Mass, Momentum and Energy Balances in Engineering Analysis
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Lecture - 42
Tutorial on Macroscopic Energy Balance

Welcome. After learning the macroscopic and microscopic balances for the energy exchange, mass exchange and momentum exchange, in this lecture, we shall be looking into a few problems on the applications of the Macroscopic Energy Balances.

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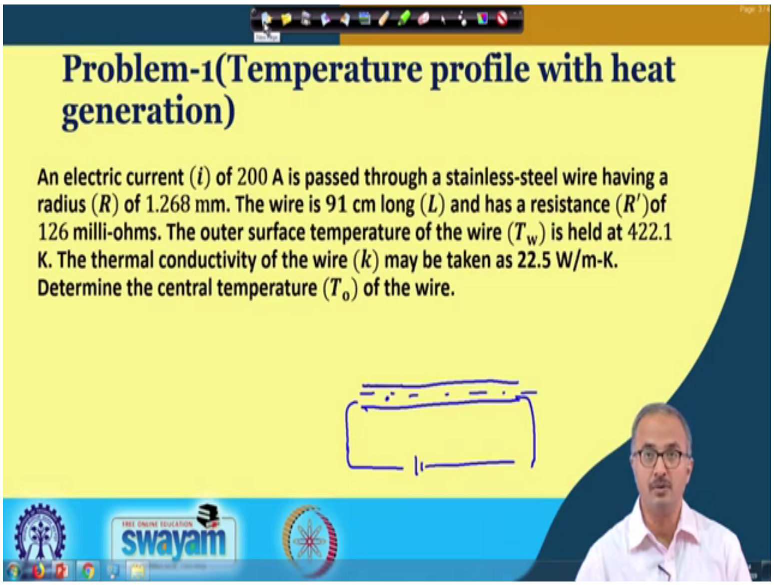

So, first problem, let us go to it concerns the heating of a wire. So, it reads like this that an electric current of about 200 amperes is passed through a stainless-steel wire having a radius of 1.268 meter, the wire is 91 centimetre long and it has a resistance of 126 milli ohms. The outer surface temperature of the wire this is designated by T w is held at 422.1 Kelvin. The thermal conductivity of the wire denoted by k is taken to be 22.5 watt per meter per Kelvin. We have to determine the central temperature of the wire ok.

Now, first you have to understand this particular process, we are heating the process and we are holding the outside temperature of the wire constant. So, first what you can do for your sake of your understanding, you can draw the process like this suppose you make this wire ok. And you can say that this is suppose your central line ok. Now, on this we are heating this particular wire, may be the heating is done through some kind of battery or cell, if may be there is some battery or cell is there by which of passing the current. And due to this there is a heating and that is the joule heating we call it ok. And the outside temperature is given. So, this temperature we have asked that due to this heating of the wire what is the temperature at the core of the particular wire ok.

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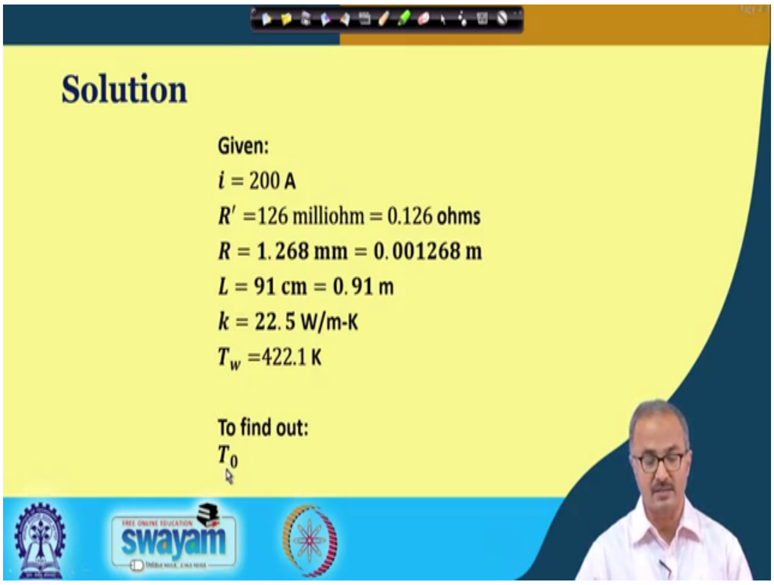

Now, let us see how we are going to solve this particular problem. Now, first let us what all is given to us. So, here we have written the amount of current we are passing, resistance of the wire. Please understand the generally the resistance is not a property of any substance, but the resistivity is the property, and resistance is calculated you might be knowing from the length and area of cross section of the particular wire ok. So, here it is given in terms of the resistance.

So, sometimes you may find that you have given the resistivity instead of the resistance ok. So, in that case you have to convert resistivity to resistance. Here this is the radius of the particular wire. You have see that the resistance we are denoting by r prime, whereas radius of the wire is given by only R. This L denotes the length of the wire, and then this is the thermal conductivity and this is the wall temperature outer wall temperature.

Again mind it that before you would start solving the problem please have the consistent unit. So, for this what we are doing, we are also converting the unit into the standard SI system, after that only you should go on solving. So, what we need to find out we need to find out the inner that means at the centre code temperature of the wire then that we are denoting by T naught.

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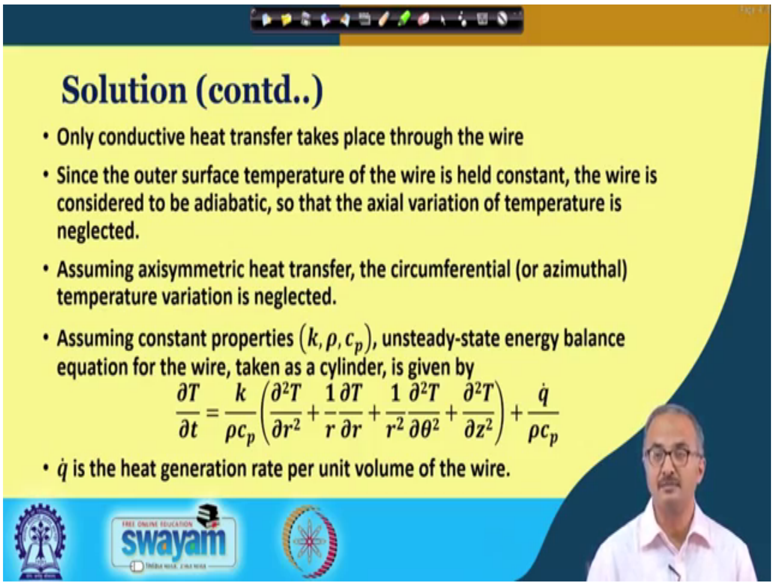

So, it is as per this particular problem only conductive heat transfer takes place through the wire, because as we know for solid systems there will be only conduction, and there will not be any kind of convection, and radiative heat transfer we are neglecting. Since the outer surface temperature of the wire is held constant, the wire is considered to be adiabatic, in the sense that we are assuming that there is no heat dissipation or heat accommodation from the our ambient; only whatever temperature is held only that is the temperature through which that wire is passing, there is no other external temperature effect from the ambient.

