Mass, Momentum and Energy Balances in Engineering Analysis

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Lecture – 09

Macroscopic Balances – III

Welcome. We shall now further look into the Macroscopic Balances.

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CONCEPTS COVERED

Concepts Covered:

- ❑ Energy conservation in integral form
- ❑Momentum conservation
- ❑Application of mass, momentum and energy balance

So, in this particular lecture we shall be doing these concepts that energy conservation in the integral form, some more things will be told to you and how we can simplify this energy conservation for various types of cases. Then we shall look into the momentum balance. And we shall end of this lecture with some very basic applications of this mass balance, momentum balance and the energy balance.

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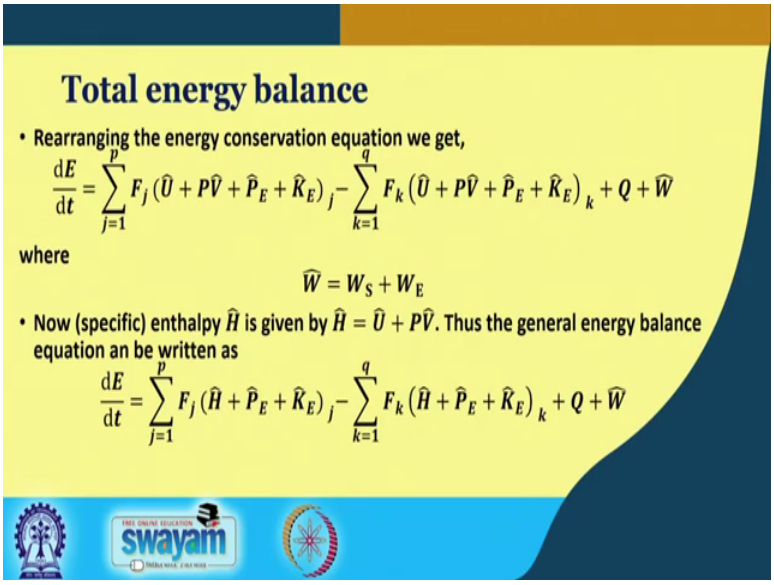

Total energy balance

- Rearranging the energy conservation equation we get,

$$\frac{dE}{dt} = \sum_{j=1}^{p} F_j(\hat{U}+P\hat{V}+\hat{P}_E+\hat{K}_E)_j - \sum_{k=1}^{q} F_k(\hat{U}+P\hat{V}+\hat{P}_E+\hat{K}_E)_k + Q + \hat{W}$$

where

$$\hat{W} = W_S + W_E$$

- Now (specific) enthalpy $\hat{H}$ is given by $\hat{H} = \hat{U} + P\hat{V}$. Thus the general energy balance equation an be written as

$$\frac{dE}{dt} = \sum_{j=1}^{p} F_j(\hat{H}+\hat{P}_E+\hat{K}_E)_j - \sum_{k=1}^{q} F_k(\hat{H}+\hat{P}_E+\hat{K}_E)_k + Q + \hat{W}$$

So, start with let us look into the total energy balance which already we have looked into our last lecture it is just for recapitulation because, we are now going to simplify this form of equation by taking various kinds of assumptions. So, taking this as the basis now we shall go to first assumption; here it is.

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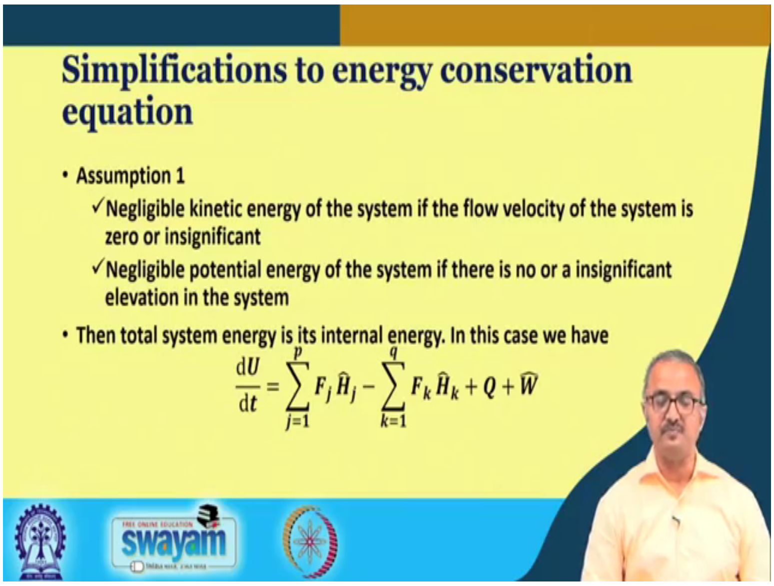

Simplifications to energy conservation equation

- Assumption 1
  - ✓Negligible kinetic energy of the system if the flow velocity of the system is zero or insignificant
  - ✓Negligible potential energy of the system if there is no or a insignificant elevation in the system
- Then total system energy is its internal energy. In this case we have

$$\frac{dU}{dt} = \sum_{j=1}^{p} F_j\hat{H}_j - \sum_{k=1}^{q} F_k\hat{H}_k + Q + \hat{W}$$

Now, if we can neglect the kinetic energy of the system and when we can do that? We can do that if the flow velocity is very very small perhaps then we can do that or we say that there is hardly any change of the flow velocity along the flow direction. So, in that case we can take the change in the kinetic energy to be 0 or we can say that it is itself is 0 change or the value itself is 0.

And second assumption is that if the potential energy of the system is also 0; that means, there is not a significant elevation of the system. So, in that case what we see that all neglecting the changes in the kinetic energy and the potential energy between the inlet and the outlet of the system what we are left with is this that we are left with the internal energy of the system and that is given only in terms of the this enthalpy of the ingoing and outgoing streams and the a heat interaction between the system and the surroundings and the work ok. So, this is one simplification.

