Mass, Momentum And Energy Balances In Engineering Analysis

Prof. Pavitra Sandilya
Cryogenics Engineering Center
Indian Institute of Technology, Kharagpur

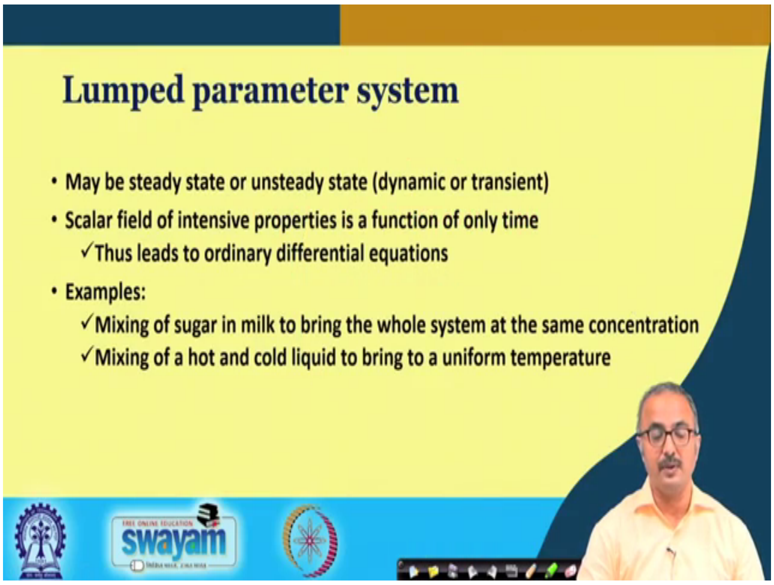
Lecture – 08
Macroscopic Balances – II

Welcome, today we shall look more into the Macroscopic Balances, which we started in the last lecture.

(Refer Slide Time: 00:26)

In this lecture, we shall know something more about the lumped system and then we shall move on to the mass conservation in integral form and energy conservation in integral form.

(Refer Slide Time: 00:40)



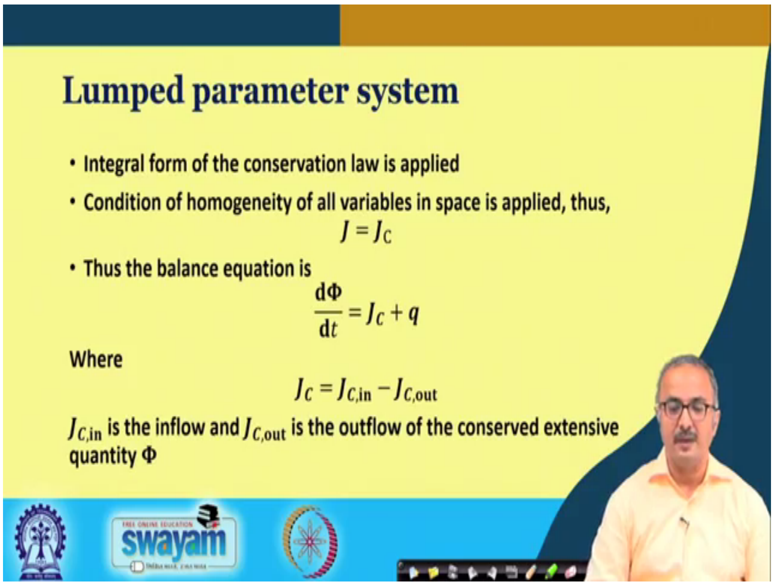
So, a lumped parameter system may be at steady state or at unsteady state, this unsteady state is also called dynamic or transient, now what it means is that under steady state the value of any parameter or property will not change with time at any given location. However, it may be so the parameter value can change from various one location to another location, but in any of the locations the values will not be changing then it is called steady state. And in case of unsteady state that in at any given location, the parameter value will change with time. So, what happens during the startup of a process or at the shutdown stage of a process, we encounter such kind of unsteadiness in the process.

Now, generally in the lumped parameter domain these systems consist of intensive properties, which are functions of only the time because, we are assuming some uniform value of the particular intensive property within the system. So, we find that the balance equation, when we write in that balance equation all the spatial derivatives are vanishing and so we are left with only the time derivative so that the partial differential equations get reduced to the ordinary differential equations.