And we are assuming that that there is only axial variation of the temperature ok. I am sorry acceleration temperature is neglected we are assuming that because it is held constant, so actually there is no temperature variation only we are having variation the radial direction and due to this difference in temperature at the surface and at the core of the wire.

And we are neglecting any kind of end effect; end effect means that if there is any kind of sources at the two ends of the wire, source of heat or some kind of sink at the two ends of the wire they are going to cause some kind of axial heat transferred. So, we are considering that that only a section of the wire we are considering and we are considering that it is long enough, so that the end effects at the two ends we are neglecting the end effects. So, there is no heat transfer from any of the given section of the wire under consideration. So, with these considerations we are assuming that there is only a radial heat transfer.

Now, next assumption is that we are assuming it is to be axisymmetric, that means there is no variation in the temperature along the circumference of the wire at any given radial axial position ok, so that means circumferentially there is no temperature gradient or temperature difference. So, this is the axisymmetric assumption. And then assuming the constant properties like k, rho and c p, and then we are talking about the unsteady-state heat energy balance first we are considering and taking the wired to be the cylinder ok. So, is the wire is taken to be cylinder, then we can take the cylindrical coordinate system.

So, here is the equation which I showed you in my earlier lecture. So, you can see that in this particular equation, we have dropped all the convective terms, because there is no flow. And we have taken this is the giving the temperature change with time, and this is the k by rho c p is the thermal diffusivity, and this is all these terms are denoting the diffusional effect of the temperature due to heat transfer.

And this is given in terms in the three directions we are seeing that this is for the radial direction this, and this is for the theta direction, and this is for the z direction and then we have a source term. And in this case the source term is coming due to the heating of the wire by some external current flow.

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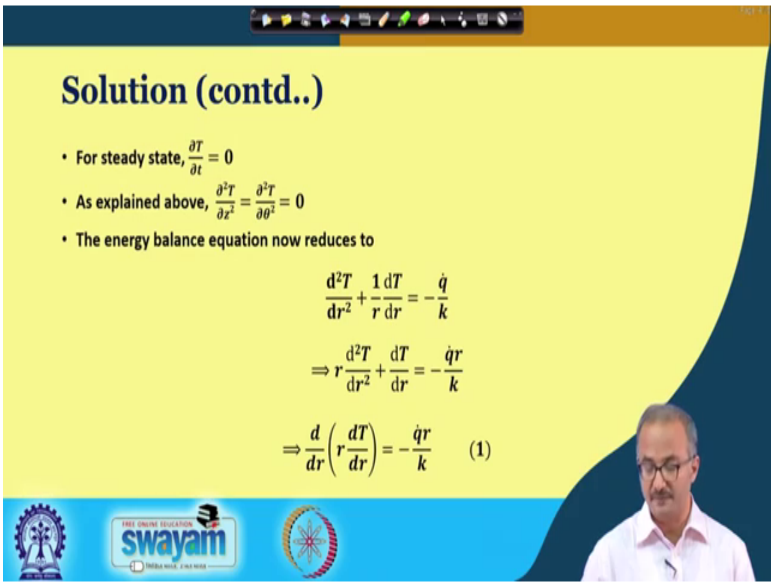

So, for steady state because it has been mentioned that it is steady state problem, so we are neglecting the time derivative of the temperature, so we are taking to be 0. And then as we have explained that because of this axisymmetry, we are neglecting the theta partial difference with respect to theta, and also we are neglecting the axial variation of the temperature.

So, with these considerations, now we are reducing the given energy balance equation in a steady state energy balance equation, and you can note here that earlier we were using the dou now we are just using d. So, this kind of transformation you must do whenever you are solving these problems. And now we have this particular equation and this one it is simple mathematics we are doing that this whole term can be reduced to this particular term ok.

Now, this is the equation finally, we are getting after making all kinds of assumptions which is going to represent our system. Now, once we have obtained this model equation, we this still not completes if we do not specify the boundary conditions.

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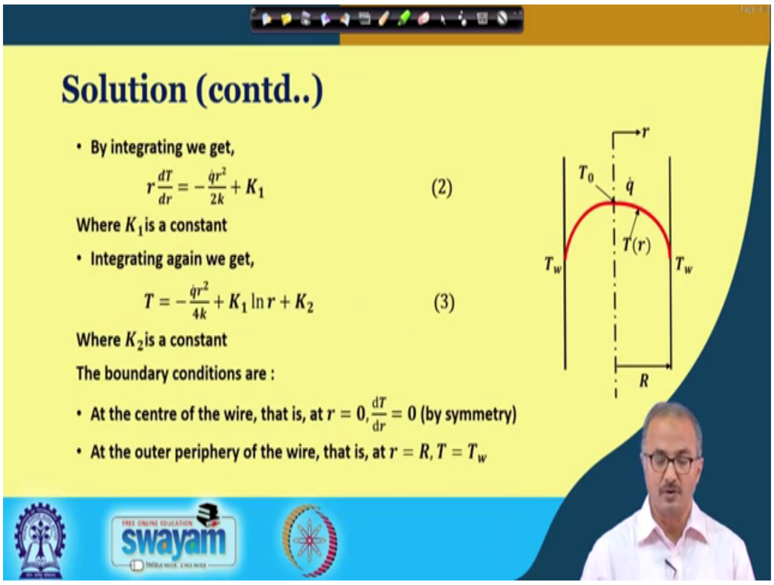

So, what we will do that we shall see that how we are going to specify the boundary conditions. First let us solve it by integration and little integration we can do. So, here first we have integrate we are getting this particular equation, and here we have some constant of integration. And again we are integrating, so we are getting this final equation this giving the temperature variation in the radial direction at any axial position ok. And here we have to get that initial boundary conditions that there is no initial conditions, there is only boundary condition. So, we see that it is a second order equation.