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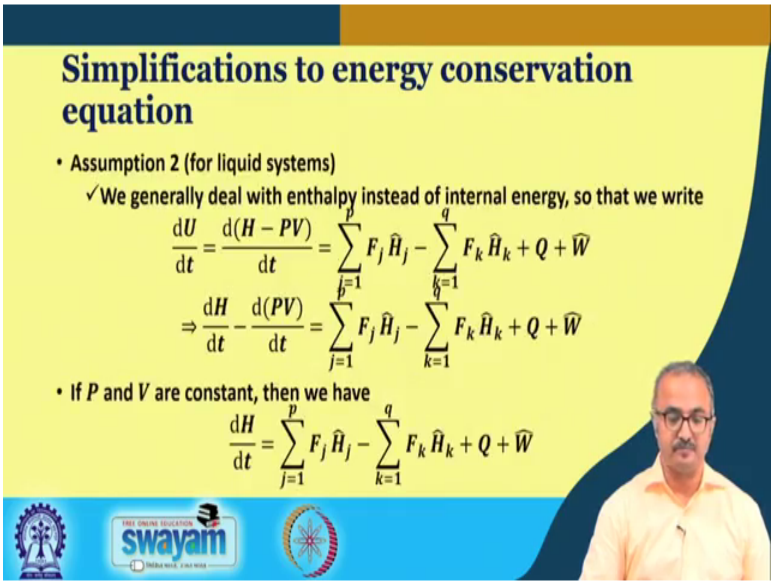

Simplifications to energy conservation equation

- Assumption 2 (for liquid systems)
  - ✓We generally deal with enthalpy instead of internal energy, so that we write

$$\frac{\mathrm{d}U}{\mathrm{d}t} = \frac{\mathrm{d}(H - PV)}{\mathrm{d}t} = \sum_{j=1}^{p} F_j \hat{H}_j - \sum_{k=1}^{q} F_k \hat{H}_k + Q + \widehat{W}$$

$$\Rightarrow \frac{\mathrm{d}H}{\mathrm{d}t} - \frac{\mathrm{d}(PV)}{\mathrm{d}t} = \sum_{j=1}^{p} F_j \hat{H}_j - \sum_{k=1}^{q} F_k \hat{H}_k + Q + \widehat{W}$$

- If P and V are constant, then we have

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \sum_{j=1}^{p} F_j \hat{H}_j - \sum_{k=1}^{q} F_k \hat{H}_k + Q + \widehat{W}$$

Another simplification may come due to this side for liquid system specifically it is not for gaseous system. Now we see that we can write this U in terms of the enthalpy, now what we do that we replace U by H minus PV. Now this is a mathematical arrangement after this ok; now right hand side remains the same as we have done in the previous slide.

Now, what we do is this we now this made separate this particular integral into two first we write d H by dt and then minus d PV by dt and rest of the things on the right hand side remains the same. Now for the liquid systems we find that P and V are generally constant. That means, we hardly find much pressure drop in the liquid systems and also because of the incompatibility of the liquids we find that the change in the volume will not be also substantial, not significant. So, we may take out this particular term and make it 0 so that we are now left with the change in the enthalpy of the system in terms of the enthalpies going in and the enthalpies going out, the heat interaction and the work interaction. So, this is another kind of simplification to the system.

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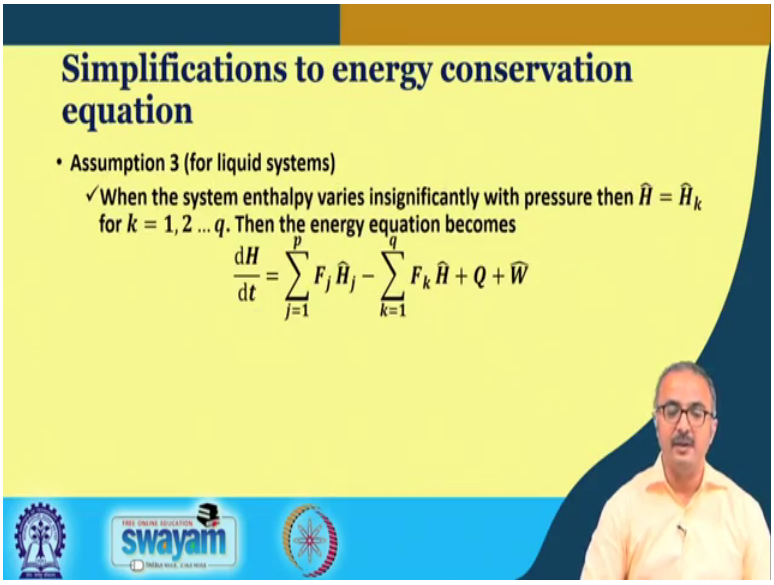

Simplifications to energy conservation equation

- Assumption 3 (for liquid systems)
  - ✓When the system enthalpy varies insignificantly with pressure then $\hat{H} = \hat{H}_k$ for $k = 1, 2 \ldots q$. Then the energy equation becomes

$$\frac{dH}{dt} = \sum_{j=1}^{p} F_j \hat{H}_j - \sum_{k=1}^{q} F_k \hat{H} + Q + \hat{W}$$

Next we come to another assumption that is for liquid systems again. So, in this case we say that when the enthalpy varies in significantly with pressure. That means we put that H this is with respect to temperature it is now varying. So, we find that we writing this H as H k in terms of all the individual streams. So, what and see that the energy equation we can now written in terms of these enthalpies which are now functions of only the temperature and they are now not the function of the pressure. So, this is something to do with the liquid systems.

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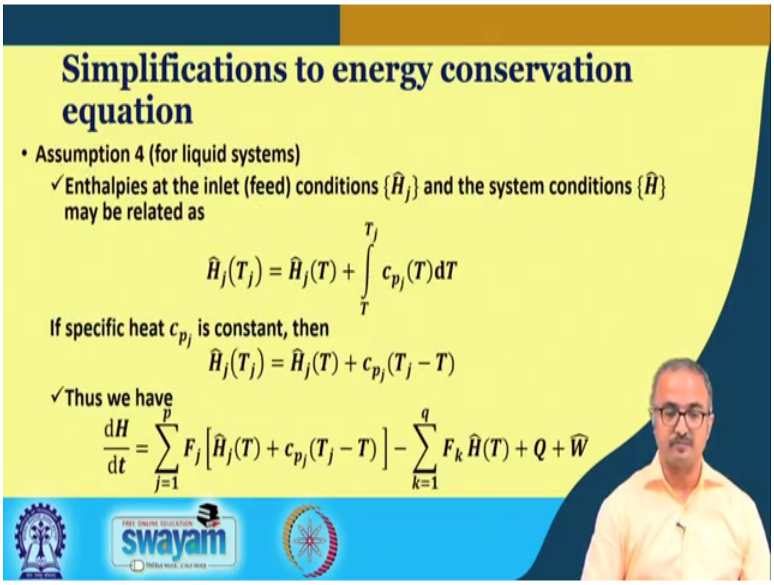

Simplifications to energy conservation equation

- Assumption 4 (for liquid systems)
  - ✓Enthalpies at the inlet (feed) conditions $\{\hat{H}_j\}$ and the system conditions $\{\hat{H}\}$ may be related as

$$\hat{H}_j(T_j) = \hat{H}_j(T) + \int_{T}^{T_j} c_{p_j}(T)dT$$

If specific heat c_{p_j} is constant, then

$$\hat{H}_j(T_j) = \hat{H}_j(T) + c_{p_j}(T_j - T)$$

✓Thus we have

$$\frac{dH}{dt} = \sum_{j=1}^{p} F_j\left[\hat{H}_j(T) + c_{p_j}(T_j - T)\right] - \sum_{k=1}^{q} F_k\hat{H}(T) + Q + \hat{W}$$

Another assumption we can do is this enthalpies this is again for liquids. So, in this case what we see that enthalpies at the inlet that is the feed conditions, and this we are giving by this and the system conditions may be related like this. That means, this is the coming with the in flowing streams and this is the actual system enthalpy. And they may be related like this that for j-th stream at the T j temperature; that means, the j-th stream has the T j temperature as we said that we are neglecting the effect of the pressure on the enthalpy ok.

