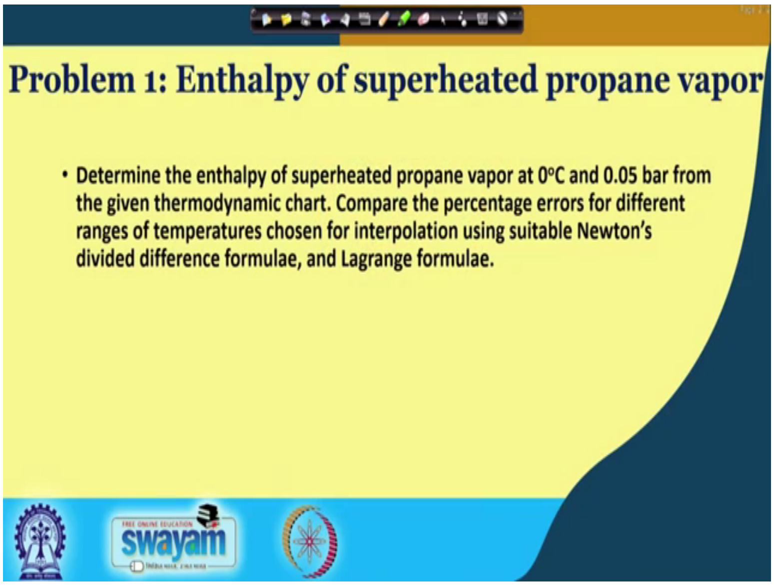
Mass, Momentum and Energy Balances in Engineering Analysis
Prof. Pavitra Sandilya
Cryogenics Engineering Center
Indian Institute of Technology, Kharagpur

Lecture - 40
Illustration of Interpolation

Welcome back. After learning something about the interpolation methods and regression in this lecture we shall be taking some examples of interpolation, and we shall see where all we need this kind of methods. I will be giving you just two examples to illustrate how we can use the interpolations.

(Refer Slide Time: 00:40)



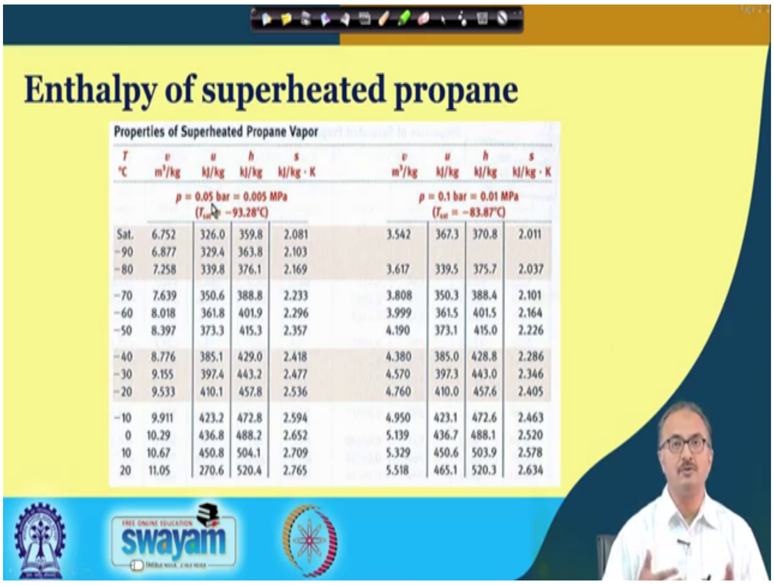
So, here we have a problem which is very much encountered in our balance equation solution. In that we find that many a times we are required to find some kind of properties like density or thermal conductivities, specific heat, enthalpy, entropy, internal energy, etcetera and then we have to resolved to either some graphical way of representation of those data or some tabular data are given to us.

And many a times we find that the temperature pressure at which we want to determine or estimate the property values are not directly given in the table. In those cases we have to have some kind of interpolation methods which will be able to give us the values of the required property at those intermediate points, ok. So, in this problem we have to determine the enthalpy of superheated propane vapour at 0 degree centigrade and 0.05

bar from the given thermodynamic chart. And we have to also compare the percentage error for different types of methods whatever we have learned so far.

