Mass, Momentum and Energy Balances in Engineering Analysis
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Lecture - 05
Balance Equations Preliminaries – II

Welcome. We now go for more Preliminaries for the Balance Equations. And in this particular lecture, we shall be looking into the following topics.

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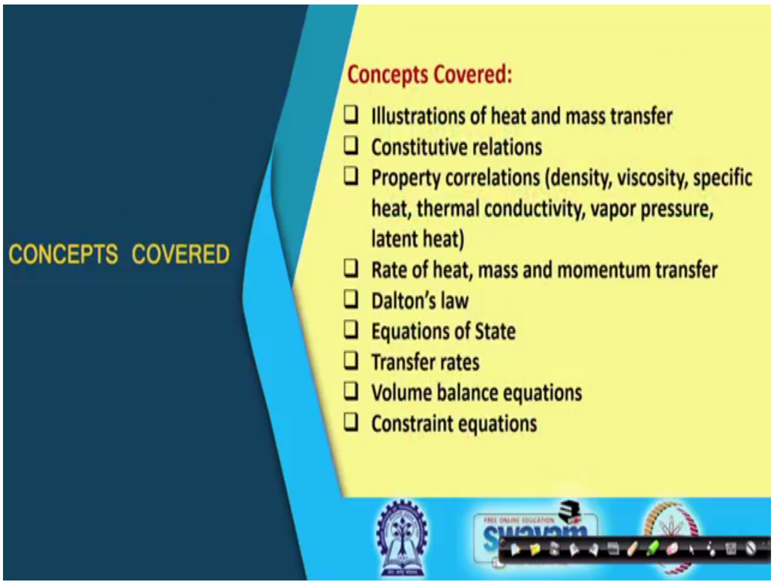

CONCEPTS COVERED

Concepts Covered:

- ☐ Illustrations of heat and mass transfer
- ☐ Constitutive relations
- ☐ Property correlations (density, viscosity, specific heat, thermal conductivity, vapor pressure, latent heat)
- ☐ Rate of heat, mass and momentum transfer
- ☐ Dalton's law
- ☐ Equations of State
- ☐ Transfer rates
- ☐ Volume balance equations
- ☐ Constraint equations

Like some illustrations of our day-to-day heat and mass transfer situations, then constitutive relations; under that we shall be studying about the various property correlations, how to find out the rate of heat transfer, mass transfer, momentum transfer, then term Dalton's law, equation of state, then transfer rates, then a some volume balance equations, and constraint equations.

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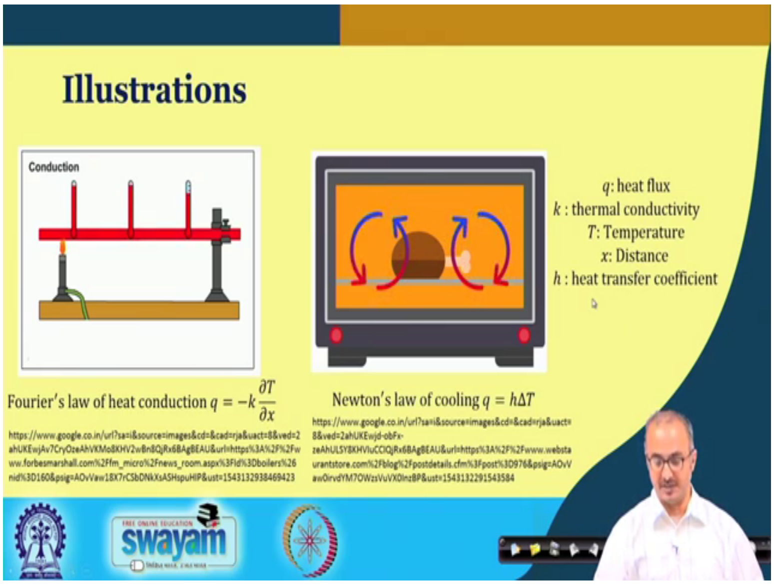

Illustrations

Fourier's law of heat conduction $q = -k\frac{\partial T}{\partial x}$

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Newton's law of cooling $q = h\Delta T$

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q: heat flux
k : thermal conductivity
T: Temperature
x: Distance
h : heat transfer coefficient

So, first let us see some illustrations of these heat and mass transfer, which in many a times you also have encountered, and you have also solved some problems in your school. So, it will be kind of recapitulation of what we already seen, what we already know. So, let us have a look at this particular situation, here we find that it is an example of conduction. And here you can see that there is a flame, if it which is heating of one rod, and here we are putting a few thermometers. And as you know that as will heat the flows from this left-end to the right-end, it will take a longer time to sense the heat at the rightmost points.

So, you see that a thermometers are showing different temperatures. And the first one on this side is getting the highest temperature earlier than the one, which is at the far end from the flame. So, this kind of problem you also have done, and you know that what kind of equations are needed. And this is the Fourier's law of heat conduction that is used to find out the heat flux, and that is given in terms of the thermal conductivity and the temperature gradient. And you might have perhaps also known that how to find out this thermal conductivity.

Now, next we go to another problem of convection in which we are finding that we are heating up some something some fluid, and due to this we find that there is a convective current going on. The red one shows the hotter one, and the blue one shows the colder one. And the hotter fluid because, it gets less dense, it tends to move upward. And the heavier liquid a fluid, which is at the top due to the lower temperature will tend to come downward, and that will create some kind of current in the fluid.

And this kind of situation also you have studied and perhaps you know that Newton's law of cooling in which you put this particular equation, therefore heat flux. And this is in terms of the heat transfer coefficient, and the delta t that is a temperature difference between the source, and the kind of the material that is getting heated up. And here are the notations, which are used in these equations, q for heat flux, k for thermal conductivity, T for temperature, x for distance, and heat h for heat transfer coefficient.