So, we need two, we have two constants of integration. So, we need two boundary conditions. And here we are exploiting something what we call the symmetry of the heat transfer, that means because this wire is cylindrical and there is some axis. And on the two sides that is along the circumference of the wire, there is only one temperature that is T w, so it is expected that from the centre to the particular outer wall whatever temperature profile we obtain, the same temperature profile will be also obtained on the other side of the axis, that means, it is kind of a reflection of this particular temperature distribution. So, this is called symmetry, symmetry along the central axis ok.

So, with this symmetry assumption, what we are putting that if we take now this particular axis to be the origin ok, from here we are taking the radial distance radial distance. So, this is r equal to 0 ok. So, at this r equal to 0 by symmetry we are saying this dT by dr is equal to 0 that is the first boundary condition. And second boundary condition is that at the outer wall, it has been specified in the problem that this temperature is constant held constant and that we are designating by T w. Now, you can see that this is particular thing is the normal boundary condition, and this condition is the Dirichlet boundary condition which we learned in our earlier lecture.

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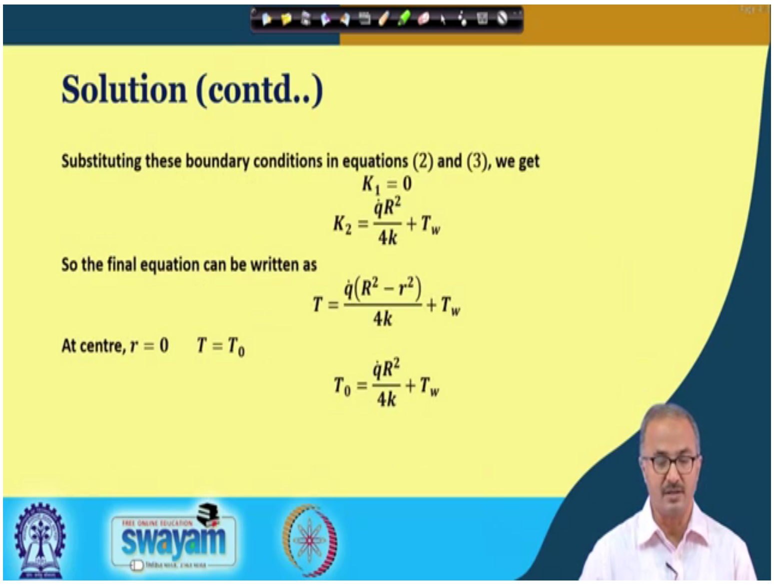

Now, once we apply these two boundary conditions on the model equation, we get the values of the two constants of integration. So, we here find that K 1 is equal to 0, and K 2 is like this. Now, we put these values of this K 1 and K 2 in the given equation. And we find this is the equation, we are ultimately obtaining to get the temperature variation along the radial direction ok.

And if we want to know as we have asked in the question what is the temperature at the core of the wire, we can simply put r equal to 0, and we find that this is the temperature at the wall. And from this particular equation, we find as we move away from the wall the temperature is going to reduce if we are going to add this particular heat and depending all also the value of the T w ok.

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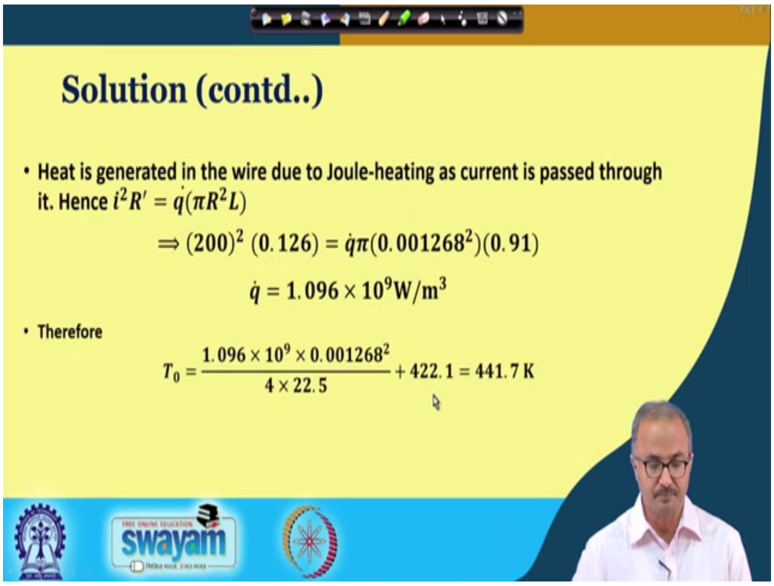

Next we come to the thing that we have to ask that how what is the heat generated, let us see that we have to find the exact value with the given information in our problem. So, let us find that whether we can find the heat generated here what we let us see that this radial distance is nothing but from the joule heating that is i square R. So, i square R is the joule heating. So, this is equal to the q dot, q dot is the heat generated per unit volume, so we are multiplying with the volume of the wire that is pi r square into the length ok. So, from this we can find out the value of the q dot, and this is coming to this much watt per cubic meter ok.

And now we put this value in this equation, we also put the value of the capital R, we putting the value of the thermal conductivity, and we also plug in the value of the outer wall temperature. And from this we find this is the temperature of the centre, at the centre this temperature ok. So, there is about 19 degree difference or (Refer Time: 13:12) degree difference between the inner and the outer wall.

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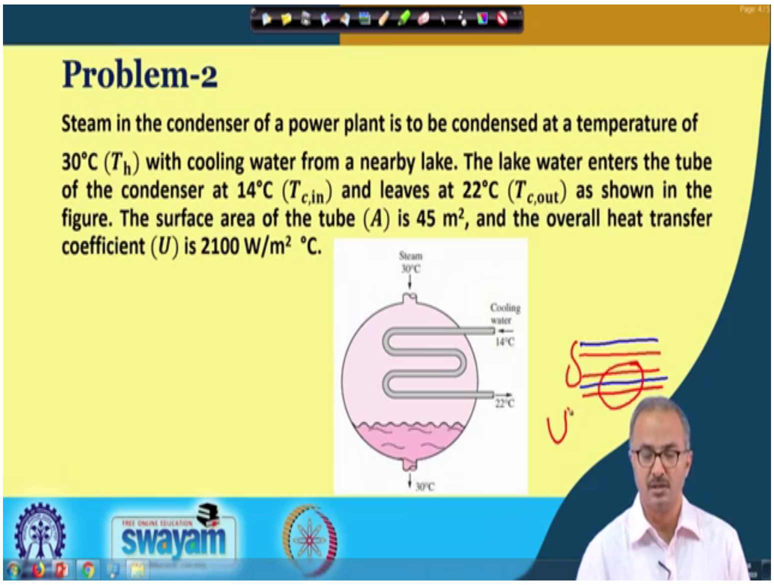

Next problem is concerns with the cooling of a steam. So, here you can see this particular system. So, steam in the condenser of a power plant, condenser is a devise it is basically the heat exchanger in which we are condensing some vapour, and generally this vapor is water vapor and in we many a times we are using such condensers in the power plants ok. So, this condensation for this condensation what we are doing we are using some cooling water, and this cooling cooled water is used from some natural sources, it could be from river, it could be from some pond, it could be from some lake.