Now, you see that whenever you are about to go for interpolation it becomes a very critical in judging that which kind of interpolation formula you should use. Generally, we want to keep the interpolation very simple, and in most of the cases we go for the linear interpolation, whenever we are having a small range to consider. But many times, you also find that whenever some of the property values are given at very sparse way, ok, in those cases we have to be very careful to figure out that whether we can go for linear or we have to go for some kind of non-linear interpolation methods, ok.

(Refer Slide Time: 02:36)



Properties of Superheated Propane Vapor

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	p = 0.05 bar = 0.005 MPa (T_{sat} = −93.28°C)				p = 0.1 bar = 0.01 MPa (T_{sat} = −83.87°C)			
Sat.	6.752	326.0	359.8	2.081	3.542	367.3	370.8	2.011
−90	6.877	329.4	363.8	2.103				
−80	7.258	339.8	376.1	2.169	3.617	339.5	375.7	2.037
−70	7.639	350.6	388.8	2.233	3.808	350.3	388.4	2.101
−60	8.018	361.8	401.9	2.296	3.999	361.5	401.5	2.164
−50	8.397	373.3	415.3	2.357	4.190	373.1	415.0	2.226
−40	8.776	385.1	429.0	2.418	4.380	385.0	428.8	2.286
−30	9.155	397.4	443.2	2.477	4.570	397.3	443.0	2.346
−20	9.533	410.1	457.8	2.536	4.760	410.0	457.6	2.405
−10	9.911	423.2	472.8	2.594	4.950	423.1	472.6	2.463
0	10.29	436.8	488.2	2.652	5.139	436.7	488.1	2.520
10	10.67	450.8	504.1	2.709	5.329	450.6	503.9	2.578
20	11.05	270.6	520.4	2.765	5.518	465.1	520.3	2.634

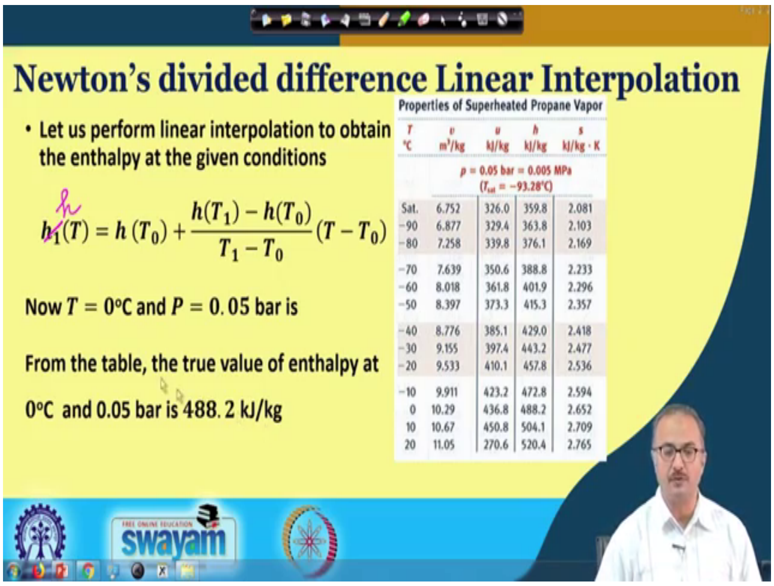
Now, with this in mind first let us see that we are given this kind of data to us. So, here you see that the properties are like this, that first row column is telling us the temperature in degree centigrade and then we have the specific volume that is small v, then we have the internal energy, then the enthalpy and then the entropy and all those things are given for some specific pressure.

So, pressure is given both in bar as the as well as in mega Pascal. And here it is mention that at this particular pressure the saturate pressure temperature is this value, all this is the boiling point.

Now, you can see in this particular table, like the first is set that means, this is T set corresponding to minus 93.8 degree centigrade. Now, because we are talking about super heated propane vapour that means, we have to consider all the temperatures which are above T set. So, that is why you find that in this particular table we have the temperatures which are more than the T set of the particular material at the given pressure. So, this kind of chart you will get for different types of components.