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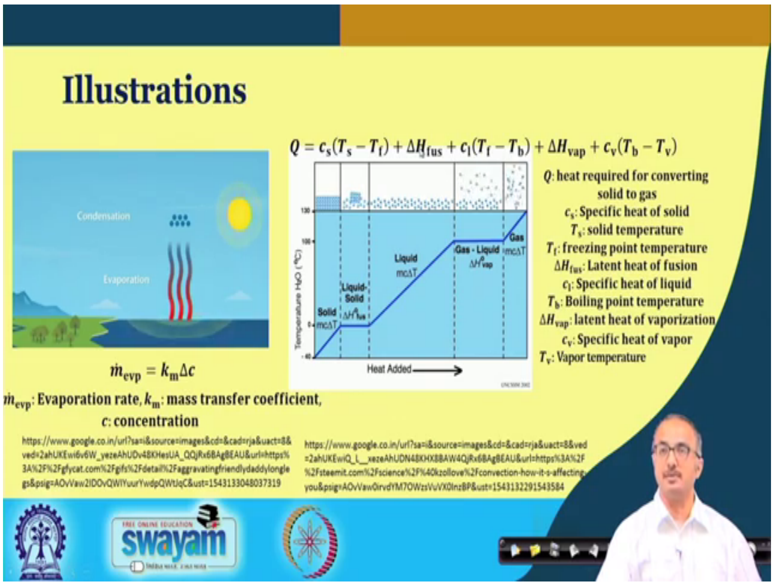

Illustrations

$$Q = c_s(T_s - T_f) + \Delta H_{fus} + c_l(T_f - T_b) + \Delta H_{vap} + c_v(T_b - T_v)$$

Q: heat required for converting solid to gas
c_s: Specific heat of solid
T_s: solid temperature
T_f: freezing point temperature
ΔH_{fus}: Latent heat of fusion
c_l: Specific heat of liquid
T_b: Boiling point temperature
ΔH_{vap}: latent heat of vaporization
c_v: Specific heat of vapor
T_v: Vapor temperature

$$\dot{m}_{evp} = k_m \Delta c$$

$\dot{m}_{evp}$: Evaporation rate, k_m: mass transfer coefficient, c: concentration

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Now, next we come to another situation, which is very common that how these clouds are formed, how we get the rain, it is basically that all the water bodies are the source of the water from which water gets evaporated due to the sun solar heat. And here at in the sky, there is a condensation of the water vapor, and then they fall back to the earth to give rain.

So, in this case you will find that we generally use this kind of an expression to find out the rate of mass evaporation. And this is given in terms of another coefficient, which we call the mass transfer coefficient. And the rate of evaporation depends on the delta C that is the difference in the concentration of water in the sea or the water body, and the atmosphere.

Next is another situation in which if sometimes we are required to find out the total amount of heat required to heat some material, which is changing its phase. So, for example, we are having some material that is initially unsolid state, and then we want to convert it into the vapor state. And as you know that as you heat up the solid, it will first go to the melting point, it will melt into the liquid. And then the liquid again will be getting heated up, and then it will try to get evaporated. And then ultimately when the whole of liquid gets evaporated, then again it will be getting heated up further in the vapor state.

Now, in this case you know that whenever there is a phase change, and here we find phase changes are occurring once from the solid to liquid, and the second from the liquid to the vapor. Now, in these two cases, we find that there will not be any change in the temperature, because it is happening due to the latent heat ok. And only during whether the non-phase change power that is here, and this and these portions, we have the sensible heat part.

And you perhaps also determined that how to find out the total amount of heat energy required to convert the state from the solid to the superheated vapor ok. So, we here you know that this is a solid sub cooled liquid, then saturated liquid superheated vapor. So, this is a mixture of gas and liquid, you will find and here we have the mixture of liquid and solid ok.

So, you are using the latent heat of fusion, and latent heat of vaporization. And you perhaps also unaware of this particular equation to find out the total amount of heat required. So, first is giving the sensible heat from this temperature to this temperature from the solid temperature to the fusion temperature, then the latent heat of fusion that is to melt the solid, then the sensible heat again to heat of the fluid from the this melting temperature to the boiling point temperature. And then again we have the heat of vaporization. And then we ultimately, we have from boiling point to the vapor state vapor temperature for superheated vapor we have the sensible heat.

And you see that here we are using different types of specific heats that is one for solid, one for liquid, and one for the vapor. So, you see that all these properties, which you need they may or may not depend on the pressure, temperature. Generally the dependent pressure temperature for simplification many a times, (Refer Time: 07:01) assume them to be constant, and go ahead with the calculations ok.

Now, all these things you see that we need some properties, and all these things where do we get them from. And all these properties are obtained generally from the experimental data and through some correlations. And sometimes for some simple systems, for some gases, theoretical expressions are also derived to determine these properties.

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Definition of constitutive relation

- A constitutive relationship correlates
  - ✓The properties of a material to the material structure, and
  - ✓The rate of a process to the operating conditions
- Examples: Ideal gas law, conductive flux through a solid, correlation to compute any material property, Hooke's law etc.

Now, all these things lead us to what we call the constitutive relation. And what are these relations? These relations are correlating the properties of a material to the material structure or they give the rate of the process under the operating conditions. For a so there are some examples, like we are you are aware of ideal gas law, then there is conductive flux through a solid as I have shown you, then correlation took compute any material property. Then Hooke's law you are also aware of that is we are using in the mechanical engineering sciences, we are using the Hooke's law to find out the stress strain relationships.

So, all these kind of relationships pertain to the constitutive relationships, which are related to the constitution of the structure of the material. And, these are important whenever you are writing any material balance or energy balance or momentum balance one or the other relationships come into the picture.

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Types of constitutive relations

- Thermodynamics relations
- Reaction rates
- Transfer rate equations
- Volume balance relations
- Equipment and control relations

So, there are various types of the constitutive relations. First is the thermodynamic relations, then reaction rates, then transfer rate equations, then volume balance equation, and equipment and control relations. And, we shall be looking into each of these one by one.

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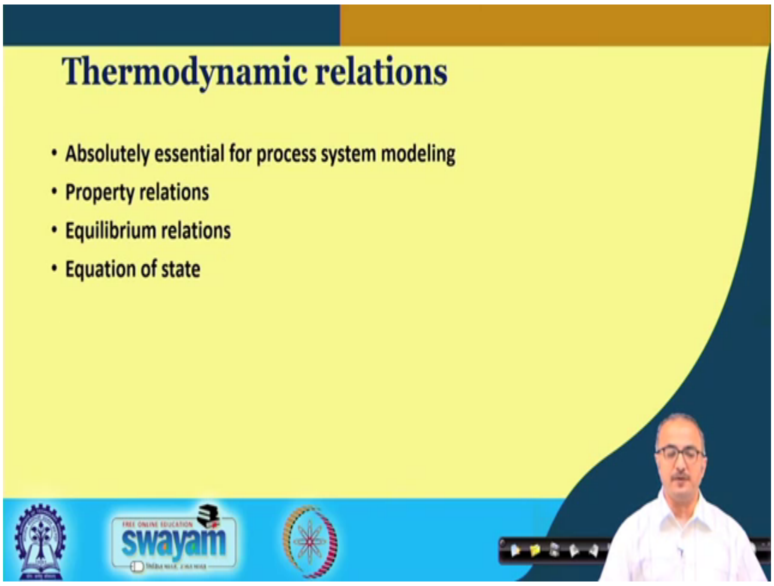

Thermodynamic relations

- Absolutely essential for process system modeling
- Property relations
- Equilibrium relations
- Equation of state

So, first let us go to the thermodynamic relations. Here we find that these are absolutely essential for any process modeling ok, because these relationships give us the many properties, and how the pressure, volume, temperature are related ok. So, these are very essential.