Now, there are many examples of such kind of lumped systems in our day to day life, one example is that of that mixing of sugar in milk. Now in this case what happens that if we mix the sugar with milk, we want that the milk should be attaining same sweetness all through the mass. So, by stirring is helping us to make this uniformity faster, we can even do the same thing by not stirring it, but we know that if you dont stir, then it will take longer time to get the uniform sweetness. So, when we are not stirring; that means, we are going through some kind of unsteady state behavior and then we are reaching the steady state, when the whole solution becomes of the same sweetness. On the other hand, when we are mixing it, we are trying to reduce the time of the unsteady period, so that we can reach the steady state quickly.

A similar example can be given from the heat transfer domain in that case, we can consider the mixing of a hot and cold fluid, if we leave the system alone then it will take it is own time through convection and some part of radiation for the fluid to get mixed and attain a single temperature. On the other hand, if we are stirring this whole mixture then we will find that the whole mixture will be attaining a single temperature or it will reach a steady state quicker then without stirring.

(Refer Slide Time: 03:46)

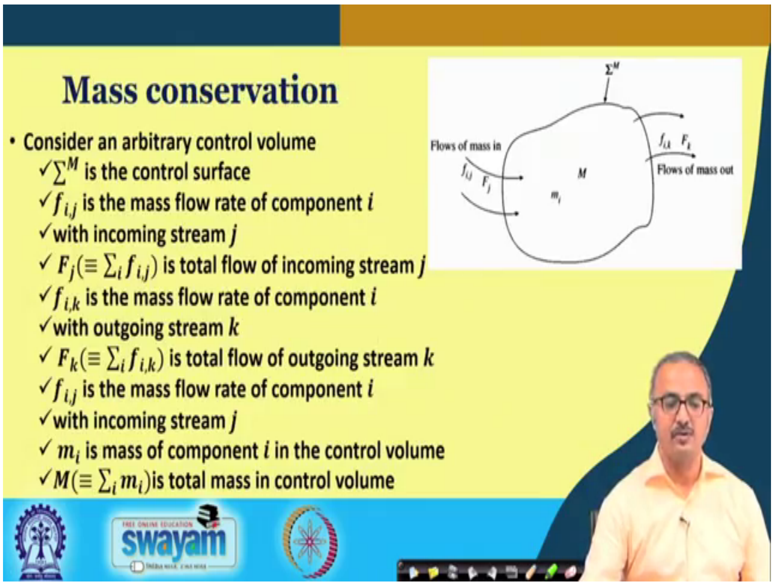


Now, let us see that how this conservation law when applied to a lumped parameter system, what form it takes? So, we know that because it is a homogenous mixture, we are talking about for the lumped parameter. So, the flux becomes a contribution only of the convective part and there will not be any diffusive part, if you recall you know that diffusive part depends on the gradient ok. So, if there is no gradient whether of concentration, whether of temperature or whether of any other parameters then we will find that there is no question of any kind of diffusive transfer. So, we are left with only some kind of convective transfer.

Now, then this whole equation gets reduced to this and this phi on this side comes by the integration of the volume specific property over the given volume. For example, if we consider the mass balance equation in that case, the specific volume or the density is what matter. So, if you are integrating the density over the volume you gets the mass. So, this total mass is represented by this thing for the system, on the other hand, on the right hand side we find this flux is governed by only the convective term whereas, this is the kind of some source term here.

Now, the flux we can see it consists of again 2 parts some may be coming into the system and some will be going out of the system. So, this flux is the net flux of the incoming and the outgoing streams into or from the particular system. So, that is how we are able to obtain this net flux.