So, H j T j is equal to H j T and then we are putting this H j T is some kind of a datum value you can say and that datum is taken as the system temperature and then we are integrating c p dt we are doing. We are doing from a system temperature to the temperature of the incoming streams because we know this relation is coming because we know that the enthalpy is not having any absolute value, till always dH we write dh equal to cp dT. So, in this case we are just expanding that d H equal to c p dT in this way that we are finding that at the feed condition or the conditions of the incoming streams what is the enthalpy in terms of the enthalpy of the system ok. So, that is why we are doing it this can kind of a manipulation.

Now, we know that the specific heat may can be taken to be constant for the liquids many a times. So, if we do so then this particular integral will be reduced to simply c p j into T j minus T ok. So, this is another simplification we can do for the liquid systems.

So, once we have done these simplifications what we can do now? We can put it in the a energy balance equation and putting these kind of things what we find that see here we have putting only for the incoming streams and the outgoing streams are taken to be same as the system conditions. So, this outgoing streams are having the same temperature as the system temperature it, you can see this like this that suppose you are having some kind of a liquid in a particular vessel and there is some liquid is coming from the outside.

Now that incoming liquid may have some different temperature and if we are stirring the liquid within the vessel the whole liquid within the vessel can come to a single temperature. Now what happens whatever liquid you are now withdrawing from the particular vessel you will find that outgoing stream will have the same temperature as the liquid which is inside the vessel. So, this is the way we see that the outgoing streams have the same condition as the condition of the system ok. So, that is why we are not putting a there is no need to expand this outgoing stream enthalpy like the way we have done it for the incoming streams.

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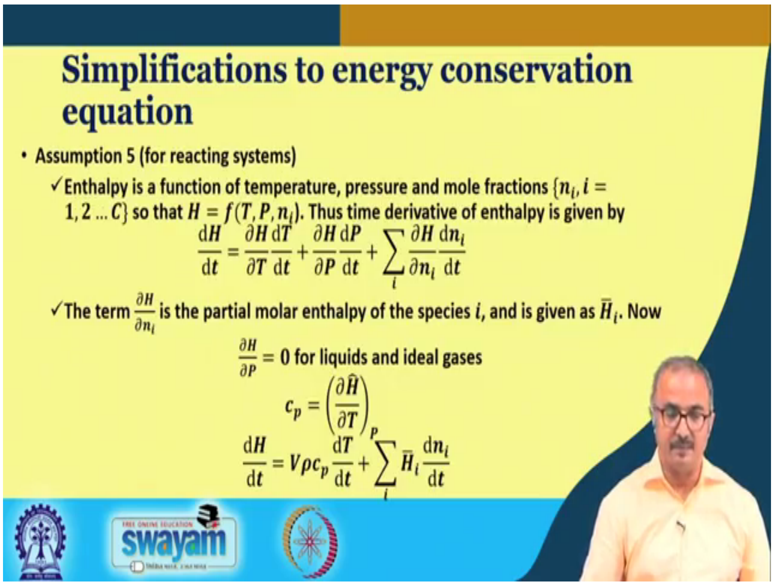

Simplifications to energy conservation equation

- Assumption 5 (for reacting systems)
  - ✓Enthalpy is a function of temperature, pressure and mole fractions $\{n_i, i = 1, 2 \dots C\}$ so that $H = f(T, P, n_i)$. Thus time derivative of enthalpy is given by

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{\partial H}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}t} + \frac{\partial H}{\partial P}\frac{\mathrm{d}P}{\mathrm{d}t} + \sum_i \frac{\partial H}{\partial n_i}\frac{\mathrm{d}n_i}{\mathrm{d}t}$$

  - ✓The term $\frac{\partial H}{\partial n_i}$ is the partial molar enthalpy of the species i, and is given as $\bar{H}_i$. Now

$$\frac{\partial H}{\partial P} = 0 \text{ for liquids and ideal gases}$$

$$c_p = \left(\frac{\partial \hat{H}}{\partial T}\right)_P$$

$$\frac{\mathrm{d}H}{\mathrm{d}t} = V\rho c_p \frac{\mathrm{d}T}{\mathrm{d}t} + \sum_i \bar{H}_i \frac{\mathrm{d}n_i}{\mathrm{d}t}$$

Now, this is special case for the reacting systems. Now in the reacting systems we first we just see that enthalpy is a function of temperature, pressure and mole fractions this mole fraction is given by some mole is given by n and this moles for all the components

C. So, we find that for any kind of system this is the way we can write in general for the enthalpy.

And now if we take the time derivative of this we know from our basic mathematics that how we write time derivatives that dH by dt will be equal to dou h by dou temperature into dT by dt. That means, it this H is a function of the temperature and temperature is a function of the time ok. And similarly this enthalpy is a function of the pressure and the pressure is a function of time and this is the enthalpy of function of all the components moles and this is the all the components moles are changing with time why because of reaction ok.

Now, you see this dou H by dou n i what it means it is the change of the enthalpy with respect to the change in the number of moles of a given component, keeping the number of moles of other components constant ok. So, in this particular thing we call it this is the partial molar enthalpy for species i, this particular definition of partial property may be extended for any other property. So, this means that we keep the concentration of one species constant varying and other species constant at a given temperature pressure and this particular thing is replaced by this H over bar into i.

Now, with this what we can do now, we can replace this with this H over bar and many a times for the liquids and ideal gases the enthalpy is independent of the pressure. So, that we can write dou h by dou p is equal to 0. Next is a dou H by dou T we know that dH equal to c p dT. So, in that case we can write dou H by dou T as equal to the specific heat.

