade reaction. It is also mentioned that the deflection of that soil medium at any point on the on the surface is directly proportional to the stress applied at that point and is independent of the stress applied at other locations.

As the springs are not connected, a load on spring will cause deformation on that particular position alone. So, any load applied on a spring will not cause deformation in another spring. The stress applied, p will be equal to k × w, where k is the spring constant or modulus of subgrade reaction and w is the deflection.

Units of p will be kN/m^2 and that of k, $kN/m^2/m$. This is the model that is proposed by Winkler. So, if the k value is known, then for a certain magnitude of stress applied at a particular point, the deformation at that point can be calculated. So, if p and k are known it is very easy to calculate the deformation.

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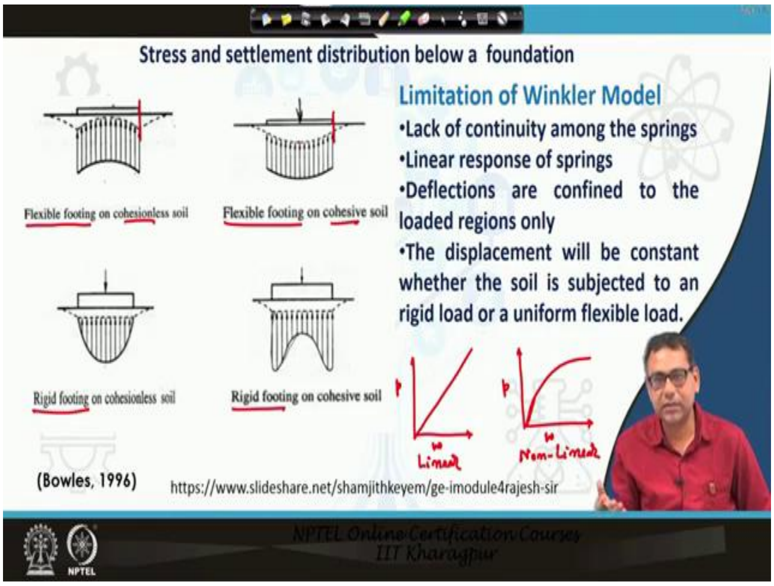

There are some limitations of the Winkler model, the first limitation being the lack of continuity among the springs. So there is no connection between the springs which means the load applied at one place cannot affect the deformation even at the nearby areas. Also, the deformation at a point is dependent only on the spring stiffness beneath it and independent of the nearby springs.

The second limitation is the linear elasticity of springs which means the load settlement curve or the load deformation curve will be perfectly linear. But in actual case, for a soil medium it is not linear. Even in case of nonlinear responses, a certain initial behaviour would be linear for which this model can be applied, but beyond certain deformation, where the response becomes non linear, Winkler model cannot give accurate results.

The third limitation is that the deflections are confined only to the loaded region i.e., if a load is applied on a particular spring only that spring deforms and the other springs do not. So the region beyond the loaded region will not be affected which is not the actual case. Another assumption is that the displacement will be constant whether the soil is subjected to a rigid load or uniform flexible load. The distribution of settlement beneath a footing depending upon the loading type and soil is given in the slide. Under a rigid footing, the settlement would be uniform but it is not the case under a flexible footing. So, Winkler model cannot capture the settlement pattern beneath a flexible footing which results in erroneous predictions. Also, if the settlement patterns are closely observed, the varying settlement beyond the footing can be seen which also cannot be modelled by this method.

These are the limitations of Winkler model which need to be modified. So, to remove these limitations further models were developed. Besides all these limitations, there is one more important aspect that the Winkler model neglected which is the time-dependent response. As per this model, the deformation is not a function of time even if the soil is clay or cohesive. But here there is no time-dependent settlement meaning that it only gives the immediate settlement. These limitations are overcome by other models which will be discussed further.

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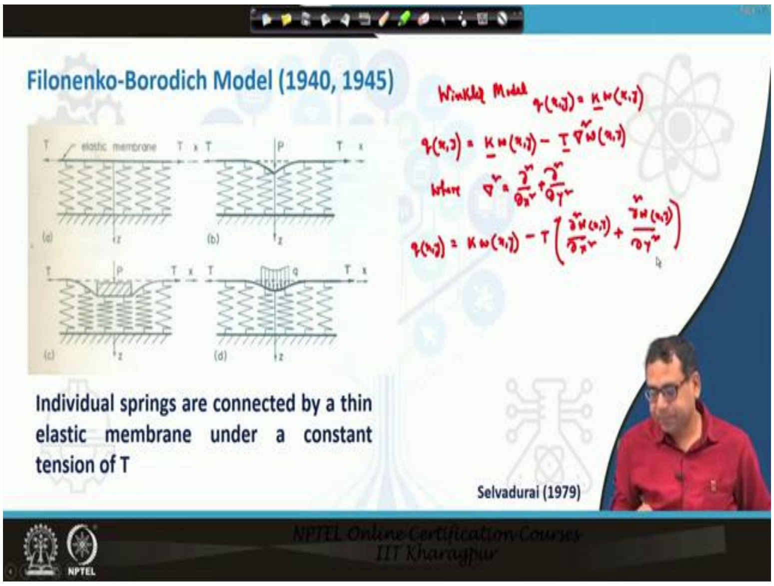