(Refer Slide Time: 05:41)

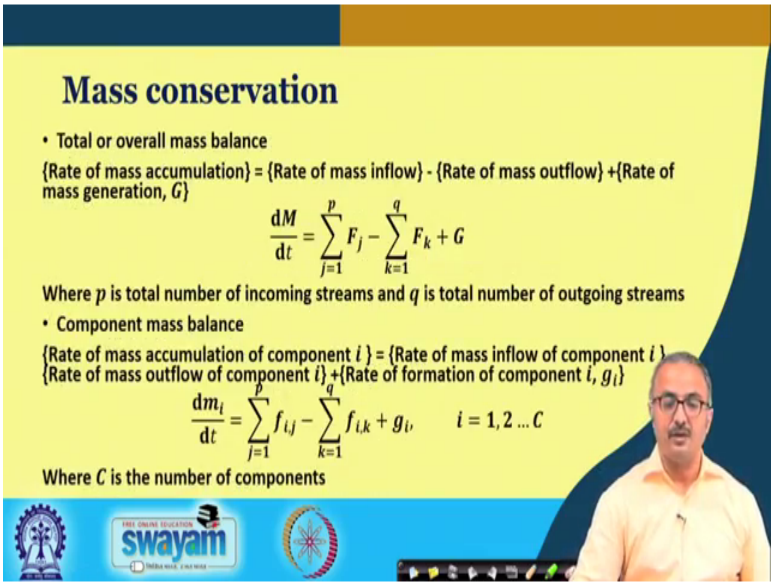


Now, coming to application of these concepts for mass conservation, let us consider a control volume as shown here in this control volume, we have certain mass M and also this mass is taken to be a mixture of various components and here I represents an ith species or component. So, this small m i represents the mass of the ith component and this particular sigma M represents the control surface.

Now, in this control volume what we find that there could be one or more than one incoming streams and these incoming streams are denoted by capital F J and here we also have something small f ij, it represents the incoming flow of the ith component in the jth stream ok. So, you can see that the total mass of a stream will be the summation of the component masses. So, for if for a FJ stream, it is equal to the summation of all the fijs ok.

Similarly, we can have outgoing streams from the control volume and in that case, we represent the outgoing flow or flow rate by Fk and this small f i k is the mass of the ith component in the kth stream ok. So, we can see that this ith this one will also be this Fk is a summation of the individual component masses and also we can see the total mass in the system is the summation of the masses of the all the components ok.

(Refer Slide Time: 07:36)

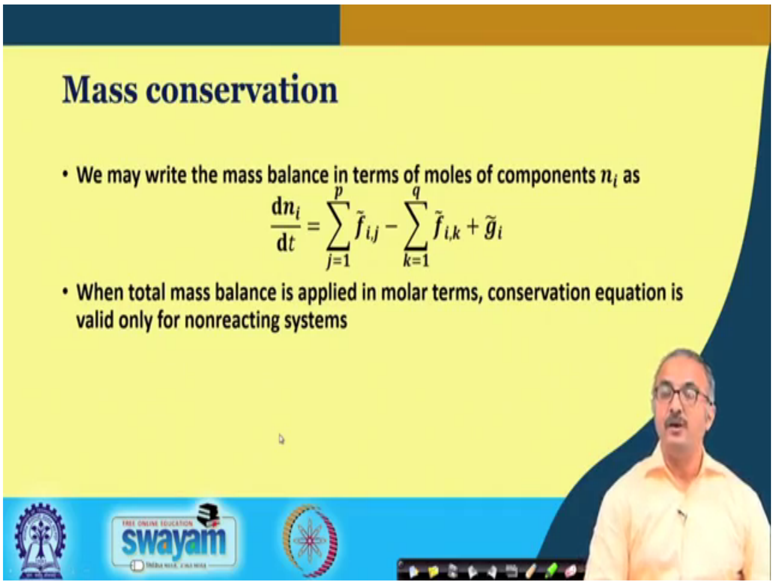


Now, let us apply our in mass balance, now what we see that rate of mass accumulation is equal to rate of mass inflow minus rate of mass outflow and plus rate of mass generation and what happens generation as I told noted in our earlier lecture that generation comes due to may be some reaction is taking place ok. So, in general we can write that this is the mass of the system n capital M. So, the mass of the system changes with time and how that is there will because, there is some kind of inflow and let us assume that there are p number of inflowing streams and let us assume there are q number of out flowing streams. So, in that case, if we add up the flow rates of all the p streams and then subtract the flow rates of all the out stream and then we are adding some kind of generation term within the system. So, this is how we are able to make a mass balance over the particular system.

Now, this is a total or overall mass balance, we can also write the mass balance for individual components in that case, what we do similar to this one we simply put the rate of mass accumulation of ith component and the rate of mass inflow of the ith component rate of mass outflow of ith component and rate of formation of component i that is given by small g i.