So, with these kind of simplifications, now we can go back to this original equation we can now see that dH by dt is equal to V rho c p into dT by dt plus all these things ok. So, here we find that how the partial molar enthalpy and these things are coming, please mind it this v rho is coming why because this was a specific value this was dependent for unit mass and the total mass is nothing, but the volume and the density. So, that is how we are getting the v rho term here.

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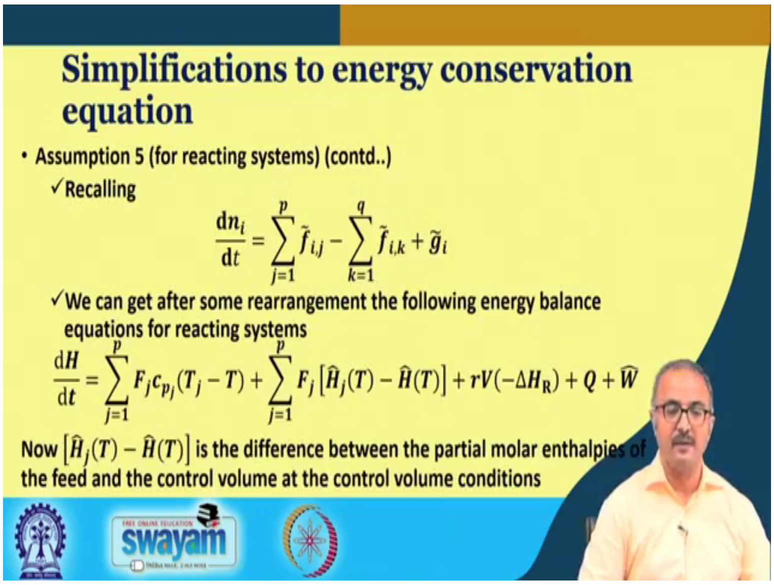

Simplifications to energy conservation equation

- Assumption 5 (for reacting systems) (contd..)
  - ✓Recalling

$$\frac{\mathrm{d}n_i}{\mathrm{d}t} = \sum_{j=1}^{p} \tilde{f}_{i,j} - \sum_{k=1}^{q} \tilde{f}_{i,k} + \tilde{g}_i$$

  - ✓We can get after some rearrangement the following energy balance equations for reacting systems

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \sum_{j=1}^{p} F_j c_{p_j}(T_j - T) + \sum_{j=1}^{p} F_j\left[\hat{H}_j(T) - \hat{H}(T)\right] + rV(-\Delta H_\mathrm{R}) + Q + \widehat{W}$$

Now $\left[\hat{H}_j(T) - \hat{H}(T)\right]$ is the difference between the partial molar enthalpies of the feed and the control volume at the control volume conditions

Now, we know that we wrote it in our previous lecture the mass conservation in terms of the number of moles of the species and here we have the same equation rewritten. So, we find that if we have this equation we can again get after some rearrangement the following equation. So, this dH by dt if you just combine this equation with this equation you will find the dH by dt will be this particular term that is the c p delta T for with associated with all the incoming streams.

Now please understand this is delta T this is no more dt, delta T is the finite difference between two particular values ok. So, here it is the a finite difference between two temperatures, and this is the one for the simplifications we made in for the enthalpy of the incoming stream, and this is the reaction rate term, here the reaction rate R is the reaction. That means the rate of change of moles of a species per unit volume.

So, we have to put in terms of we multiply this with the total volume to get the total change due to reaction and total change and you know that any reaction may be associated with some heat of reaction. And that heat of reaction will decide whether the system is exothermic that is liberating heat or endothermic that is absorbing heat ok.

So, this particular whole term signifies the total amount of energy change due to the heat of reaction and rest of the things are as usual that the heat interaction due to the conduction and radiation and this is the work due to the expansion and the shaft work;

that means, the only change we are extra term we are having is due to the reaction and this will be absent when we have a non reacting system.

Now, you see this particular term is the difference between the partial molar enthalpies of the feed and the control volume at the control volume conditions. That is both are at the same temperature as that of the control volume.

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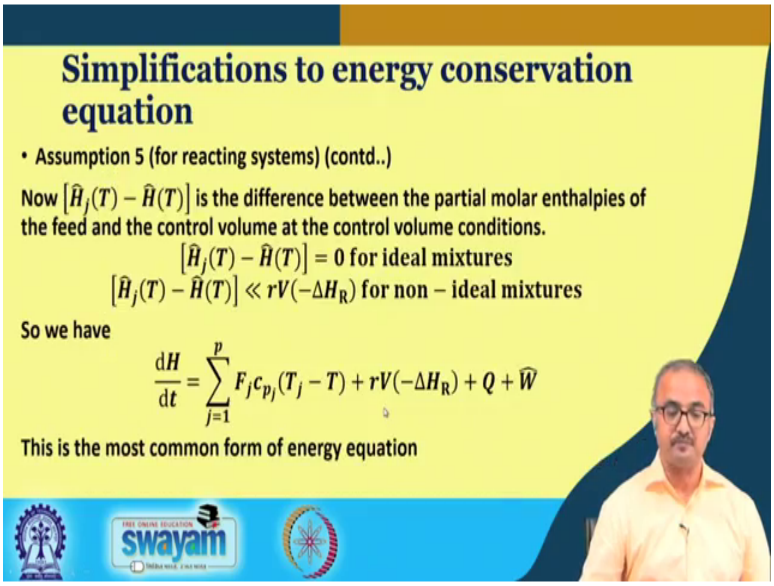

Simplifications to energy conservation equation

- Assumption 5 (for reacting systems) (contd..)

Now $\left[\hat{H}_j(T) - \hat{H}(T)\right]$ is the difference between the partial molar enthalpies of the feed and the control volume at the control volume conditions.

$$\left[\hat{H}_j(T) - \hat{H}(T)\right] = 0 \text{ for ideal mixtures}$$

$$\left[\hat{H}_j(T) - \hat{H}(T)\right] \ll rV(-\Delta H_\mathrm{R}) \text{ for non} - \text{ideal mixtures}$$

So we have

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \sum_{j=1}^{p} F_j c_{p_j}(T_j - T) + rV(-\Delta H_\mathrm{R}) + Q + \hat{W}$$

This is the most common form of energy equation

Now, this particular thing may be 0 for ideal mixtures or may be very very less than the total amount of heat that is generated for non ideal mixtures ok. So, in that case we can neglect this particular term and we are left with now this particular equation ok. So, that is another simplification we have made for the reacting systems. And this is the most common form of the energy equations and you will see that we are using this kind of equations many a times ok.