So, in this particular problem, it is said that we are fetching the cold water from some lake, and it is available at 30 degree centigrade. And you know that the steam will be at a quite higher temperature; if the steam at 1 atmosphere you know that the superheated steam will be at a temperature more than 100 degree centigrade, because hundred degree centigrade is the boiling point of water which will give us the saturated vapour, above that it will be superheated vapor.

Anyhow we find that there is a big difference between the steam temperature and the temperature of the from the lake what we are obtaining, and this is how we are going to condense the particular steam. So, lake water enters the tube as you can see here, it is entering the tube, and then it is going out. So, we want the steam to be condensed at 30 degree centigrade. And 14 at 14 degree centigrade, we are getting the lake water, and it is getting heated up to 22 degree centigrade. And we as we see here that the steam in the temperature is 30 degree centigrade, and it is going outside at 30 degree centigrade. It means what, it means that the steam is at its saturated condition ok.

And the surface area of the tube through which the heat transfer is going on is given as 45 square meter, and the overall heat transfer coefficient is given as 2100 watt per meter square per degree centigrade. And please understand this overall heat transfer coefficients coming because it has to take care of the heat transfer instances between the wall means within the tube that liquid and the outside steam.

So, in this particular heat transfer coefficient as we have seen in our lectures that we say that if this is the tube, if this is the tube ok, then we say that there are various resistances possible. One resistance is like this on the tube side, one resistance within tube ok, and the outer side there will be other resistance and due to some convection. So, all these resistances are taken care of by the overall heat transfer coefficient that is U ok.

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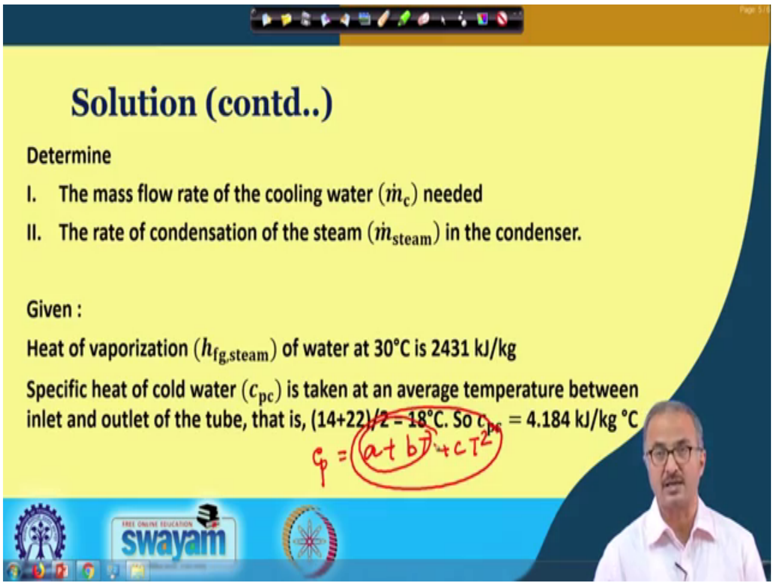

Now, we have to determine the mass flow rate of the cooling water needed, and the rate of condensation of the steam in the condenser ok. Now, given is the heat of vaporisation as 2431 kilojoule per kg at 30 degree centigrade, and specific heat of cold water is given at an average temperature, and that is 14 plus 22 divided by 2 that is 18 degree centigrade, because this is kind of an assumption. We are assuming that the specific heat is given at a average temperature. If not for more rigger what you should do, you should find the specific heat from some correlation.

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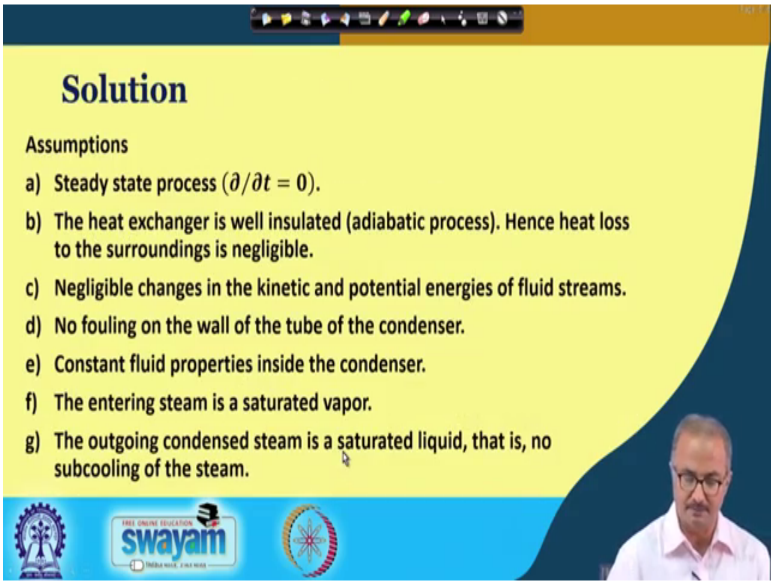

Suppose, the specific heat given like this, it is given like this that c p is equal to some function of say temperature ok, some function. You can have various function; you can have linear function, you can have non-linear function like this. So, depending on the type of the function, you may also consider the variation of the specific heat with temperature, so that is quite easy to take care of. In this particular problem, for sake of simplicity, what we are doing we are assuming the specific heat to be constant and the value is taken at the average temperature between the inlet and outlet of the tube side water, and average temperature is simply taken to be the arithmetic average.