Then these constitutes of the property relations, equilibrium relations, and equation of state. So, whatever I shall be showing you, now these are not exhaustive. You will find there are some selective examples, I shall be showing you, so that you can understand that how to extend all these concepts for some other problems.

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Property relations

- Generally given in terms of intensive properties like temperature (T), pressure (P) and composition
- May range from simple linear expressions to complex polynomials
- Some properties:
  - ✓Density
  - ✓Viscosity
  - ✓Specific heat
  - ✓Thermal conductivity
  - ✓Latent heat of vaporization/fusion/sublimation
  - ✓Vapor pressure
  - ✓Mass diffusivity
  - ✓Enthalpy

So, let us first come to the property relations. Generally, these property relations are given in terms of the intensive properties like temperature, pressure, and composition. And, these relationships may be very simple expressions in a linear form or they may be some very complex and polynomial ok.

And these are some of the representative properties for which we need these relationships, and which we generally encounter in our mass momentum and energy balance equations. So, we are we have density, viscosity, specific heat, thermal conductivity, latent heat of vaporization or fusion or sublimation, then vapor pressure, mass diffusivity, enthalpy and so on and so forth. So, all these things are given in terms of some kind of equations.

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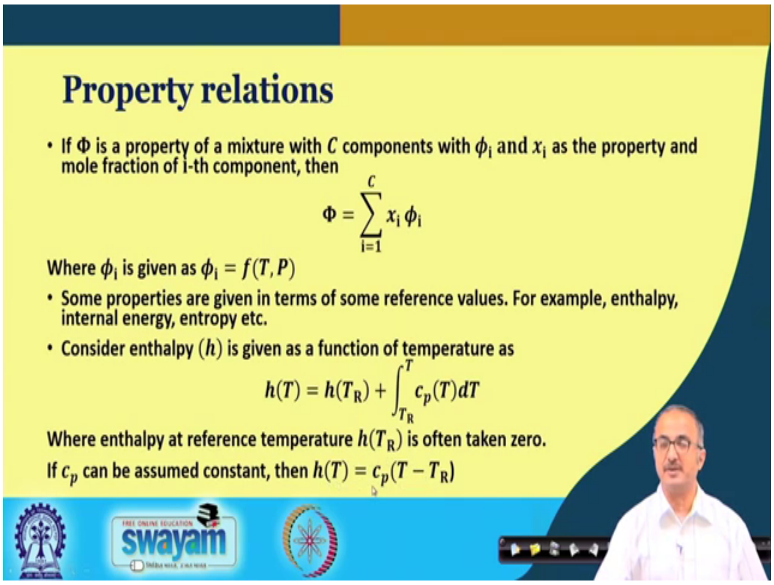

Property relations

- If Φ is a property of a mixture with C components with ϕ_i and x_i as the property and mole fraction of i-th component, then

$$\Phi = \sum_{i=1}^{C} x_i \phi_i$$

Where ϕ_i is given as $\phi_i = f(T, P)$

- Some properties are given in terms of some reference values. For example, enthalpy, internal energy, entropy etc.
- Consider enthalpy (h) is given as a function of temperature as

$$h(T) = h(T_R) + \int_{T_R}^{T} c_p(T)dT$$

Where enthalpy at reference temperature $h(T_R)$ is often taken zero.

If c_p can be assumed constant, then $h(T) = c_p(T - T_R)$

Now, suppose we have this capital phi is a property of a mixture, and that mixture contains C number of components. And each of these components has a property that is small phi i and x i is the amount that is given in terms of the say mole fraction of the i-th component.

So, with these notations, we see that the property of the mixture is given as some kind of an average. And this average is a mole fraction weighted average. So, what we are doing basically, we are summing up the product of the mole fraction, and the particular property value for each of the components that is how we are getting the overall property of the particular system. And here we find the overall property is now a function of the temperature, pressure, and also the composition.

Now, some now in one once you understand that these pure component properties are functions of only one temperature, pressure, whereas the total property is a function of the composition as well. Now, some properties are given in terms of the reference values. For example, enthalpy, internal energy, entropy. So, for these properties we need some reference, because there is no absolute value of these properties.

And one example is that if you talk about the enthalpy. So I am not going into any kind of derivation, because these are taught in full detail in thermodynamic ok. And you have separate course dedicated for this, I am just taking the important relations to demonstrate it ok.

So, here we have for enthalpy that as a function of temperature. Here is some reference enthalpy at some reference temperature T R. And rest is the integral from the reference temperature to the actual temperature at which we want to find out the enthalpy. And this is the C P dT that is the specific heat at constant pressure into dT. So, this is a generalized expression to find out the enthalpy; and enthalpy variation with temperature.

And many times this reference enthalpy is taken to be 0 ok. So, you find that whenever you are looking into literature, and you find that there are enthalpy values, you will also see that the enthalpy values will be related to some reference values. And we may be very careful in this that different literature may have different reference values at different reference temperatures. So, whenever you are taking the values from different sources, make sure that the reference temperatures are the same; if not, then you are going to commit mistake. And whatever results you obtain will also be erroneous, so that is the very important point one should note about the reference state.

Now, if we assume this specific heat to be constant, then we can take it out of this integral, and we get this particular expression. And many a times, you will find that we are using this kind of an expression to find out the enthalpy value, when this is not available from any thermodynamic chart or table.