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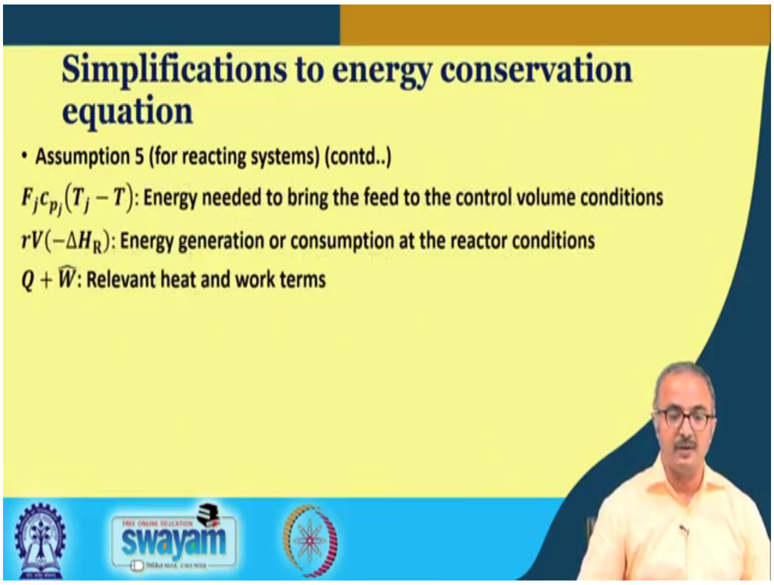

Simplifications to energy conservation equation

- Assumption 5 (for reacting systems) (contd..)

$F_j c_{p_j}(T_j - T)$: Energy needed to bring the feed to the control volume conditions

$rV(-\Delta H_R)$: Energy generation or consumption at the reactor conditions

$Q + \widehat{W}$: Relevant heat and work terms

So, here are the ones that this is, this particular term signifies the energy that is needed to bring the feed to the control volume conditions ok. That means, if the feed is at a lower temperature then we need this kind of a sensible heat to raise its temperature. Or if the feed is at a higher temperature than the control volume then we need again a sensible heat to cool down the feed. So, this particular term represents that sensible heat and this term represents the energy generation or consumption at the reactor conditions and this particular a term represent the relevant heat and work terms ok.

So, whenever we are making the simplifications please do not forget to mention all the assumptions associated with the particular simplification, if you do not read the assumptions then you are going to commit mistakes. And also please see to it that from system to system, from the process different types of processes which kind of assumptions are valid or not valid ok.

According to the process you decide that which assumptions need to be taken or need not to be considered and it is that is why it is very very important for us to understand the process properly. If we do not have a proper understanding of the process the operating conditions the thermodynamic properties of the various components associated in the process then we are going to commit some mistake in the assumptions. And with wrong assumptions we shall be landing up with wrong results, and it will be giving us wrong analysis or wrong behavior performance prediction for the given systems, and will lead to wrong design for the systems ok. So, you have to understand all these concepts very carefully.

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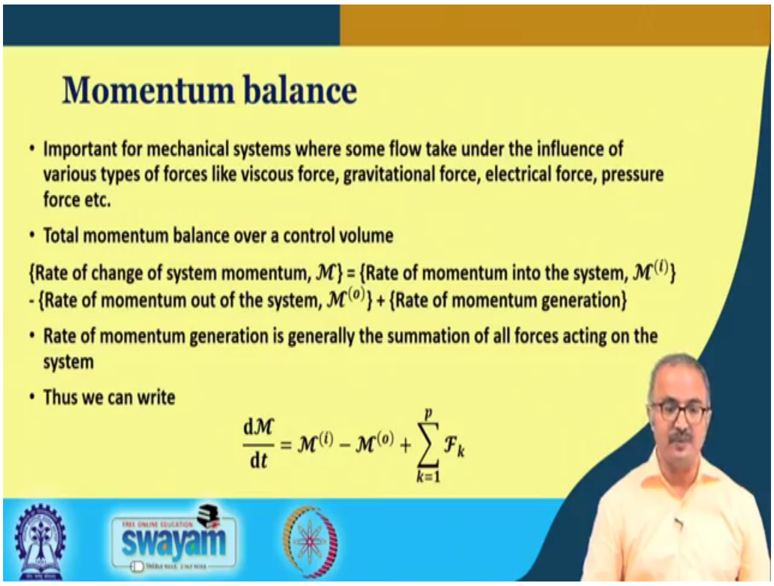

Momentum balance

- Important for mechanical systems where some flow take under the influence of various types of forces like viscous force, gravitational force, electrical force, pressure force etc.
- Total momentum balance over a control volume

{Rate of change of system momentum, $\mathcal{M}$} = {Rate of momentum into the system, $\mathcal{M}^{(i)}$} - {Rate of momentum out of the system, $\mathcal{M}^{(o)}$} + {Rate of momentum generation}

- Rate of momentum generation is generally the summation of all forces acting on the system
- Thus we can write

$$\frac{d\mathcal{M}}{dt} = \mathcal{M}^{(i)} - \mathcal{M}^{(o)} + \sum_{k=1}^{p} \mathcal{F}_k$$

Now, after looking into the energy balance we now go to the momentum balance and momentum perhaps that is another property that is conserved and is nothing but the product of the mass and the velocity ok. So, in general the momentum is a vector because velocity is a vector. So, this momentum balance is important for mechanical systems where some flow takes place under the influence of various types of forces like viscous force, gravity force, electrical force, pressure force. So, there can be various types of forces associated with various types of processes in the in the nature ok.

So, you have to consider appropriate forces for example, if there is something coming down from some height the gravity will play the effect ok. Viscous force comes when? When a fluid is moving over some surface then due to the viscosity of the particular fluid there will be some kind of drag force between the surface and the fluid and the surface will try to retard the fluid motion ok.

So, there you get this viscous effect, similar effects you can also find when you are walking on some rough surfaces. For example, sand then you will find because, there will lot of drag and it will be difficult for you to walk on the sand. On the other hand the drag will be quite less if you are walking on some smooth surface or some ice. So, in the mountains when you go you should be very careful on the whenever you are walking on the ice because you may slip because of the very very less amount of drag force on you ok.

So, we know that this kind of drag forces are kind of a necessary evil at times to save us from any kind of calamity. But in case of this process also similar manner we find these drag forces they generate some kind of pressure drop in the system ok. Now let us go to the statement of the total momentum balance over a control volume here we have it that rate of change of system momentum. And we are putting it this kind of M, will be equal to rate of momentum into the system that is given by M into this (Refer Time: 18:56) into this kind of subscript i in a parenthesis to signify the input. And this is the rate of momentum out of the system given by this kind of superscript that is a in parentheses o.