Next we make all these assumptions. First we assume that this is steady state, so that we can make all the time derivative to be 0. The heat exchanger is taken to be insulated, that means, it is adiabatic process. So, we are neglecting all the heat exchangers between the systems and the surroundings. Then we are also neglecting any changes in the potential and kinetic energy of the fluid streams, and we are assuming that there is no fouling that is any deposition of any material on the wall that is fouling, so we are neglecting any fouling on the wall of the container or the tube and the condenser.

And then we are assuming that all the fluid properties are remaining the constant this particular assumption is not straight if there is any kind of change in the particular property of the fluid it also does not matter, you can also take care of this. It is always change that you may have to integrate over the particular temperature range.

Then we have the entering steam to be at saturated vapor; and the outgoing condensed steam is also saturated liquid, that is, there is no subcooling, that means, we are saying that we are getting the steam at saturated vapor condition, and it is just getting converted into the saturated liquid. So, that only heat of vaporisation is involved and getting evolved during this condensation. And there is no sensible heat transfer if we are having superheated steam that superheated steam first we should be first converted to saturated steam, and that will involve some loss of sensible heat. And again if the saturated liquid is getting converted to subcooled liquid, again we have to release some amount of sensible heat. So, all these two sensible heat effects are neglected.

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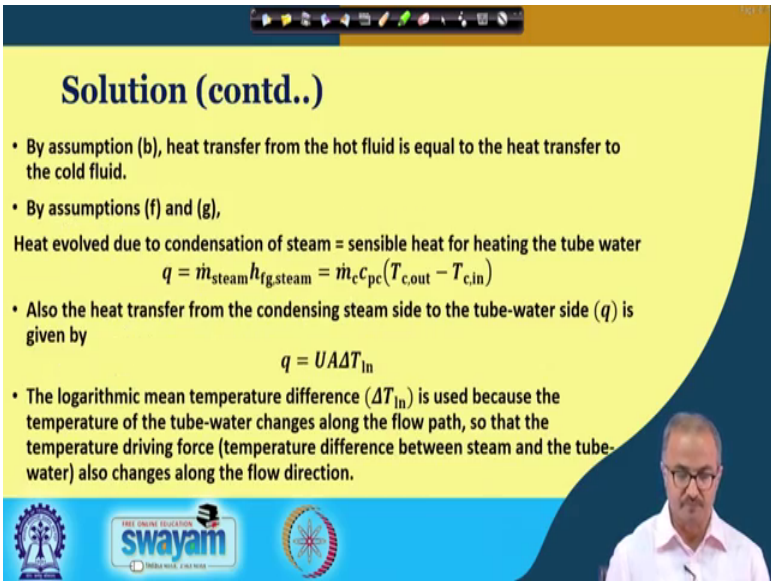

Now, by the assumption b, we have just mentioned heat transfer from hot fluid is equal to the heat transfer to the cold fluid, that means, whatever heat is being lost energy is being lost by the hot fluid is being completely taken up by the colder fluid. And heat evolved due to condensation of steam is equal to sensible heat for heating the tube water. This is by assumptions for f and g

So, here we have that this is the amount of heat that is evolved m that is the mass rate of the steam into the enthalpy of vaporisation is equal to this is the m c pc, m c pc is T c minus T out that is this much of the amount of the heat is been gained by the liquid. And here is the heat transfer from the condensing steam to the tube water and that is also given by U A delta T.

Now, you see that in this is we are here we are using LMTD that is logarithmic mean temperature difference, why, because as we see from the particular situation that the temperature the tube water is getting heated up, and whereas the steam is remaining constant at a constant temperature. Due to this what is happening the driving force for heat transfer is changing constantly, temperature difference is changing constantly.

So, instead of taking this change in the delta t temperature difference at all the actual locations doing the doing the flow of the tube water, what we are taking we are just considering the inlet and outlet temperature of the tube water, and finding the logarithmic mean temperature. And about this also we you can find it out more in my lectures or in some other books that why and when do we need this logarithmic mean temperature.

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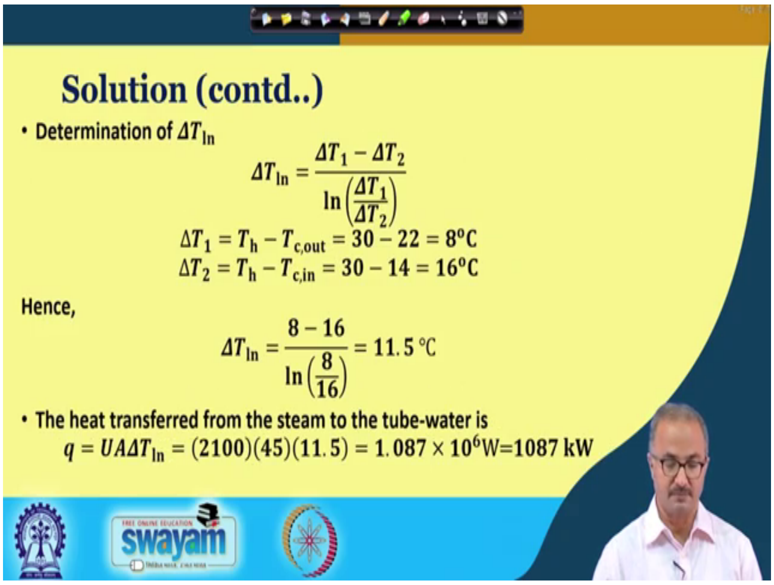

So, here it is the formula to find out the logarithmic mean temperature. And here we find that what is delta T 1, delta T 2 we plug in this values, and we get this is the delta T ln, ln is signifying the logarithmic mean. So, here we find that how much heat transferred UA delta T, we put the value of U when A given in the problem delta T ln is taken from this ok. And please understand here it does not matter should we should not convert this to

Kelvin, this is a difference in temperature is not temperature itself ok. So, difference in temperature in Kelvin and in centigrade will be the same ok. So, never convert this delta t into by means just by adding 273, no never do that that will be wrong ok. So, we are simply taking this temperature difference here, and this is how we are getting the q, and this is this much of kilowatt.