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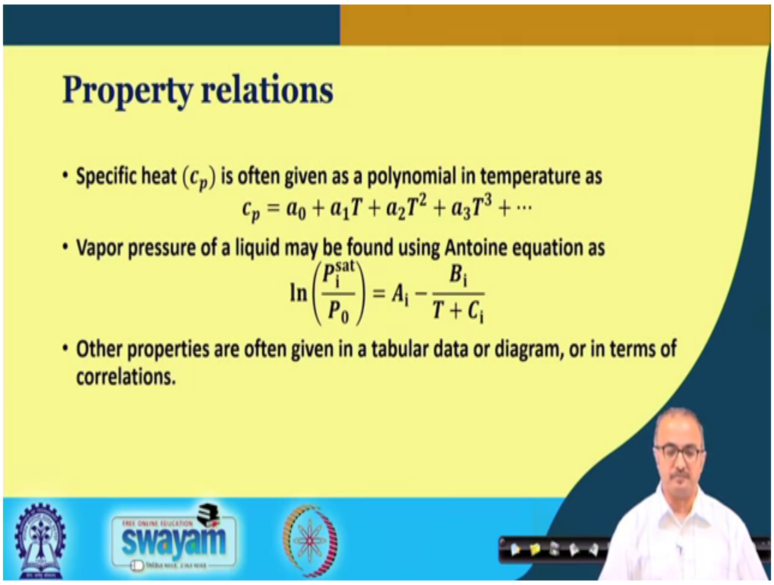

Property relations

- Specific heat (c_p) is often given as a polynomial in temperature as

$$c_p = a_0 + a_1 T + a_2 T^2 + a_3 T^3 + \cdots$$

- Vapor pressure of a liquid may be found using Antoine equation as

$$\ln\left(\frac{P_i^{sat}}{P_0}\right) = A_i - \frac{B_i}{T + C_i}$$

- Other properties are often given in a tabular data or diagram, or in terms of correlations.

Now, you see that this specific heat itself is a property. And that again is a function of temperature, and you will find in literature that such kinds of polynomial equations are given to compute this specific heat. And this you can see that it can be in a big series ok.

And similarly, we have another property that is vapor pressure. And many a times there is a common equation that is the Antoine equation is used to find out the vapor pressure of a liquid. And the expression of this Antoine equation is something like this that you find that logarithm of the saturated pressure or the vapor pressure divided by some standard pressure.

And then this A, B, C these three coefficients are there in this particular form. And again you will find in the literature in the data source, these values of the A, B, C are given ok. So, this is another example of a constitutive relationships. So, similarly you will find that many other properties will be given to us in tabular form or in some kind of diagram or in terms of some correlations.

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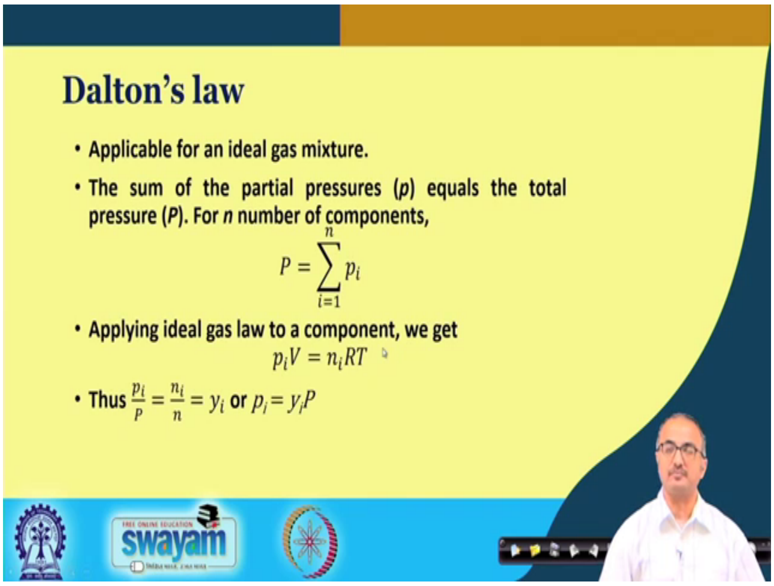

Dalton's law

- Applicable for an ideal gas mixture.
- The sum of the partial pressures (p) equals the total pressure (P). For n number of components,

$$P = \sum_{i=1}^{n} p_i$$

- Applying ideal gas law to a component, we get

$$p_iV = n_iRT$$

- Thus $\frac{p_i}{P} = \frac{n_i}{n} = y_i$ or $p_i = y_iP$

So, Dalton's law is another equation and under the constitutive relationship, which you perhaps are aware of that this using Dalton's law, you can find out the total pressure in a system for a gas. And if the gases are ideal, and perhaps that the total pressure is given as the sum of the partial pressures of each of the components.

So, here you see that the partial pressure can be obtained from the particular system at a given volume and given temperature. And you also know that if you take the ratio of the partial pressure to the total pressure, it gives you the mole fraction of the particular component in the vapor phase; so that the partial pressure can be given as the product of the mole fraction of the particular component in the gaseous phase and the pressure. So, this is the way you see that the partial pressure and the total pressure are related through the composition.

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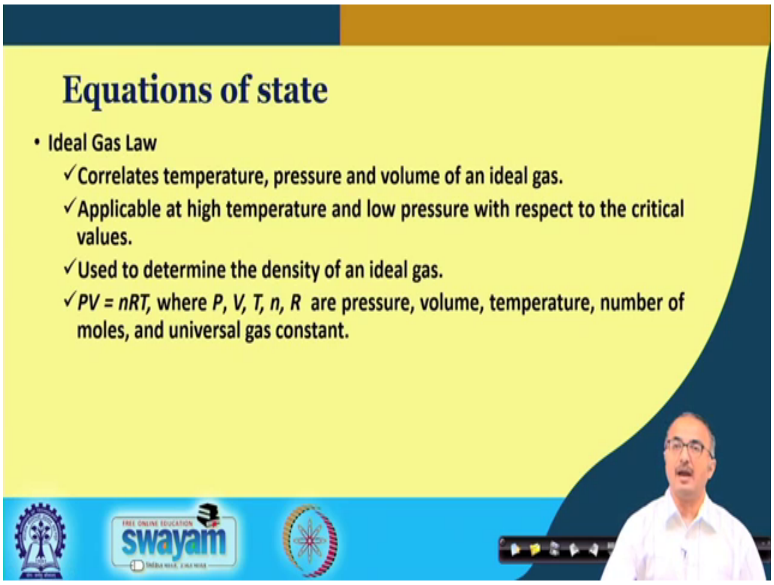

Equations of state

- Ideal Gas Law
  - ✓Correlates temperature, pressure and volume of an ideal gas.
  - ✓Applicable at high temperature and low pressure with respect to the critical values.
  - ✓Used to determine the density of an ideal gas.
  - ✓$PV = nRT$, where P, V, T, n, R are pressure, volume, temperature, number of moles, and universal gas constant.