The first limitation of the Winkler model was the lack of continuity among the springs which was overcome by Filonenko-Borodich model, proposed in 1940 and 1945 where the individual springs are connected by a thin elastic membrane under a constant tension of T.

The equation of Winkler model was: q(x,y) = kw(x,y) which had to be modified for the Filonenko-Borodich model as:

$$q(x,y) = kw(x,y) - T\nabla^2 w(x,y) \text{ where, } \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

So, the final expression for Filonenko-Borodich model can be written as:

$$q(x,y) = kw(x,y) - T\left(\frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2}\right)$$

In Winkler model only one parameter was involved but here two parameters, k and T are involved. So, it is called a two parameter model. The values of both the parameters should be known to apply this model.

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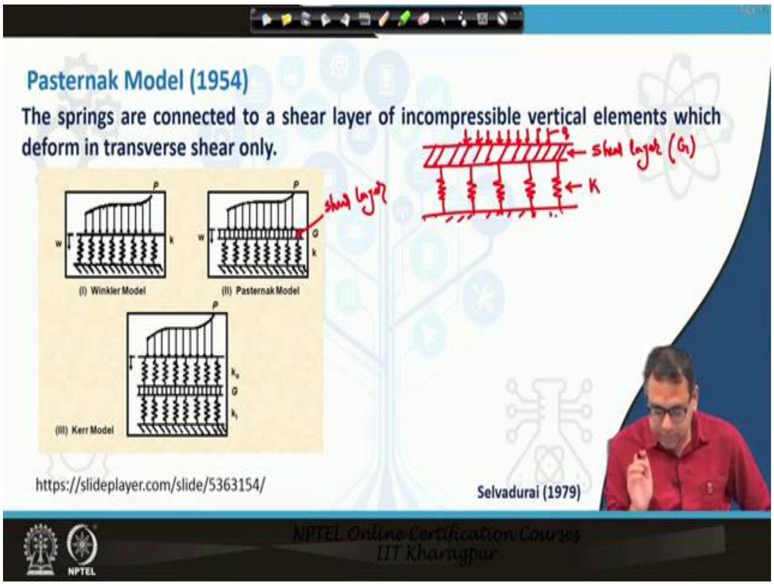

The next model to be discussed is the Pasternak Model (1954) in which the springs are connected to a shear layer of incompressible vertical elements that deform in transverse shear only. The shear layer lies above the springs connecting all of them and the load, q would be applied on the shear layer. The parameter required to be known pertaining to the shear layer is the shear modulus, G and that of springs is k.

Now, if a load is applied, there will be some deformation beyond the loaded region because of the connectivity among the springs. In other words, the deformation of a spring depends also on the load on another spring. Besides, the load may cause non-uniform deformation because of the presence of shear layer depending upon the type of footing (flexible or rigid). So, these three limitations were improved by using this model but still the spring constant is linear.

Though non-linearity was not incorporated in this model, I will discuss in the later stages about incorporating the non-linearity in this model also. But now the discussion would be about the linear springs with connectivity and the deformation extending beyond the loaded region along with the non-uniform deformation scenario.

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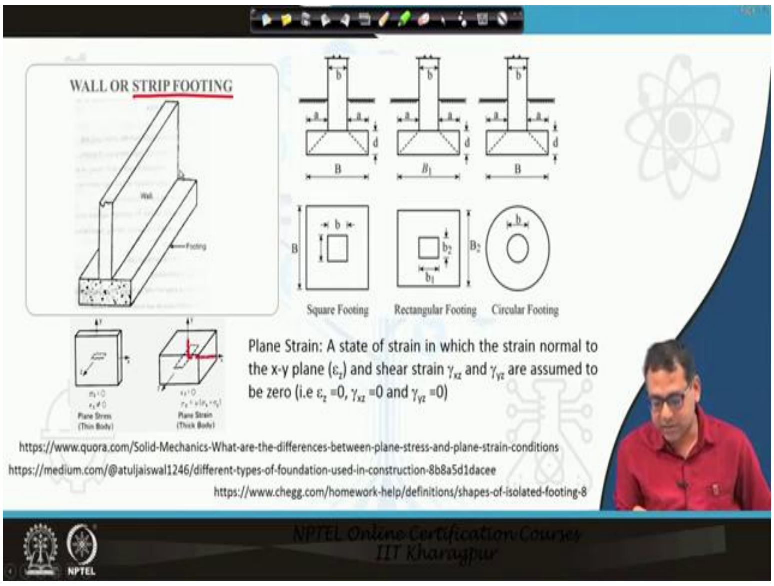