So, here you see the these are similar only thing is this, now we are replacing the total mass by the component mass the total flow by the component flows and in this case what we are finding, we are just summing up the flow rate of ith component in all the streams in all the incoming streams or the p number of incoming streams and similarly we are subtracting from this the summation of the flow of the ith component from all the outgoing streams and that that is numbering 2 q number of outgoing streams and this is the g i that is the rate of generation of the ith component and here please note, that in this case this particular equation has to be applied for all the components. So, if there are c number of components, we have to write c number of mass balances.

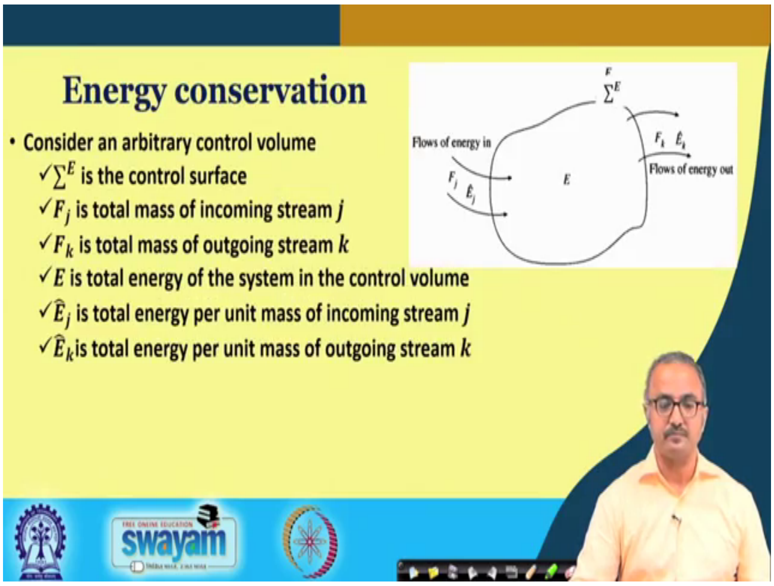
(Refer Slide Time: 10:03)



Now, this mass balance form may also be extended in a molar form because, this molar form is many times used for reacting systems. So, in that case what happens that the mass is replaced by the total number of moles of the ith stream inside the system and these are; this tilde represents the molar flow of ith stream. So, this I am showing the component material balance in a similar manner, you can write the overall material balance in terms of the moles.

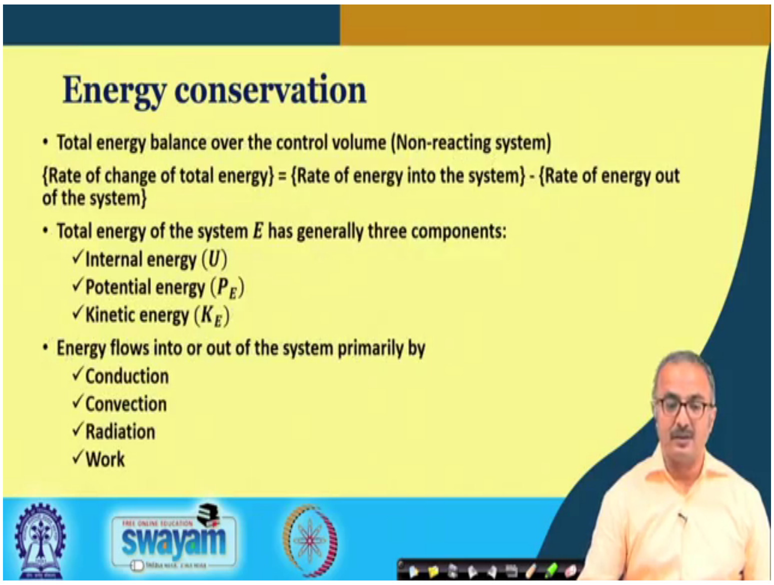
Now, please note that whether whereas, mass is conserved mass is conserved because, we know that mass can neither be destroyed nor be created. So, total mass will always be conserved ok; however, it is not necessary that the number of moles will be conserved and you will find many a times in many of the reactions, you will find if you add up the number of moles on the reactant side they may not be adding up to the number of moles on the product side. So, we can say that the number of moles will not be conserved on the other hand mass will always be conserved ok. So, only for non reacting systems, we find that the mass balance and the mole balance will be valid, otherwise for reacting systems mass balance will be valid.