Another type of thermodynamic relation is the equation of state. This also you are quite aware of, and you know that this correlates the temperature, pressure, volume of an ideal gas for ideal gas law. And it can be for real gas also, we shall see that later.

And you see that ideal gas may be assumed, when the gas is at high temperature and low pressure. And perhaps also you are aware of the various assumptions made for the ideal gases, which are there in our school day textbooks. Now, this kind of equation of state is often used to find out the density of the ideal gas. And here we have the particular equation PV equal to nRT, and with the various notations here ok.

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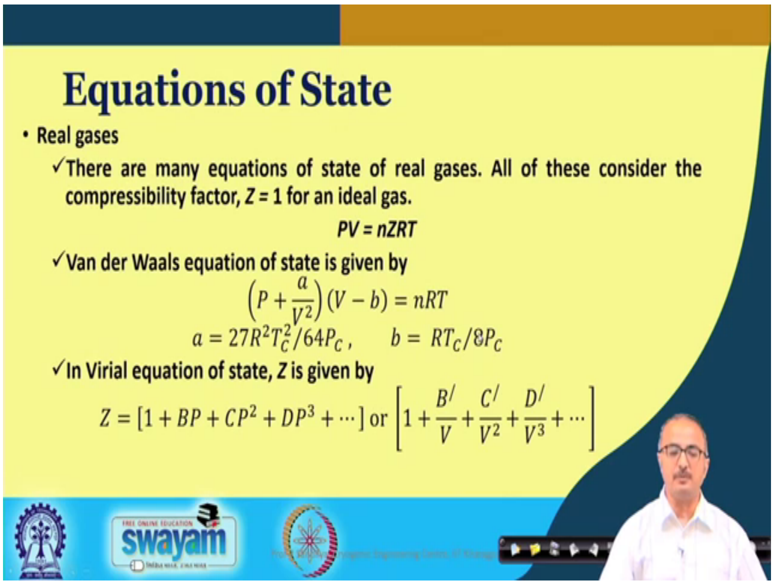

Equations of State

- Real gases
  - ✓There are many equations of state of real gases. All of these consider the compressibility factor, $Z = 1$ for an ideal gas.

$$PV = nZRT$$

  - ✓Van der Waals equation of state is given by

$$\left(P + \frac{a}{V^2}\right)(V - b) = nRT$$

$$a = 27R^2T_C^2/64P_C\,, \qquad b = RT_C/8P_C$$

  - ✓In Virial equation of state, Z is given by

$$Z = [1 + BP + CP^2 + DP^3 + \cdots] \text{ or } \left[1 + \frac{B'}{V} + \frac{C'}{V^2} + \frac{D'}{V^3} + \cdots\right]$$

And next we come to for the real gases. For real gases again there are various approaches. One of the simplest approaches is to use the PV equal to nZRT. Here, we have put another factor to account for the non-ideality that is the compressibility factor or super compressibility factor this Z value ok.

And another thing, you also have encountered perhaps the Van der Waals equation of state worth of that is the first equation of state for the real gases or non-ideal gases. And this takes this particular form, and you see that there are some pressure correction, and some volume correction for the interactions between the molecules. And you see that these are these a and b are given in terms of the critical temperature and pressure.

This z may also be obtained from some Virial equation of state is which is given in this particular form in terms of pressure, on this form in terms of the volume. So, I am not going into any of the detail now. Whenever they are they come to us, we shall look into the details there.

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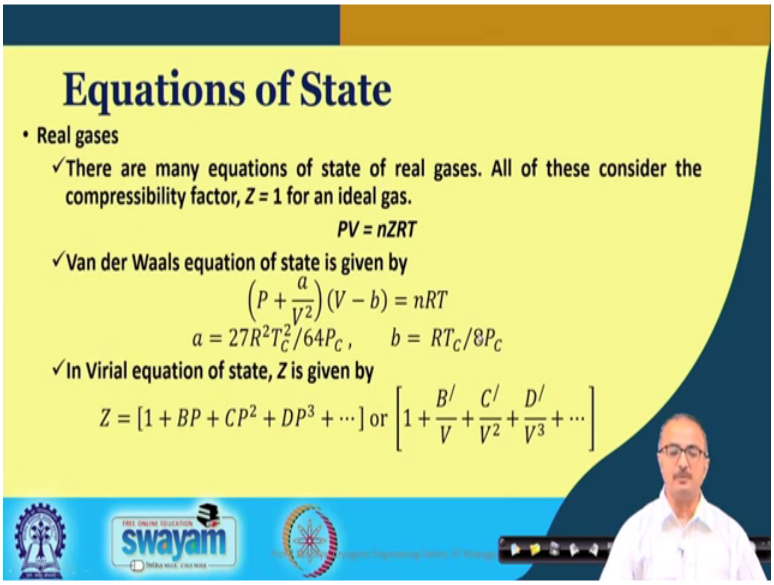

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Now, here you see that how this compressibility factor depends for with pressure. And as you can see for different gases, I have plotted it here that for ideal gas, it should be one ok. And whenever it deviates from 1, either on is more than or less than 1, it means we are going towards non-ideality.

And here you can see for different types of gases. The variations of the Z with pressure is different. And you can see that as we increase the pressure, the gas becomes more and more non-ideal. And in this particular figure, you again you see that here we have plotted the Z versus the pressure, but at different temperatures. And you can here see again that as the temperature rises, the gas moves more and more towards the ideality; the less the temperature the more the deviation from the ideality.