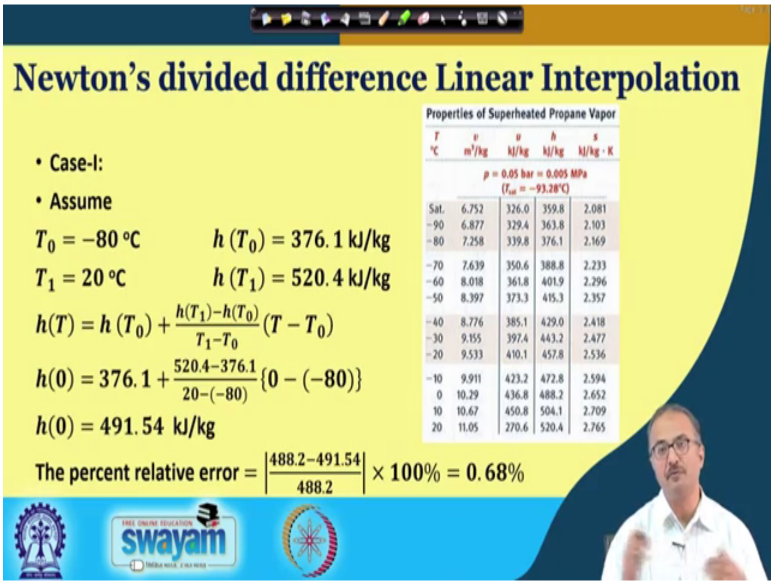
So, we have been asked to find out that at 0.05 bar and at 0 degree centigrade what is the enthalpy. Now, you see that in this particular table the values of enthalpy are given, but for sake of finding out the percentage error by considering different types of interpolation methods what we are doing is this we shall be using some interpolation method and try to estimate the value of the enthalpy at this temperature. And whatever value is given to us here, this value will be our standard value with which we shall be comparing the estimated value from the interpolation formulae and from there we can figure out that how good or how bad is our interpolation formula, ok.

(Refer Slide Time: 04:35)



So, let us go ahead. So, here we are using the Newton's divided difference linear interpolation that is the easiest we can think off and as earlier we see that this is the particular formula we use. So, here it will be h T this will be h only. So, this h T we have to find out in terms of some other enthalpy values at some other temperatures like T 0 and T 1, ok. So, this is the formula. So, here the T is 0 degree centigrade and the P is 0.05 bar, and from the table we find the true value as 488.2 kilojoule per kg.

(Refer Slide Time: 05:24)

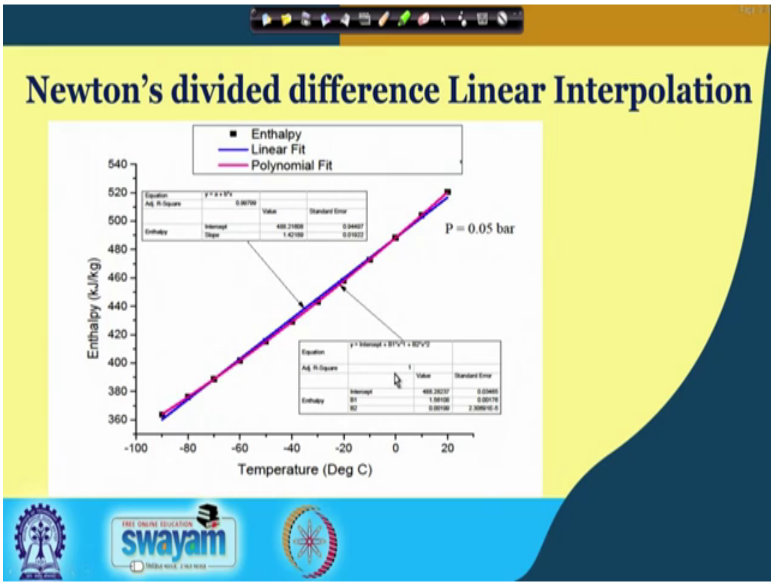


Now, first what we do? We go to estimate we must first assume that the limits like minus 80 degree centigrade and 20 degree centigrade that means, here we choose this 20 degree centigrade and minus 80 degree centigrade to arrive at a value for this 20 degree centigrade enthalpy value, ok. Let us see what happens.