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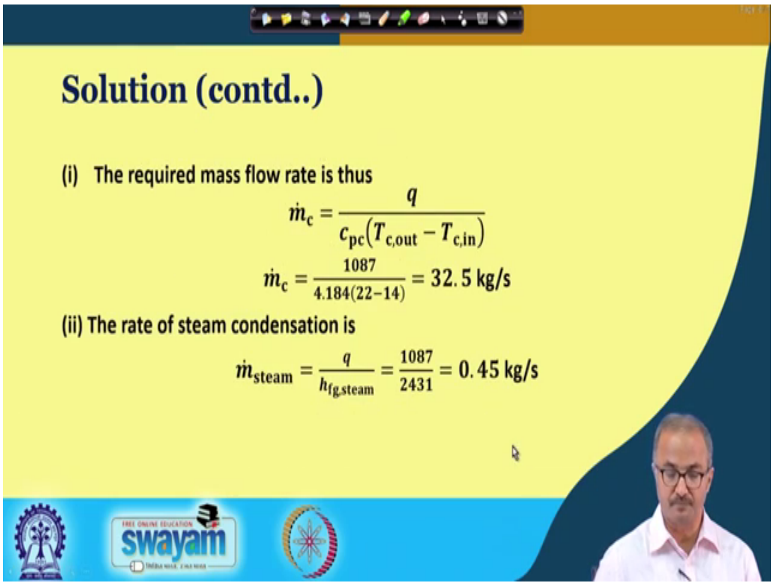

Now, the amount of mass flow rate of the water required is coming like this as q divided by c pc into T c out minus T c in, we plug in the values of this various variables, and this is the mass flow rate of the cold water in the tube side. Now, the this once we get this q, we can also find out that how much condensation of steam has taken place, and this is this q divided by h f g, and this is the enthalpy vaporisation, that means, 0.45 kg about 450 grams of the steam has condensed ok. So, this is very direct approach to find out the heat transfer analysis to find out this kind of flow rates.

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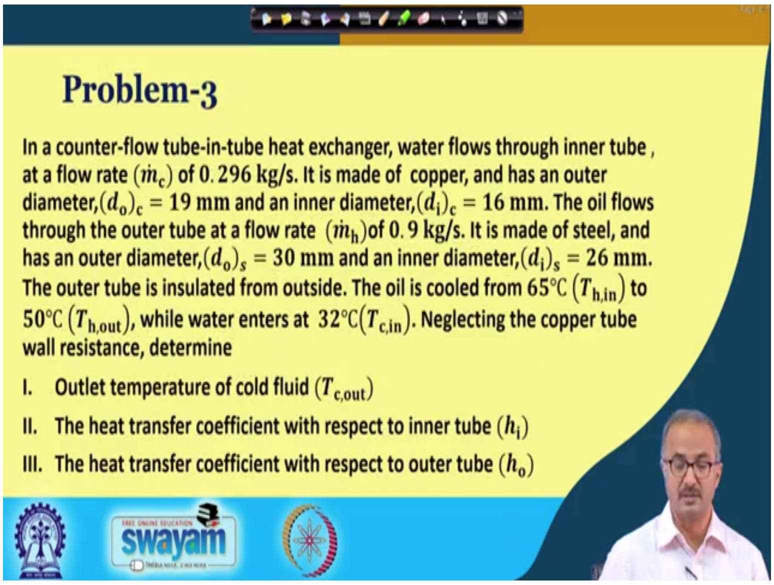

Now, next problem is about a tube-in-tube heat exchanger with a counter-current flow. And in this water flows through the inner tube with this is the m dot c is the notation we are using for the water flow in the inner tube, and this is the rate of the water flow. And the tube is made of copper and has an outer diameter of 19 millimetre, and inner diameter of 16 millimetre. So, from the this you can see that thickness of the tube will be 19 minus 16 divided by 2 that is 3 by 2 that is about 1.5 millimetres.

The oil flows through the outer tube at a flow rate of 0.9 k g per second and denoted by this particular symbol. And it is made of steel. And the outer diameter is this and inner diameter is 26 mm. Again you can find out the thickness is 2 millimetre ok. Now, and you can see that obviously the because you want the heat exchange between the hot and cold fluid to be very efficient, so you are making the inner tube with copper which has a very high thermal conductivity, and still you do and the outer tube is of steel because you do not want the system to exchange any kind of heat with a surroundings ok. So, we are choosing a material which has a low enough conductivity ok, and it is much lower than the copper conductivity ok, so that is why you are making the inner tube from copper and not from steel.

The outer tube is insulated from outside. So, we can see the reason that why we are using steel and not copper, and the oil is cooled from 65 degree centigrade to 50 degree centigrade ok. And the water enters at 32 degree centigrade. So, here it is said that neglect the copper tube wall resistance. It may be neglected because the copper as very high thermal conductivity ok. And with this assumption we are asked to determine the outlet temperature of cold fluid, the heat transfer coefficient with respect to inner tube, the heat transfer coefficient with respect to the outer tube.

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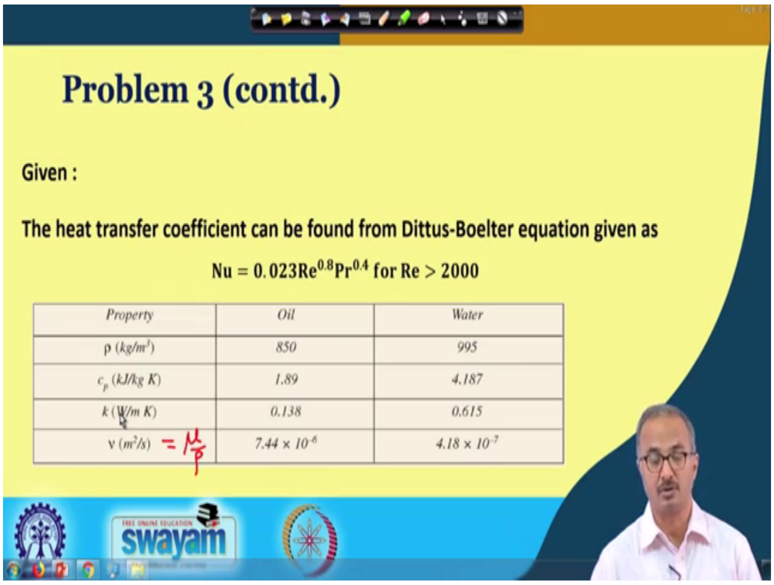

So, let us see how we go do this problem. Now, we have been also given some data that this is a correlation, if it is used to find out the heat transfer coefficient in turbulent flow in a circular tube. So, this is the Dittus-Boelter equation, this is very famous equation. And this is the kind of these 2000 is the critical Reynolds number for a circular tube. So, if Reynolds number is more than 2000, then this Dittus-Boelter equation is used to find out the heat transfer coefficient.