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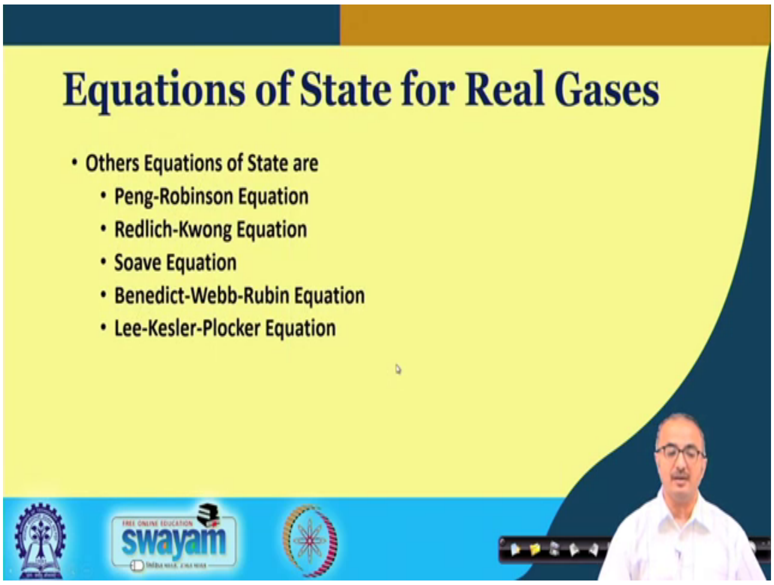

Equations of State for Real Gases

- Others Equations of State are
  - Peng-Robinson Equation
  - Redlich-Kwong Equation
  - Soave Equation
  - Benedict-Webb-Rubin Equation
  - Lee-Kesler-Plocker Equation

Then we have many equations of state, I have just named a few here some of them are empirical, some of them are semi-empirical. And all these equations of state their expressions are can be found out in if some standard thermodynamics book or in some other data sources, I shall not be going into the detail of these equations here.

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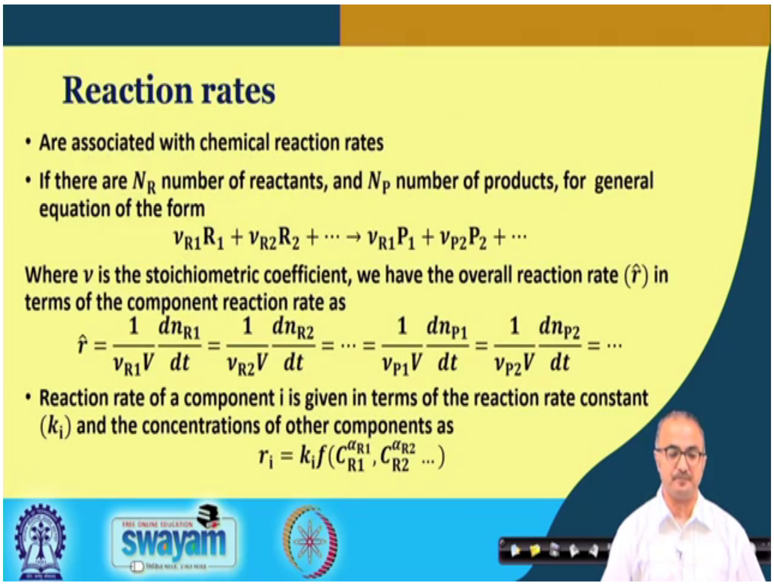

Reaction rates

- Are associated with chemical reaction rates
- If there are N_R number of reactants, and N_P number of products, for general equation of the form

$$\nu_{\mathrm{R1}}\mathrm{R}_1 + \nu_{\mathrm{R2}}\mathrm{R}_2 + \cdots \rightarrow \nu_{\mathrm{R1}}\mathrm{P}_1 + \nu_{\mathrm{P2}}\mathrm{P}_2 + \cdots$$

Where ν is the stoichiometric coefficient, we have the overall reaction rate $(\hat{r})$ in terms of the component reaction rate as

$$\hat{r} = \frac{1}{\nu_{\mathrm{R1}}V}\frac{dn_{\mathrm{R1}}}{dt} = \frac{1}{\nu_{\mathrm{R2}}V}\frac{dn_{\mathrm{R2}}}{dt} = \cdots = \frac{1}{\nu_{\mathrm{P1}}V}\frac{dn_{\mathrm{P1}}}{dt} = \frac{1}{\nu_{\mathrm{P2}}V}\frac{dn_{\mathrm{P2}}}{dt} = \cdots$$

- Reaction rate of a component i is given in terms of the reaction rate constant (k_i) and the concentrations of other components as

$$r_\mathrm{i} = k_\mathrm{i} f(C_{\mathrm{R1}}^{\alpha_{\mathrm{R1}}}, C_{\mathrm{R2}}^{\alpha_{\mathrm{R2}}} \ldots)$$

Next is the reaction rates: and these reaction rates come whenever we are having some kind of reaction happening in the system. And here, we is find a generalized expression of any reaction given in terms of the reactants. And the products are represents the reactants, and the P represents the products. And this a nu R 1 nu, R 2 all these nu values are the stoichiometric coefficient.

So, this kind of equations, you have been taught in your school, in the chemistry, and we also have been taught how to find out the overall rate of reaction, in terms of the individual rates of reaction. So, all these d n R 1 by d t shows the how the amount this these things are changing this these reactants are changing with time for with respect to each of these individual components you find, these are the various rate expressions ok.

And for the individual component, you see this is the kind of rate expression given that it is given in terms of some coefficient, this we call the reaction rate constant ok. This reaction rate constant into some function of the various reactants, and here you can see that these are raised to some kind of power that is alpha. And this alpha is determined from the experiments. And whenever we are summing up these alpha values, these give the order of the reactions ok. So, these are the some very basic things, we have done. But, these are shown, because these are also forming the constitutive relationships like this is the constitutive relationship ok.

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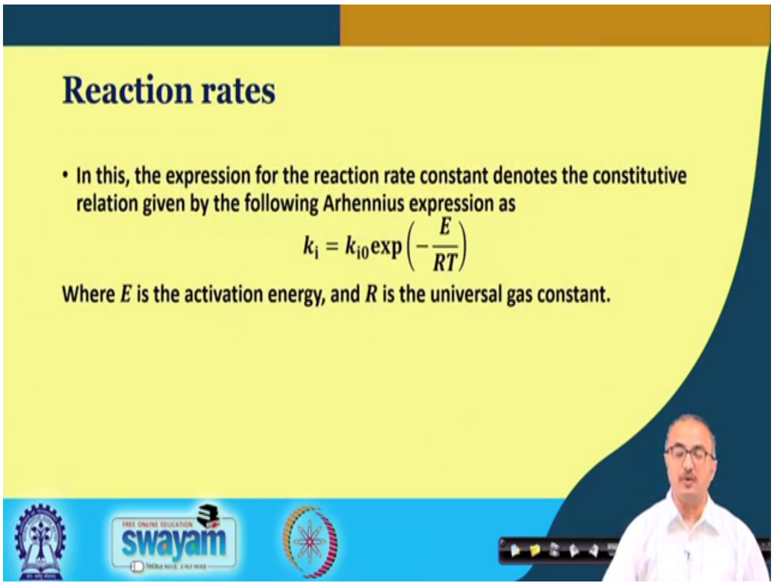

Reaction rates

- In this, the expression for the reaction rate constant denotes the constitutive relation given by the following Arhennius expression as

$$k_i = k_{i0}\exp\left(-\frac{E}{RT}\right)$$

Where E is the activation energy, and R is the universal gas constant.