And the rate of momentum generation the how do we generate momentum this momentum is generated due to the forces, you know that rate of change of momentum is the force. So, any kind of force acting on the system will be generating some moment some momentum change in the system and that is how the flow behavior of the system will be different. So, this is the mathematical representation of the momentum balance the dM by dt is equal to incoming momentum, rate of incoming momentum minus rate of arguing momentum and these are all the forces which are acting on the system.

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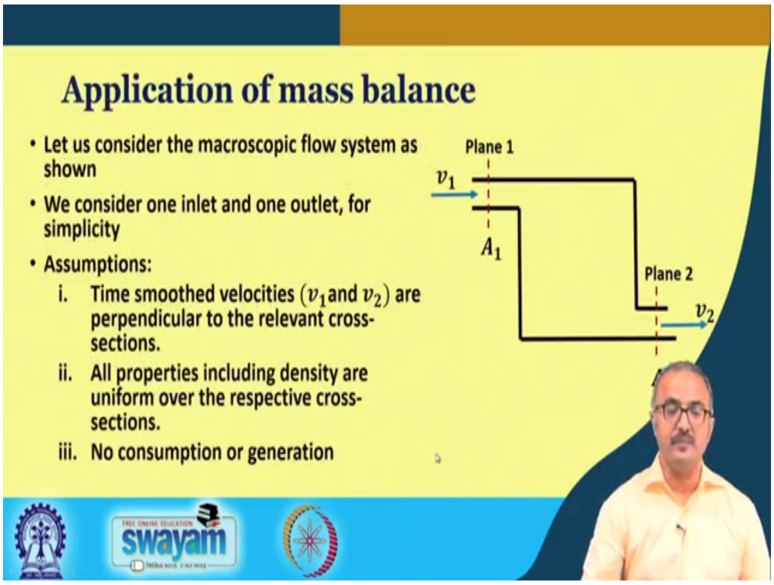

Application of mass balance

- Let us consider the macroscopic flow system as shown
- We consider one inlet and one outlet, for simplicity
- Assumptions:
  - i. Time smoothed velocities (v_1 and v_2) are perpendicular to the relevant cross-sections.
  - ii. All properties including density are uniform over the respective cross-sections.
  - iii. No consumption or generation

Now, after learning some basic concepts about the mass balance, energy balance and momentum balance let us do a very simple analysis on the very simple system to see that how we can apply these concepts.

So, let us for that we have this particular system here you see that the flowing system in this case we are considering only one inlet and only one outlet ok. Now we are doing what? We are having these two planes at the inlet and at the outlet and we are considering that the inlet and outlet need not have the same area of cross section. So, we are representing these areas of the cross section by A 1 and A 2 respectively. And we are having the fluid moving into the system with a velocity v 1 and coming out of the system with a velocity v 2 ok.

Now, to analyze this first we apply the mass balance with some assumptions. The first assumption is that we are considering time smoothed velocities. That means, these v 1 and v 2 are even if there is some small substitutions we are not considering that we are taking a an average velocity for this two and the two ports ok. And all properties including density are uniform over the respective cross section. That means, along this particular cross section here we are seeing that there is no variation of the property ok.

Similarly, at this cross section also there is no version of property. So, in that case what we find this velocity will be uniform at this particular inlet and this particular outlet. And these velocities are sometimes obtained from the division of the volumetric flow rate by the cross sectional area. And that is called the superficial velocity of the fluid and we are also assuming that there is no consumption or generation.

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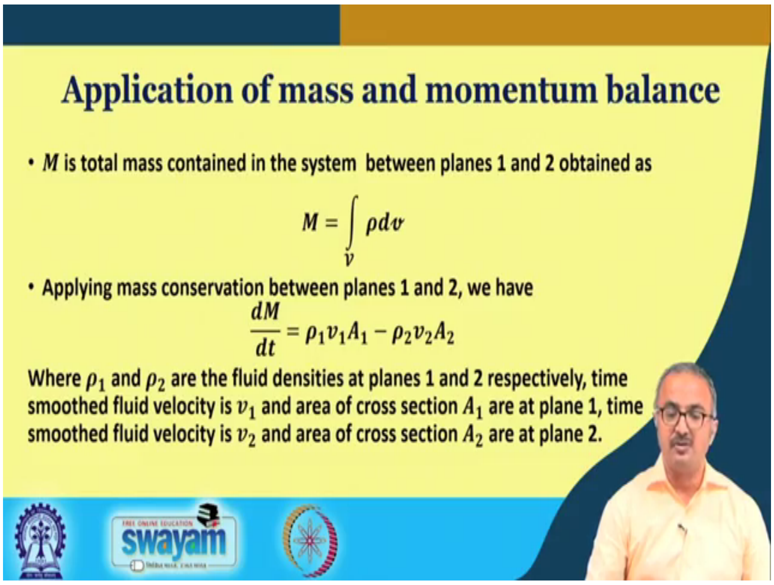

Application of mass and momentum balance

- M is total mass contained in the system between planes 1 and 2 obtained as

$$M = \int_{V} \rho \, d\upsilon$$

- Applying mass conservation between planes 1 and 2, we have

$$\frac{dM}{dt} = \rho_1 v_1 A_1 - \rho_2 v_2 A_2$$

Where ρ_1 and ρ_2 are the fluid densities at planes 1 and 2 respectively, time smoothed fluid velocity is v_1 and area of cross section A_1 are at plane 1, time smoothed fluid velocity is v_2 and area of cross section A_2 are at plane 2.

So, with these assumptions we see that the mass balance between these planes one and two is reduced to this particular equation and here we are simply having this mass contained within the system is rho dv. So, the mass balance mass balance is coming to like this that, this is the total mass in the control volume rho dv. So, we are putting it like this and here we have the two terms which are signifying the incoming and the outgoing flows. And here are the notations we are putting the subscripts as per the particular port. So, one here represents the incoming port and two here represents the outgoing port so density into velocity into the area of cross section.

Next we come to the momentum balance, it is also quite simple that this is a rate of change of momentum and this is the rate of momentum into the system. How we are obtaining it? Here you see that rho 1 into v 1 into v 1. So, rho one into v 1 that is density into velocity is the rate of change of the mass ok; that means, mass flow rate; mass into the velocity is the momentum. So, that is M into v or rho 1 v 1 into v 1 is the rate of momentum going into the system and into the area and as you know this is the unit normal that is coming out of the system ok. So, this is the mass flux into the area that is giving me the rate of change of the momentum.