(Refer Slide Time: 11:36)



Now, let us applied it is lumped parameter analysis for energy conservation. Now again for this, we again consider certain control volume and with this sigma E is the control surface and here we find again, we have the energy coming in through some kind of flows. So, again we have those Fj flows representing the incoming streams and here we have the Fk flows representing the outgoing streams and here we have the Ej cap that is showing the energy associated per unit mass of the jth stream. Similarly, we have this E cap k this is showing the energy associated per unit mass of the kth stream ok; that means, for the total energy for all the streams going in will be the summation of the product of this particular mass flow rate and the specific energy ok. So, these are the notations for our energy balance.

(Refer Slide Time: 12:46)



Now, let us write the energy balance and let us write for non reacting systems first. Now what we find here? We have the rate of change of total energy is equal to rate of energy going into the system and rate of energy going out of the system and what we find here? The total energy of a system is now we will be dependent on many other energies and how do we calculate it that any energy of any system consist of basically, these 3 parts one is the internal energy then the potential energy and the kinetic energy.

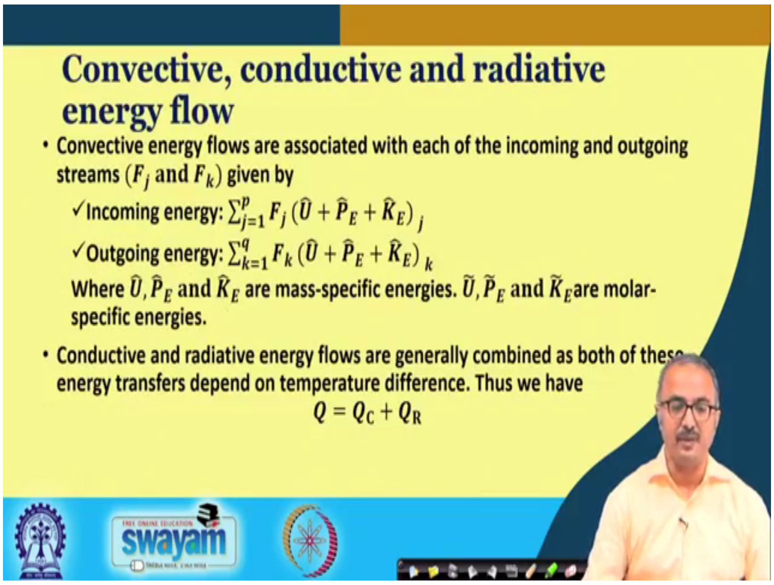
Internal energy as you know is coming due to the structure of the particular system and due to the various kind of interactions between the constituting particles within the system that is the internal energy of the system and then we have potential energy that comes due to the elevation of the particular system with respect to some datum ok. Now this elevation could be positive or negative similarly, we have kinetic energy that comes due to the motion of the particular system.

Next what are the modes by which energy can flow in or flow out of the system? So, these are the various modes, we know that energy may be flowing due to conduction, due to convection, due to radiation and due to work flow. Now conduction perhaps all of you know is coming, whenever there is a temperature gradient across a solid the energy will flow by conduction and in that case, we use the Fourier's law of heat conduction.

Now, when talk of conviction it is again due to some kind of a temperature difference, but now it is going through some kind of fluid and in that case you know perhaps that we use the Newton's law of cooling then we go to radiative heat transfer in that case, we use a stiffen Boltzmann equation and that is also coming whenever there is a temperature difference ok.

And next is the work, now we shall look more into the work let us see what kind of works may be there in a given system.