That means, if we certainly take a very large range to make the interpolation. So, as I told you in my earlier lecture that range of interpolation have to be chosen carefully depending on the linearity or non-linearity of the particular problem. Now, sometimes we can figure it out from our physics of the problem and sometimes we may not be able to figure it out from the physics of the problem, then we have to have this kind of data chart with us to do those things. So, we shall see these things one by one.

So, considering these two temperatures and then figuring out that enthalpy values at these two temperatures we do the iteration and find that this is the value of the enthalpy we are getting and we what we find that from the true value if you look at we find that we are getting a very small percentage error that is about 0.68 percent. Now, you can see that at the very first go even after considering a very large range of temperature we have been able to get a percentage error very very small it is less than 1 percent that means, we are able to get a very good estimate of the enthalpy or the particular property. Now, this should trigger us that why it happened. It could only happen if we find that we have a very linear relationship. In that case what we find? The slope will be constant. So, that whatever range we choose we are going to arrive at a very good estimate value.

(Refer Slide Time: 07:14)



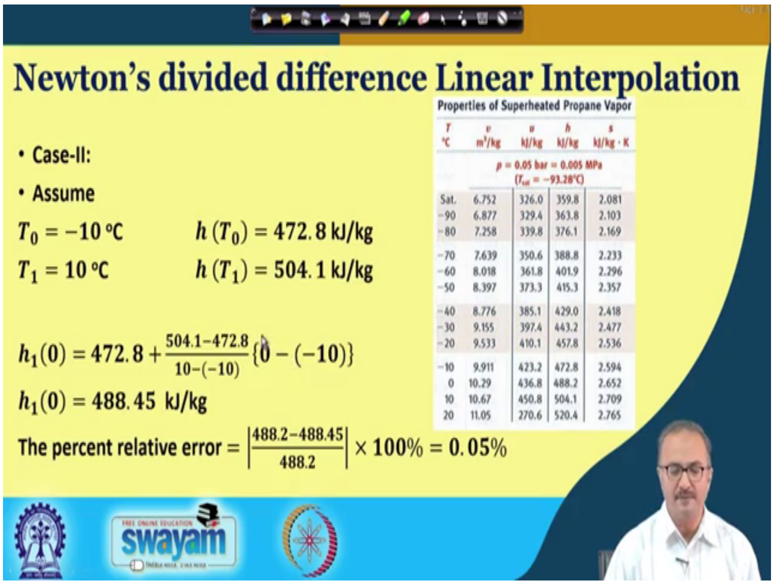
So, to understand this, what we do? We just simply plot it to plot this data whichever given to us at 0.05 bar and these are these black colour things are the actual values which are given in the table. And then what we do? We first fit a linear curve here we have fitted the linear curve and this can be fitted using some package like origin, like Excel, you can use any of this packages or MATLAB and if you feed them it will also give you the correlation coefficient how much you are getting and you are you can find that R square error coming to be a 0.99799.

Now, this particular value tells as that this is a very good fit even with a linear thing that means, is it if it is as if near as unity we take the fit to be very good. So, we are already getting a very good fit, with the linear fit within this particular zone, ok. And that is the reason we are finding that even if you are choosing a very large range of the temperatures to estimate the value we are able to get a very good percentage error very less percentage error. And just for sake of testing we can also fit one polynomial fit, ok.

Here we have taken a quadratic fit, and here we see this particular line is giving us the quadratic fit and here we find that the R square value as come to 1. So, there little difference between this 0.99799 and 1. So, in order to reduce our computation time we can just go with the linear fit without going for this, ok. So, that is what we are finding that there is no extra gain we are getting by increasing the degree of the polynomial for getting the interpolated value, ok.

So, in a whenever you are doing this kind of interpolation either you try out with the linear interpolation if you find the range is quite small and all the particular independent variable value, ok. If the range is large only then perhaps you can think of going for a non-linear interpolation, but it is always good to check that by this graphical way that how this linearity or the whether the things are linear or not, ok. So, that will you know reduce your time of iterations.