And these are this table gives us the properties of this oil and water like density, the specific heat, the thermal conductivity and the kinematic viscosity. This kinematic viscosity is the ratio of the dynamic viscosity and the density that means nu is equal to nu is equal to mu, this dynamic viscosity and the density ok. So, this value has been given, and this is all these you can see are given in SI unit. And one thing you must be taking care of is this that here you see that c p is given in kilojoule per kg, whereas this k is given in terms of watt it is not in kilowatt.

So, you will find that in our problem solution sometimes these two things will come together, at that time you should first convert either kilojoule to joule or watt to kilowatt before you start solving the problem.

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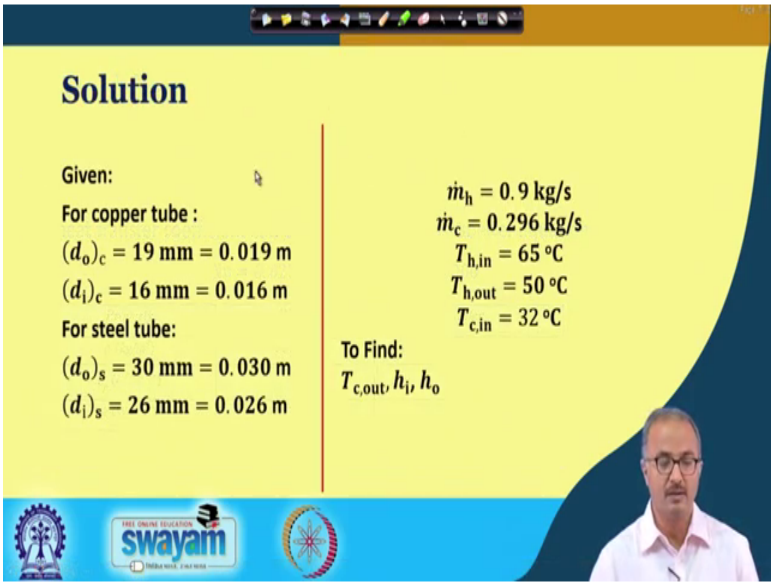

Now, we write this whatever information is given to us in the problem, so these are the information given to us for the copper tube outer diameter, inner diameter, steel tube - inner diameter, outer diameter, inner diameter. And these are the mass flow rate of the oil and the water, and this is the inlet temperature of oil, outer temperature of oil and inlet temperature of water. And we have to find out the outlet temperature of water, the heat transfer coefficient based on the inner diameter and this is the based on the outer diameter.

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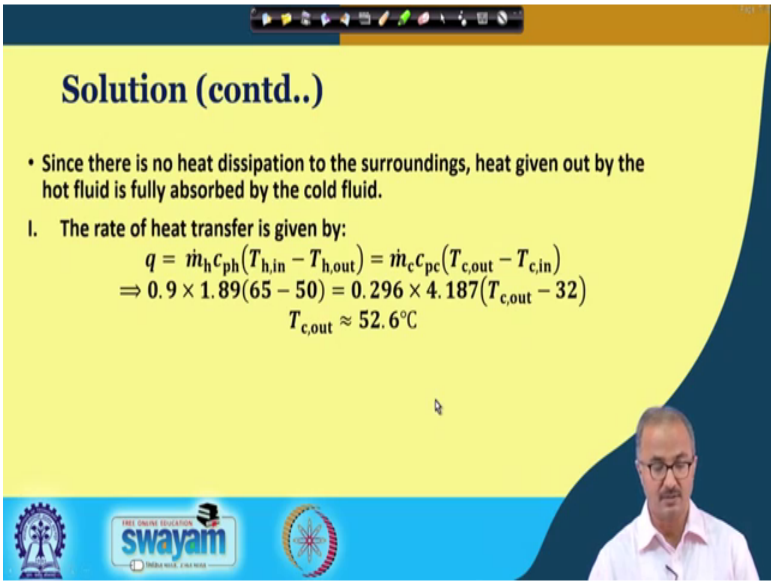

Now, here we are writing because there is no heat dissipation to the surroundings. So, heat given out by the hot fluid is fully absorbed by the cold fluid ok. So, with this we write this particular equation for the heat transfer between the two fluids. And now what we are doing we simply are plugging in the values of all these particular variables except that T c out which we are we have to find out. And by plug in the values, we find we are getting the value of the T c out as 52.6 that is the cold water is getting heated from 32 degree centigrade to 52 degree centigrade about by 20 degree centigrade.

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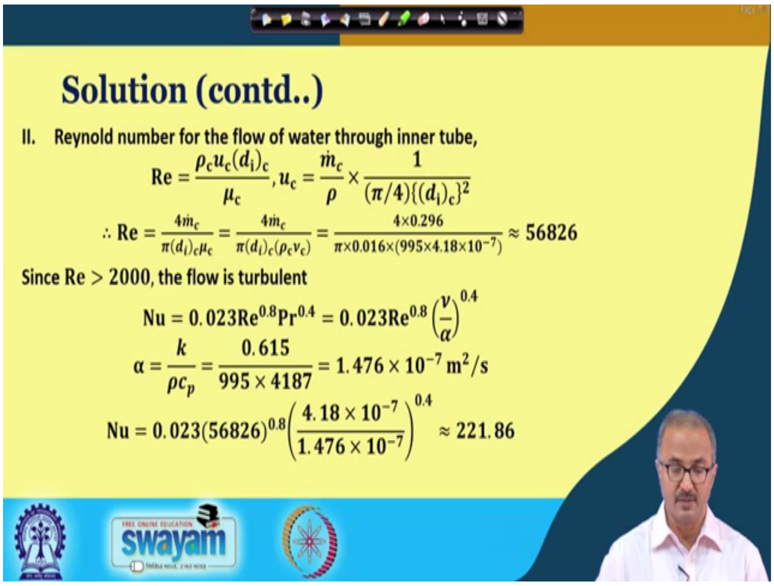

And now to find out the heat transfer coefficients, we need to find out the Reynolds number. So, for the inner tube, we are finding Reynolds number like this that rho u d by mu c. And now you see that u c that is the flow rate, superficial velocity of the cold fluid it is not given directly, but this we can find out from the mass flow rate, and this is the particular equation this mass flow rate divided by the density that is given the volumetric flow rate, and this is the cross sectional area of the inner tube ok.