(Refer Slide Time: 14:59)

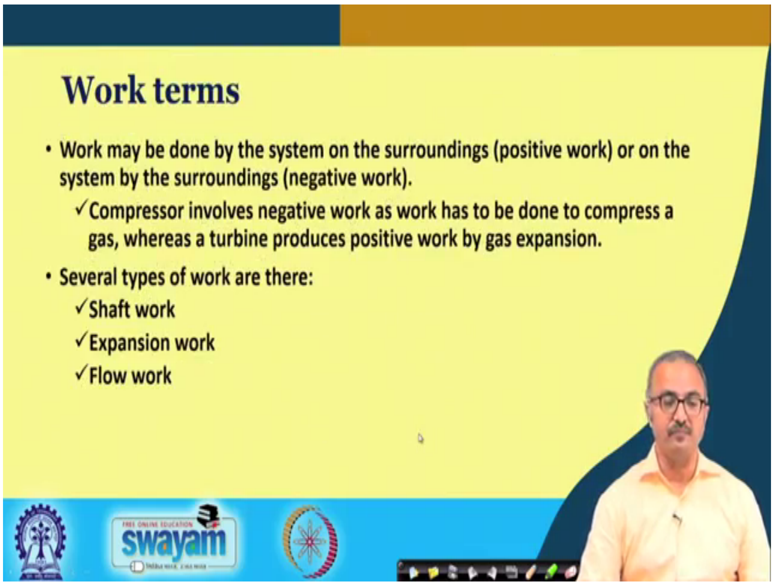


So, as I said that convective conductive and radiative energy flows are can be first taken together and first we see that the incoming energy is due to this internal energy potential energy and the kinetic energy and this is associated with certain stream j and then these are the specific values that is per unit mass. So, we have to multiply them with a mass flow rate to get the total amount of energy going per unit time.

Similarly, for the outgoing energy we also multiply the mass flow rate that is going out of the system with it is internal energy, the potential energy and the kinetic energy taken together ok. So, this cap thing represents the mass specific energies whereas, we may also use molar specific energy which will represented by this kind of tilde.

Next generally the conductive and the radiative energy flows are taken together, we put Q and this Q has 2 components QC and QR.

(Refer Slide Time: 16:15)



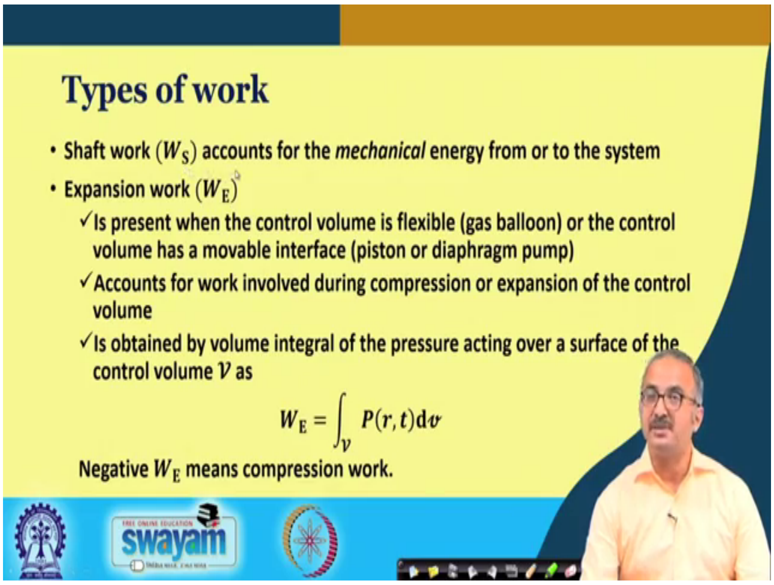
Now, you see that work may be done in various manner, now a system may do work on the surroundings or the surrounding may do work on the system. Now, because of these two kinds of interactions some kind of sign conventions are followed in the literature. Now these sign conventions may differ for us the sign convention is like this, if the system is doing work on the surroundings; that means, we are able to derive some work from the system that can be utilized or some other useful purpose, then we say this is a positive work. On the other hand, if the surroundings is working on the system then we have the negative work.

Now, understand this again I tell you there is this particular sign convention may be taken in the opposite manner in some other literature, but it does not matter that we sign convention you follow as long as you are consistent in the sign convention the results will be the same ok.

Now, let us give an example of this positive and negative work. Now you see compressor, now compressor is doing what? That compressor is trying to compress some kind of gas and how does it work? That we have to give some energy to the compressor to compress it so; that means, compressor is involving a negative work. On the other hand, if you look at a turbine, turbine is doing what? Turbine is used to produce work by expansion of a fluid. So, this turbine is considered to have a positive work.

Now, several types of work are there maybe it may be shaft work, it may be expansion work or it may be flow work.