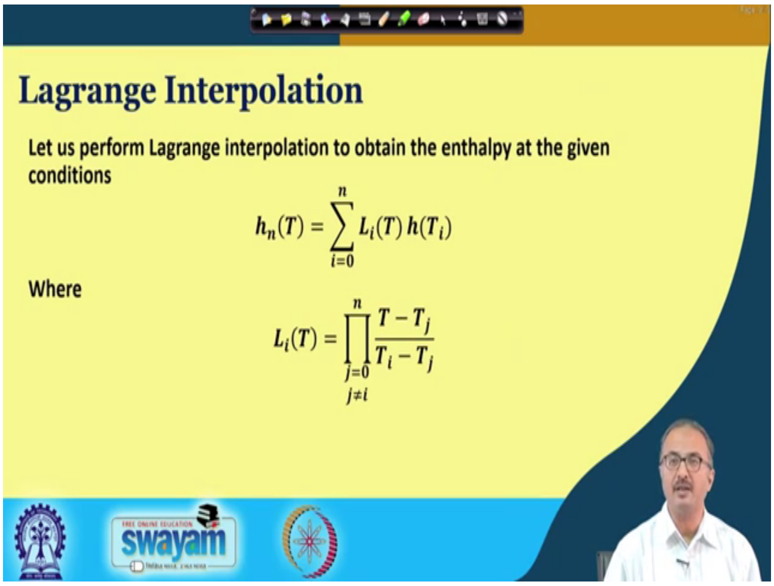
(Refer Slide Time: 09:33)



Now, we have taken the case 2 and in this what we are doing that we have those reduced the range, ok. So, already it was 80 degree centigrade minus 80 and 20 degree centigrade. Now, it is minus 10, 10 degree centigrade. Now, here we find that we are getting this value 0.05 percent.

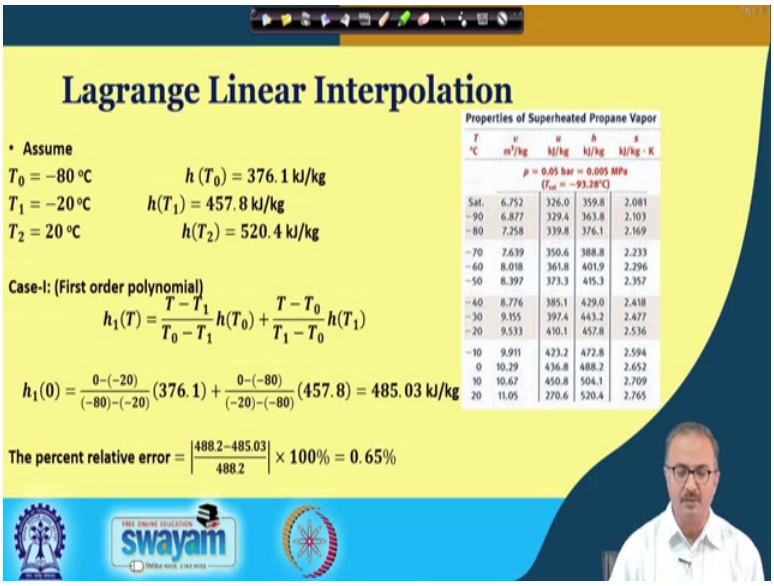
Now, whether we should go for 0.05 percent or whether we can stay with 0.68 percent that will again depend on the problem, ok. Now, it is not necessary for you always to keep reducing the percentage error. Many a times for engineering analysis we are quite used to having errors like 1 percent, 2 percent, in 5 percent errors are permissible at times, ok. So, it is not always very necessary for us to keep on increasing the degree of the polynomial or keep reducing the range of the estimation and to try trying to reduce the percentage error. It will all depend on the case by case basis whether we need to reduce or not, but in this case what we find that 0.68 percent may be taken to be a good value.

(Refer Slide Time: 10:38)



Next, we go to Lagrange interpolation. Now, this formula is different as we have seen earlier and in this case, we are again going to perform a linear interpolation with the Lagrange polynomial.