And the other than this, we also find that we are required to find out this reaction rate constant. And perhaps you know that this is given in terms of the Arhennius reaction. And there he has some kind of standard reaction rate, and this is given in terms of the temperature, and this E is the activation energy. So, by using this expression, we find out that how this reaction rate constant varies with the temperature, and this also pertains to the constitutive relation.

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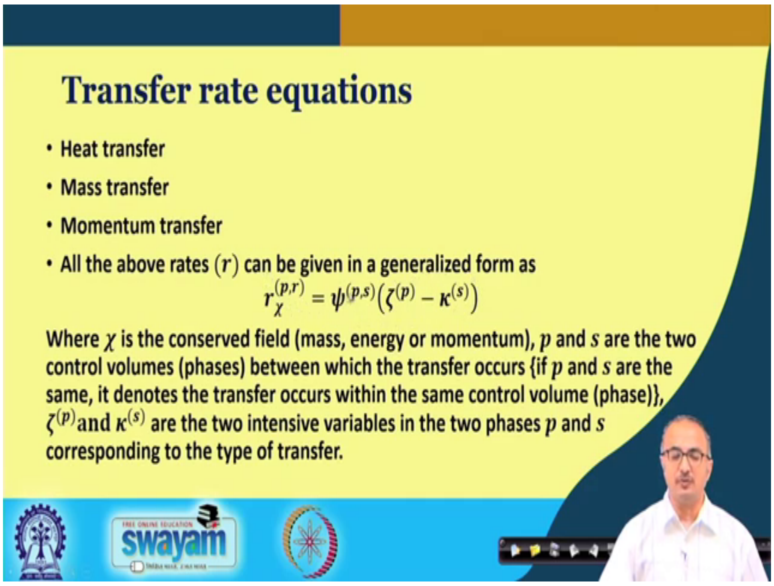

Transfer rate equations

- Heat transfer
- Mass transfer
- Momentum transfer
- All the above rates (r) can be given in a generalized form as

$$r_{\chi}^{(p,r)} = \psi^{(p,s)}\left(\zeta^{(p)} - \kappa^{(s)}\right)$$

Where χ is the conserved field (mass, energy or momentum), p and s are the two control volumes (phases) between which the transfer occurs {if p and s are the same, it denotes the transfer occurs within the same control volume (phase)}, $\zeta^{(p)}$and $\kappa^{(s)}$ are the two intensive variables in the two phases p and s corresponding to the type of transfer.

Next, we come to the transfer rate equations ok. Now, in this transfer rate, we have the heat transfer, we have mass transfer, and we have momentum transfer. And all of these are taught separately in fluid mechanics, mass transfer, heat transfer, courses. So, I shall not be going into in a detail of it, I shall be just looking into the some fundamental concepts.

Now, here you find that all these transfer rates may be given in a generalized form like this that you will find that this is the rate, and this p, and this will be s so this is the s here. So, this is s ok. Now, this p and s represent the two control volumes. And these two control volumes may represent two different phases. So, the transfer may happen within one phase or it can be from between two phases ok.

So, this is a generalized form of the rate of these three things. And here we find again, we have some kind of driving force. And these driving force denote some kind of intensive variables in the two phases p and s. And this particular zeta is the some conserved field that can be mass energy or momentum.

And this psi is some kind of coefficient, coefficient which is we called a proportionality constant. So, this may be taking different forms as we shall see after some time that but this is a general form that we have some kind of driving force of the intensive variables, corresponding to this particular conserved field ok.

So, we find that these kinds of expressions for this mass balance, this will represent the concentration. For energy balance, it will represent the temperature. And for the momentum balance, it may represent the pressure or the velocity ok. So, we have different kinds of intensive variables associated with each of these transfers.

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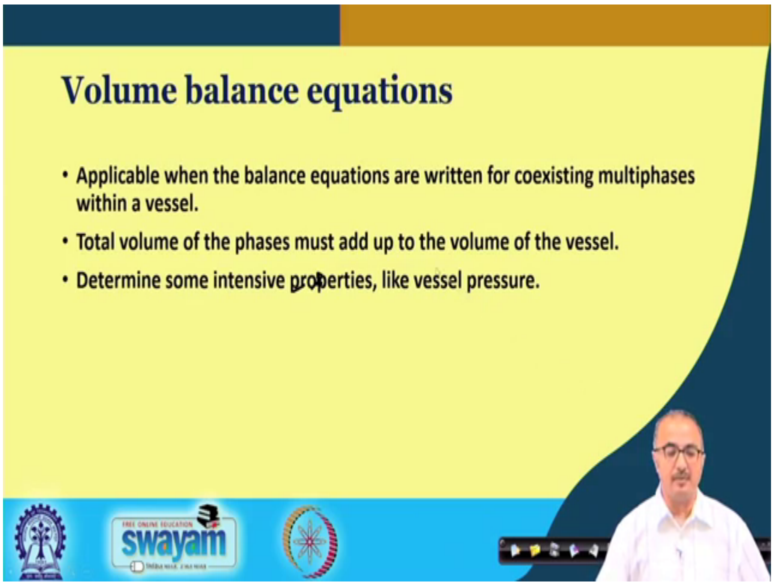

Volume balance equations

- Applicable when the balance equations are written for coexisting multiphases within a vessel.
- Total volume of the phases must add up to the volume of the vessel.
- Determine some intensive properties, like vessel pressure.

Now, we come to volume balance equation. And this is what we find basically that it is basically, we are writing these balance equations, whenever there are coexisting, multiphase within a vessel. And we know that whenever different vessels are occurring together in a particular vessel, each of the phases will occupy its own volume. But, the total volume cannot be greater than the volume of the vessel.