(Refer Slide Time: 18:10)

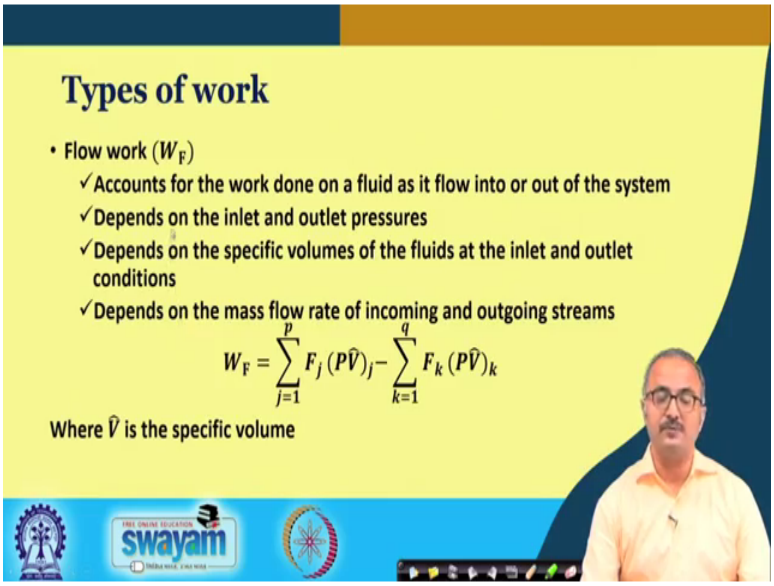


So, let us go into each of them one by one, first let us look in to the shaft work, it is denoted by say W with a subscript of s. Now it represents; what it represents? The mechanical energy of from or to the system. So, if there is any kind of mechanical energy being transferred from the system or to the system, we call it shaft work an example why it is called shaft? Because we take it like this that there is a motor on which we have a some shaft and when the motor rotates and if we are placing some kind of load on the shaft the load will also rotated, may be rotated or it can also undergo some other kind of motion ok. So, that is why we call it a shaft work, it is a purely mechanical energy being transferred.

Then we have expansion work please understand expansion and compression are opposite to each other. So, if we have negative expansion, we take it to be compression ok. Now, when do we have expansion work? When we have a control volume which is flexible by flexible, we mean that the control volume is not retaining it is shape. For example, in a balloon if we are trying to inflate a balloon, it will start enlarging. So, that is a flexible control volume on the other, we can also have some control volume with a movable interface. For example, a piston a syringe you can say the syringe, we can see syringe in our day to day life for example, in the injection we find that we are using syringe to draw the particular medicine ok. So, in that case that is a movable interface. So, that also comes under this kind of expansion work system.

Then this work accounts for any kind of compression or expansion of the control volume and this is obtained how numerically it is obtained by taking the integral of the pressure with the volume. So, this P dv we call we call it some sometimes Pdv work it means that we are taking the small integral in small control volume over week. We are putting a pressure and then it is doing some work and then we are integrating it over the whole volume and negative work of expansion is the work of compression.

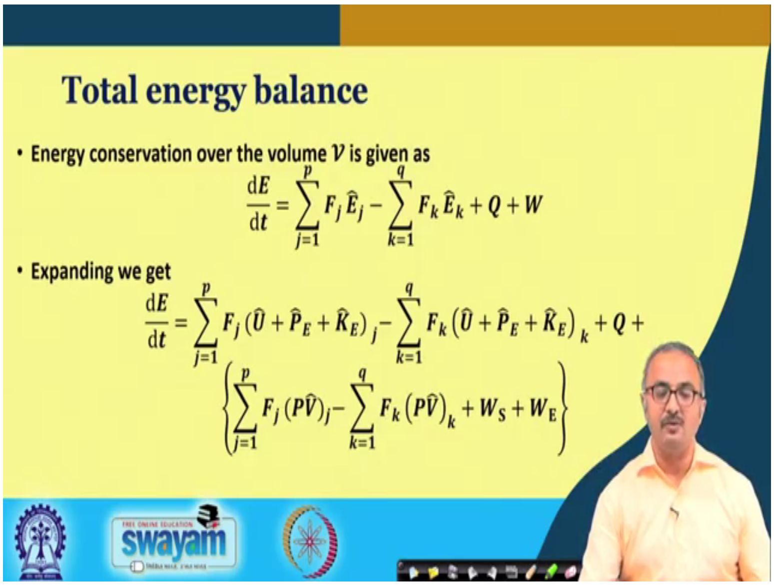
(Refer Slide Time: 20:28)