Similarly, we have the mass flux as rho 2 into v 2 as the mass flux mass flux into area is the mass flow rate and into the v 2 gives the momentum. So, we are getting rho 2 v 2 square into A 2 and again if this is associated with the direction and this direction is given by the outward normal at this plane 2. Then we are the pressure forces and for this equation please refer to in the previous lecture. So, we will see that is the pressure force coming on the system and the pressure is acting on an area. So, pressure into area is the force. So, the pressure force at plane 1. And similarly we have the pressure force at plane 2 and again we have the unit normals associated at the each plane.

Then we have the force due to the solid surface, as I was telling you there could be drag forces which are experienced by the fluids due to the surface on which they flow and next is the gravity force of the fluid ok.

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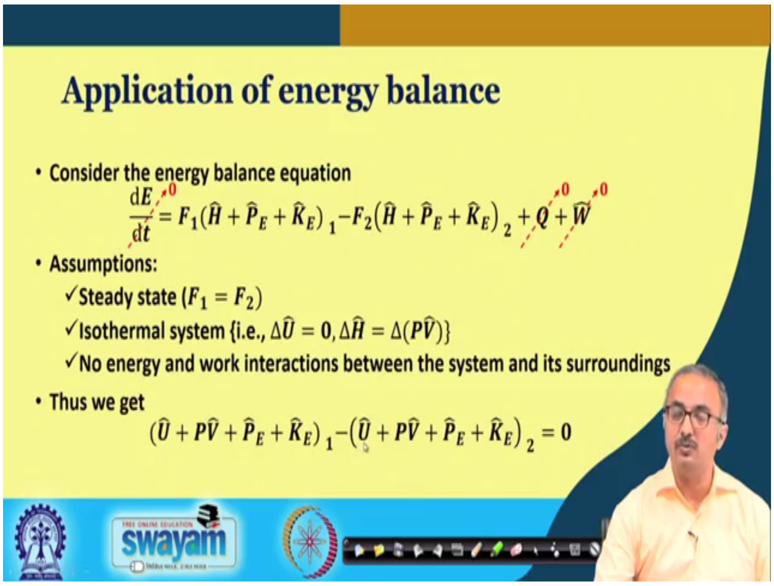

Application of energy balance

- Consider the energy balance equation

$$\cancelto{0}{\frac{dE}{dt}} = F_1(\hat{H} + \hat{P}_E + \hat{K}_E)_1 - F_2(\hat{H} + \hat{P}_E + \hat{K}_E)_2 + \cancelto{0}{Q} + \cancelto{0}{W}$$

- Assumptions:
  - ✓Steady state ($F_1 = F_2$)
  - ✓Isothermal system {i.e., $\Delta\hat{U} = 0, \Delta\hat{H} = \Delta(P\hat{V})$}
  - ✓No energy and work interactions between the system and its surroundings
- Thus we get

$$(\hat{U} + P\hat{V} + \hat{P}_E + \hat{K}_E)_1 - (\hat{U} + P\hat{V} + \hat{P}_E + \hat{K}_E)_2 = 0$$

Now, let us go into the application of the energy balance for the same system. Now in this case please note that we have only two streams and we are naming them as F 1 stream and F 2 stream, F 1 is the ingoing stream and the F 2 is the outgoing streams. So, with each of these streams we are associating all the kinds of energies we studied earlier and now we are doing some simplifications on this particular system.

So, first we assume that the system is the steady state in that case we find that there will not be any change in the energy of the system with time. So, that we are putting this left hand side derivative as 0 and also from the mass balance equation we find that the flow rates will be constant from the inlet to the outlet. And there will not be any kind of accumulation of the mass within the system.

Next we make another assumption that isothermal system in that case we know that there will not be any change in the internal energy of the system, because internal energy is a function of only temperature. So, it will not be changing. So, that the change in the enthalpy is will be purely due to this change in this particular product PV that P into the specific volume.

Next we come to another that there is no energy or work interactions between the system and the surrounding that is neither we are giving any work to the system nor we are trying to take out any work from the system ok; so no pumping no compressing etcetera. So, in that case we are putting the work interaction 0 and heat interaction can be put to 0, if we assume there the process is well insulated. So, that it is the adiabatic ok. So, in that case we put the heat interactions to be 0.

Now, with these kinds of assumptions we now write this particular expression and now we find that U 1 and U 2 will be cancelling each other out and what we shall be left with will be only these three terms and these three terms ok.

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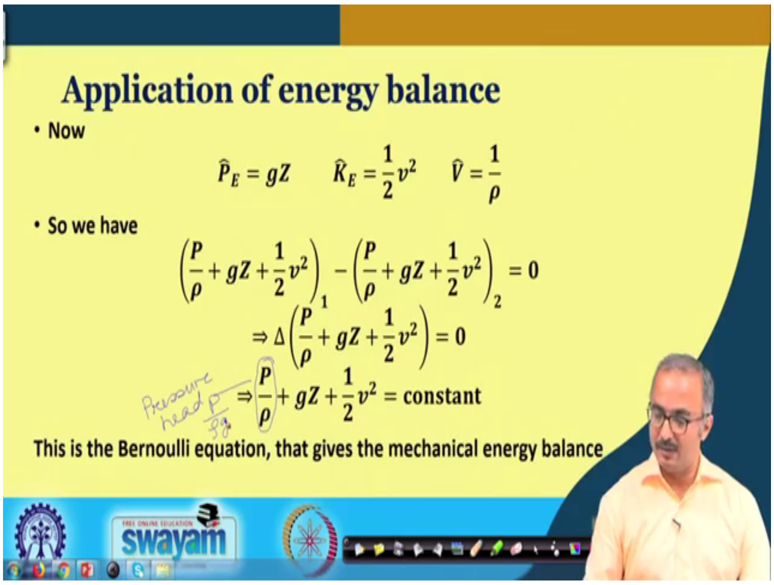

Application of energy balance

- Now

$$\hat{P}_E = gZ \qquad \hat{K}_E = \frac{1}{2}v^2 \qquad \hat{V} = \frac{1}{\rho}$$

- So we have

$$\left(\frac{P}{\rho} + gZ + \frac{1}{2}v^2\right)_1 - \left(\frac{P}{\rho} + gZ + \frac{1}{2}v^2\right)_2 = 0$$

$$\Rightarrow \Delta\left(\frac{P}{\rho} + gZ + \frac{1}{2}v^2\right) = 0$$

$$\text{Pressure head } \frac{P}{\rho g} \Rightarrow \boxed{\frac{P}{\rho}} + gZ + \frac{1}{2}v^2 = \text{constant}$$

This is the Bernoulli equation, that gives the mechanical energy balance

Now, next what we do is that we write the expressions for the potential energy, the kinetic energy and the specific volume. We know the specific volume is the reciprocal of the density the here we are writing the kinetic energy per unit mass. So, that it is coming to half into v square similarly the potential energy per unit mass is the product of the gravitational force and the Z. So, these three terms are now giving us the respective terms to be put in the energy balance equation which we have just found out by making those assumptions.