Before the derivation of expression for Pasternak model, knowledge about the different types of loading or foundations which will be encountered in this course is necessary. A foundation can in general be a strip footing or a circular or rectangular or square footing. Strip footing or strip loading is called as plane strain condition. Plane strain condition is a state of strain in which strain in the direction perpendicular to one of the planes (say, xy plane) is 0. So, the direction perpendicular to xy plane is z direction and the normal strains (ε_{zz}) along with shear strains (ε_{xz}, ε_{yz}) in that direction will be 0.

This is generally observed for a very thick body like strip footing where the strain is 0 in the direction normal to the xy plane. So, that the strip loading is treated as a plane strain condition. The strip footing will be solved using the plane strain condition, but not the other type of footings like rectangular, circular or square.

There is another special case called the plane stress condition that is valid for the thin bodies were stresses are 0 in one direction. But that will not be explained or dealt in this course as it is very rare to encounter such a condition in the soil-structure interaction problem.

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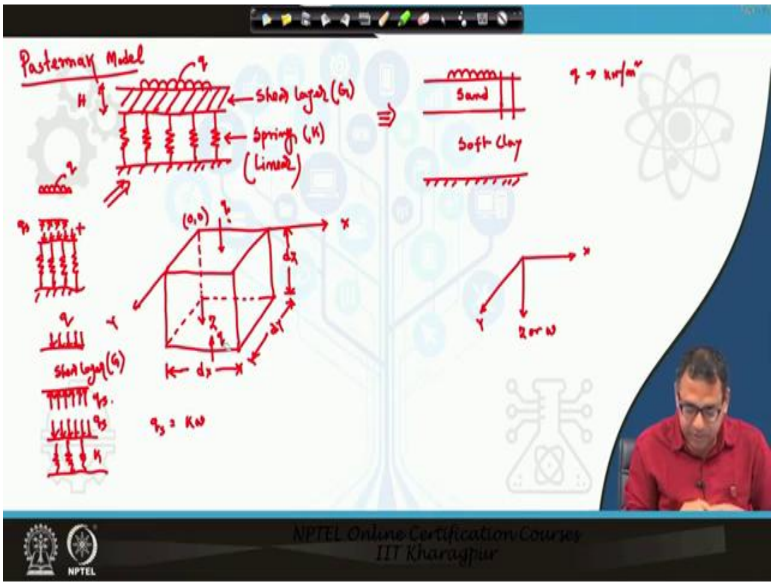

Now the expression for Pasternak model will be derived. Consider a shear layer with shear modulus, G and height H above a number of identical springs with a subgrade modulus value of k. The springs referred to here are also Winkler springs meaning they can model only linear behaviour. This idealization may be equivalent to a condition in real life where a sand layer is underlain by a weak soil layer. If the springs can capture the time-dependent behaviour, it would be apt to say that a sand layer underlain by soft (clay) soil, but as the time-dependent behaviour cannot be modelled now.

The incorporation of the time dependent behaviour or the time dependent settlement will also be discussed later on after which, clay can also be modelled using this model. For now, the assumption is that the soil is so soft or weak that it is difficult even to stand over there because of which a granular layer is placed over the soil. Over this granular layer or within it, the load or the foundation would be placed. This is the ideal condition of the Pasternak model where a soft layer or a weak layer is overlain by sand.

First the expression will be derived for the 3D condition i.e. for a rectangular or square footing and then converted to a plane strain condition. Consider a small cubical element in the shear layer with sides dx, dy and dz in the x, y and z directions respectively. The loading considered to be applied is a UDL of stress intensity q kN/m^2 over the sand layer or the shear layer.

Now, if this condition is analysed separating each entity like that of a free body diagram, there will be the UDL q acting. There will be the shear layer upon which a load intensity of q kN/m^2 is acting and this layer also experiences a reaction, q_s from the springs beneath. Finally the layer of springs upon which its own reaction, q_s acts. If these three free body diagrams are summed up or considered altogether, the original case can be obtained. Basically, as per Winkler model $q_s = k \times w$ or $k \times z$ but here, there is a shear layer along with the springs.

So, all the stresses acting on the considered cubical element are shown in the slide above. In the next class, after considering all forces acting on this cube, I will try to develop the basic equation for the Pasternak model, thank you.