Now, next we come to the flow work and what it is it accounts for the work done on a fluid as it flows into or out of the system ok. So, whenever there is a flow there is a flow work associated. So, there if there is no flow there is no flow work and it depends on the inlet and outlet pressures the specific volumes of the fluids at the inlet and outlet and also depends on the mass flow rate and without going into the derivation of this let us see that what form it takes. So, this P into V cap, this V cap is the specific volume. So, this P into V cap represent the specific flow work and for the jth stream, if we write it and multiply with the flow rate of the jth stream then we get the total flow work associated with the jth stream and then when we add it up for all the P incoming streams then we have the total amount of flow work going into the system.

Similarly, for the outgoing streams also we can do that we can take the summation over all the q outgoing streams for the product of the mass flow rate of the kth outgoing streams with it is this flow work specific flow work ok. So, and when we do this kind of an algebraic sum, we get the total flow work associated with a system.

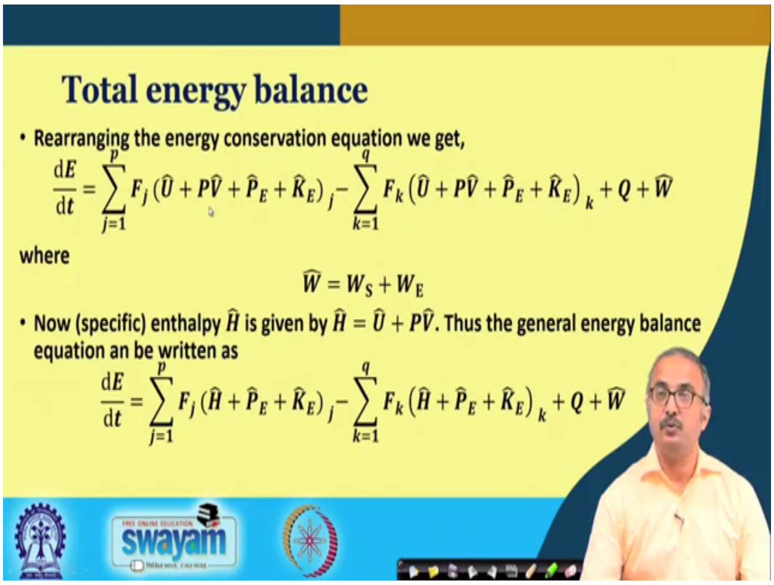
(Refer Slide Time: 22:04)



Now, after knowing about all these work and energy changes that could be there in a system, now we are ready to write the conservation law for the energy in this form. So, here we have for a control volume V, we write it like this that we have the d by dT that is the energy change within the system with time is equal to the total amount of flow work that sorry total amount of energy that is going with all the incoming streams minus total amount of energy that is going out of the system for all the outgoing streams then plus some kind of heat transfer and some kind of work interaction ok.

Now, what we do? This E we put in terms of all the individual energies that is the internal energy the potential energy and the kinetic energy for both the incoming and for the outgoing streams, this is the total of the convective and the radiative heat transfer and here we write all the source of work associated with our work that is the shaft work, the expansion work and this is the flow work for the incoming streams and this is the flow work for the outgoing streams.

(Refer Slide Time: 23:24)



Now, we rearrange this equation and all the rearrangement we find we put this like this that U, U and V with this V put the flow over PV and then rest of the things remain the same and here what we do? We just take the shaft work and the expansion work together as W. Now we know that U plus PV is the enthalpy from thermodynamic relation. So, this is the need of this rearrangement now what we do? We rewrite this expression in terms of the enthalpy of the system. So now, finally, we obtain this particular equation for the energy balance in terms of the enthalpy potential energy kinetic energy, the heat transfer by the conduction and radiation and the total work.

(Refer Slide Time: 24:21)

These are the books which you can refer to for more detail and we shall be continuing this one for some more specific examples.

Thank you.