So, volumetric flow rate divided by cross area gives us the superficial velocity. And then we are plugging the superficial velocity in this and we are finding this expression in terms of the mass flow rate, and we plugging that the various values only you mind it that the mu value is taken to be the product of the density and the kinematic viscosity which are given in the problem. So, we are putting those values and we are getting the Reynolds number like this.

So, now this is much, much higher than 2000 that means we can use the Nusselt number. And in Nusselt number, what we are doing that we are finding this thermal diffusivity alpha, and this kinetic viscosity given to us. So, this alpha value we obtained from this k rho c p. Here you have to take care that you are using the proper units of the various variables. And this is the value of the alpha we are getting. And now once we have got all these values we get the plug in the values in this equation and we get the Nusselt number value as this ok.

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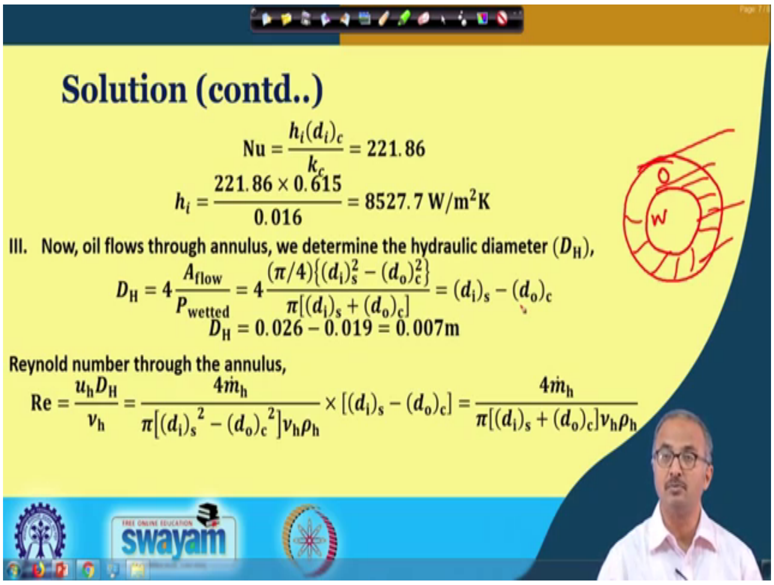

Once you know Nusselt number, then you go with the definition of Nusselt number that is h d by k ok. And here we are putting the appropriate and subscript to the h because this is for the inner tube. So, we are finding the h for the inner tube, and this is the inner diameter of the inner tube and this is the k c of the in the in the water. So, we are putting in these values, and we are getting this is the heat transfer coefficient on the inner tube side.

The similar method is being now followed for the oil side that is the outer tube side. Here we have now you see that when for the outer tube, we are not having a circular geometry. What is doing the oil is flowing to some annular. So, whenever we are deviating from any kind of circular geometry, what we do we generally go with the hydraulic diameter ok. So, you see that this is our system ok. In this system water is here and the oil is here ok. So, because oil is in the annular section, we need to find out the hydraulic diameter. And by definition of hydraulic diameter we know that this is equal to 4 into flow area divided by the wetted perimeter, wetted perimeter is the perimeter which is wetted by the fluid ok.

So, here we have this is the area of flow, that means, we have only this much area and this much area is obtained by subtracting the total area minus the inner area. So, this is the this pi by 4 d i s square, that means, the inner diameter of the outer tube, and the outer diameter of the inner tube. So, this is the area available. And this is the wetted perimeter it is wetting is being done on the inner surface of the inner tube here its wetting and this another tube ok. So, inner surface of the outer tube and outer surface of inner tube that is mean neglected.

So, this is the area pi d wetted perimeter, and now we find that after this mathematic reduction, it is basically d i s minus d o that means, it is simply the difference between the diameters inner diameter of the outer tube and outer diameter of the inner tube ok. Now, we put the values over here, and we get the hydraulic diameter as this 0.007 meter. Once we get this value, we put in this Reynolds number definition, and we plug in the values of the u, and we get the this particular expression to find out the Reynolds number. ah

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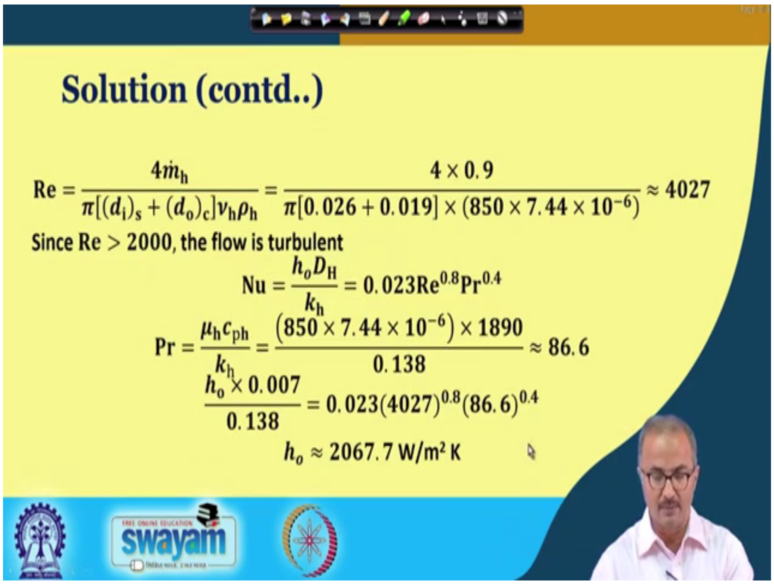

We put all these here and we find this is the Reynolds number obtained ok. And again we find this Reynolds number is more than 2000. So, we can use this particular equation to find out the value of the heat transfer coefficient, we find the Prandtl number as before ok. And we plug in these values here and we get the value of the heat transfer coefficient for the outer wall ok. Now, you see that you have to be very, very cautions whenever you are using any kind of correlation for the heat transfer coefficient.

The same applies for mass transfer coefficient and same applies also for the friction factor. So, the fluid dynamic that is the nature of the flow is very very important whenever you are selecting any kind of correlation. For example, for laminar flow had it been less than 2000, then you should not have use this equation, you should have gone for seductive equation, this is another equation for circular pipes seducted. So, those kind of equations should be used for the laminar flow. So, this is very crucial whenever you are finding this heat transfer coefficient, mass transfer coefficient and the friction factor.

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These are some of the reference books you can consult for further explanation and knowledge.

Thank you.