So, whenever we are totaling the volume of each of the phases, it must give the volume of the vessel. So, this is what we call the volume balance equation. We are making the summing up the volumes of individual phases ok. And then why do we need it that this determines some intensive properties like vessel pressure. For example, if we are having some liquid to start with in an enclosed vessel, what will happens? And you are start heating up the vessel.

Now, initially we find that there is a certain pressure in the vessel. And as you start heating up the vessel, you will find the liquid is going in the vapor phase. Now, what you find the amount the volume of the liquid is coming down, whereas the volume of the vapor is going up. Now, the total volume remain the vessel volume.

Another changes happening is this, because the liquid is evaporating. We find that the pressure is also now rising in the vessel ok. So, how this pressure will rise that will also depend on the rate at which the liquid is getting evaporated, and the volume is changing, but there will be change in the pressure. So that is why, we need this kind of volume balance equations to find out the pressure exerted within the vessel.

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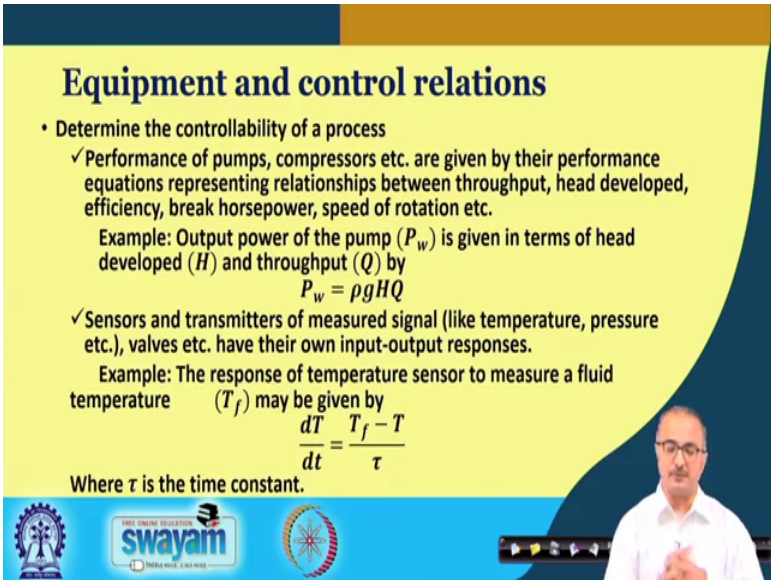

Equipment and control relations

- Determine the controllability of a process
  - ✓Performance of pumps, compressors etc. are given by their performance equations representing relationships between throughput, head developed, efficiency, break horsepower, speed of rotation etc.

    Example: Output power of the pump (P_w) is given in terms of head developed (H) and throughput (Q) by

    $$P_w = \rho g H Q$$
  - ✓Sensors and transmitters of measured signal (like temperature, pressure etc.), valves etc. have their own input-output responses.

    Example: The response of temperature sensor to measure a fluid temperature (T_f) may be given by

    $$\frac{dT}{dt} = \frac{T_f - T}{\tau}$$

    Where τ is the time constant.

Next we come to the equipment and control relations ok. Here we have to determine the controllability of a process. Now, you see that whenever we are having any kind of industrial process, we need to control; that means we have to keep many operating parameters within some specific range to get some particular performance. And, we have to maintain these particular variables at those values, because otherwise the system may either not give us the desired performance or it can also result in some hazard ok. And so this is very important for us.

So, there are many types of relationships, which we use one thing is for the equipment. So, like for example performance of pumps, compressors are given by their performance equation representing the relationships between their throughput, the head developed, the efficiency, brake horsepower, speed of rotation etcetera.

So, these are the relationships for a given pump or compressor. And this can be found in any standard fluid mechanics book in the fluid, moving machinery the chapters. So, you will see that you will get such kind of an expression ok. To find out the output power of the pump, and this is given in terms of the throughput and the head developed, g is the gravitational acceleration, and rho is the density. So, you find that these kinds of equations, which are correlating the pumping power, and the head, and the throughput also come under the constitutive relationship. This is about some equipment. And this kind of expressions will be there for other equipment also.

And 2nd is the various types of sensor, transmitters, we are using. Sensors like temperature, pressure etcetera, then valves we are using. So, we will find that for each of these components, there will be some kind of mathematical relationship, which will be correlating the input, output.

For example, suppose for a sensor temperature sensor, you see that suppose we are exposing a thermometer or any other sensor to some particular fluid ok. You find that it will take some time for the sensor to respond, and come to a steady value of the temperature. And these dynamics that how the temperature is changing, and how a particular sensor would respond to any kind of new ambience around it that will be given by some kind of relationship, which one of them I have shown here.

And here you can see this is a parameter tau that is a time constant. And this time constant tells us that how fast a particular sensor would react to any kind of changes ok. So, this kind of equations, which correlate the input, output. Another example is valve. Now, the flow rate they have suppose you are have a some having some flow rate through a valve. Now, how much valve is being opened or closed depending on that the fluid flow rate will change.

Now, you see that whenever you are changing the valve position, for example you are changing it in the tap in your day-to-day life. So, you will find that the responses are not instantaneous, there will be it is time lag. However, small it may be, there will be its time lag. So, this particular time lag how fast the change in the valve position will change the flow rate.

And especially, this becomes more pronounced in the bigger scale ok. You may not feel it in smaller scale, but your bigger scale will find it this kind of time lag between the any change being imposed on the valves, and it is effect on the output side will be felt a bit later. So, all these things can be mathematically written ok. I am not going into those things. These are again can be found in any book on the process dynamics and on control ok. So, all those relationships come under the constitutive relation.

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References

References:

- ❑ Bird, R.B., Lightfoot, E.N. and Stewart, E.W., 2007. Transport phenomenon. John Wiley and Sons.
- ❑ Brodkey, R.S. and Hershey, H.C., 2003. Transport phenomena: a unified approach. Brodkey publishing.
- ❑ Reid, R.C., Prausnitz, J.M. and Poling, B.E., 1988. The properties of gases and liquids. McGraw-Hill. International Edition.

With this, we come to the end of these constitutive relations. And we see that what all kinds of equations are used. And here are some of the references you may refer to know the detail about these particular topics.

Thank you.