Now, we put those things here we put that instead of the v cap we are putting one by rho. So, it is becoming P by rho plus gZ plus half v square minus P by rho gZ plus half v square for the inlet and the outlet and that is equal to 0. It means what that if we take the difference this delta is signifying the difference to the depth. That means, the difference between the energy of the inlet and the outlet is 0 so that we see that the total energy in the system is remaining constant. That means, there is no change in the energy from the inlet to the outlet provided we have those assumptions we have just mentioned. And this per this equation you are quite aware that this is the Bernoulli equation and we say that this Bernoulli equation is the mechanical energy balance.

Now, this mechanical energy because the all the terms in this particular expression does some kind of mechanical work. So, we are talking mechanical energy now second thing is this there could be various representations of the Bernoulli equation. Now one representation can be we divide the whole expression by the g. So, if you divide it by g P by rho g plus Z plus half v square by 2 2 g the half v square by g now what it means is this that all the terms will be in some kind of a length dimension ok.

So, whenever we are putting the various forces or the energies in the length dimension we call them head. So, accordingly we have the various heads here, the first one this, this one we call the pressure head pressure head ok; that means, P by rho g pressure head. So, you can check it also that when you take this P by rho g the dimension will be in terms of some length ok. For example, if you are putting the SI unit this will be meters ok, similarly if we look at this second term now this is will be only Z it will give rise to only Z ok. So, this we call the potential head; that means, it will see tell us the effect of the elevation of the particular system from some given datum. So, this is the potential head ok.

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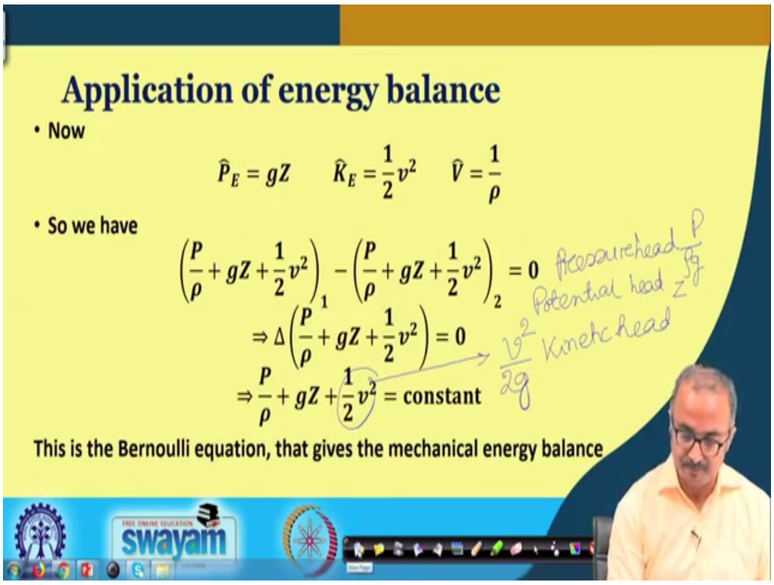

Application of energy balance

- Now

$$\hat{P}_E = gZ \qquad \hat{K}_E = \frac{1}{2}v^2 \qquad \hat{V} = \frac{1}{\rho}$$

- So we have

$$\left(\frac{P}{\rho} + gZ + \frac{1}{2}v^2\right)_1 - \left(\frac{P}{\rho} + gZ + \frac{1}{2}v^2\right)_2 = 0$$

$$\Rightarrow \Delta\left(\frac{P}{\rho} + gZ + \frac{1}{2}v^2\right) = 0$$

$$\Rightarrow \frac{P}{\rho} + gZ + \frac{1}{2}v^2 = \text{constant}$$

This is the Bernoulli equation, that gives the mechanical energy balance

Pressurehead $\frac{P}{\rho g}$

Potential head Z

$\frac{v^2}{2g}$ Kinetic head

And the third term which we shall get from this particular term this will be v square by 2 g again if you take the dimension of this particular term you will find this will be in terms of some length dimension ok. So, this we call the kinetic head ok. So, we have three heads one writing again once we have the pressure head pressure head that is P by rho g we have the potential head that is simply Z. And we have the next is the kinetic head and we find if we sum up these three heads we are having constant. That means, there will not be a total head of the system will remain constant it will remain unchanged from the input to the output side ok.

So, in this particular lecture now we have seen that we have applied these energy balance mass balance and momentum balance to a very simple system. And in future we shall do some tutorial on the various problems and through see that how else we can apply these equations.

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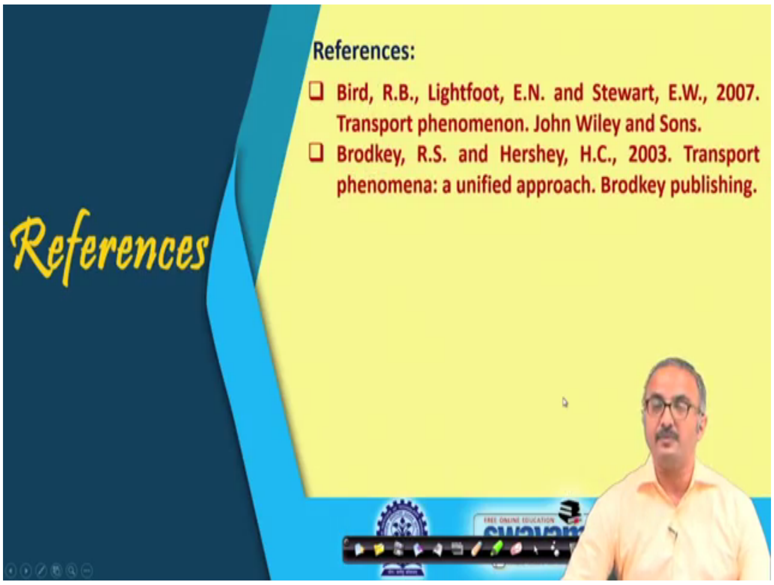

References:

- ☐ Bird, R.B., Lightfoot, E.N. and Stewart, E.W., 2007. Transport phenomenon. John Wiley and Sons.
- ☐ Brodkey, R.S. and Hershey, H.C., 2003. Transport phenomena: a unified approach. Brodkey publishing.

So, you can get more detail of this, from these two books there are very good books for to give you more detail. And we shall also continue more on this in future.

Thank you.