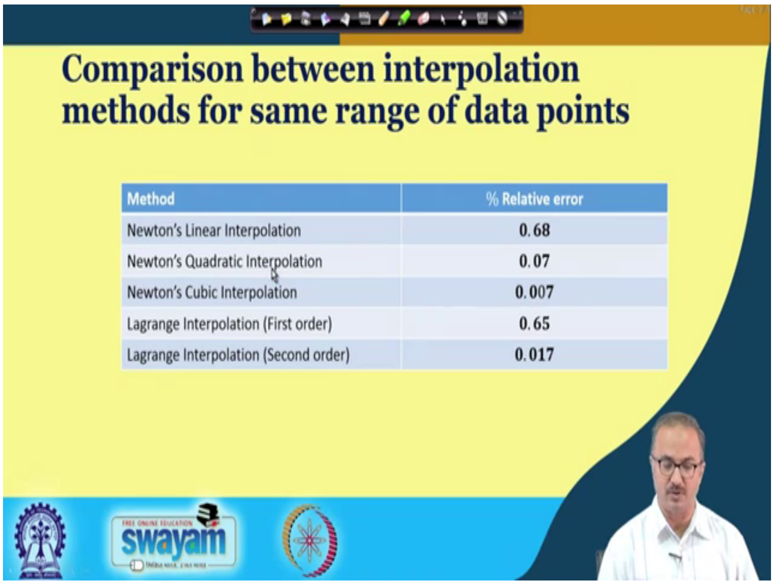
(Refer Slide Time: 10:55)



Now, here you see that we have taken as you know instead of Lagrange, we have to take these 3 things. So, we have arbitrarily chosen this 3 temperatures minus 80, minus 20, degree centigrade and plus 20 degree centigrade, and at this temperatures from this particular table we find the values of the enthalpies. And now, we put the Lagrange formula, this is a linear interpolation formula of Lagrange and we find that we are getting again the value which is about 0.65 percent having error.

So, you see that whether we are using the Newton's linear interpolation or Lagrange linear interpolation in both the cases we are finding that we are having almost the same percentage error, ok. Although now difference is this, in case of Newton's formula we needed only 2 points whereas, in case of Lagrange we are going for 3 points, ok.

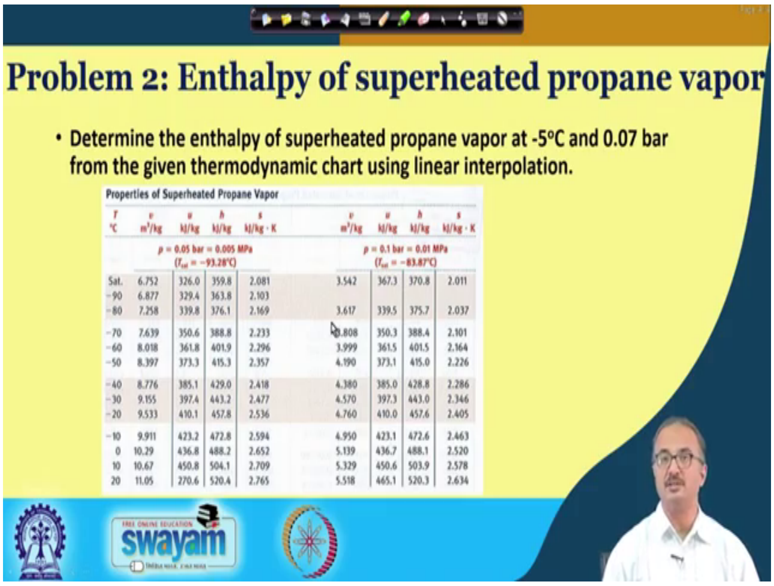
(Refer Slide Time: 11:53)



Now, without doing any more of these deductions what I have done is this, I have calculated separately using different interpolation formula this percentage error for this particular problem. And here you can see we have tabulated them like Newton's linear, then quadratic Newton's quadratic, Newton's cubic, then Lagrange interpolation first order, Lagrange interpolation second order and here you can see all the percentage errors.

So, what we are finding? That for example, you take Newton's interpolation methods. We are finding that as we are increasing the order of the interpolation we are able to decrease the percentage error, ok. But at the same time, it is coming at the cost of the computation, computation effort, ok. At the same time that even for Lagrange we find that if you go for second order is it is giving us a less percent error than for the first order, ok. So, this way we figure out that increasing the order of the interpolation we are able to get a better and better estimate. Again, as I said that we should not get tempted by these small percentage error we should be very very clear in our mind that whatever is the requirement for the particular problem accordingly we should go for the interpolation method.

(Refer Slide Time: 13:19)



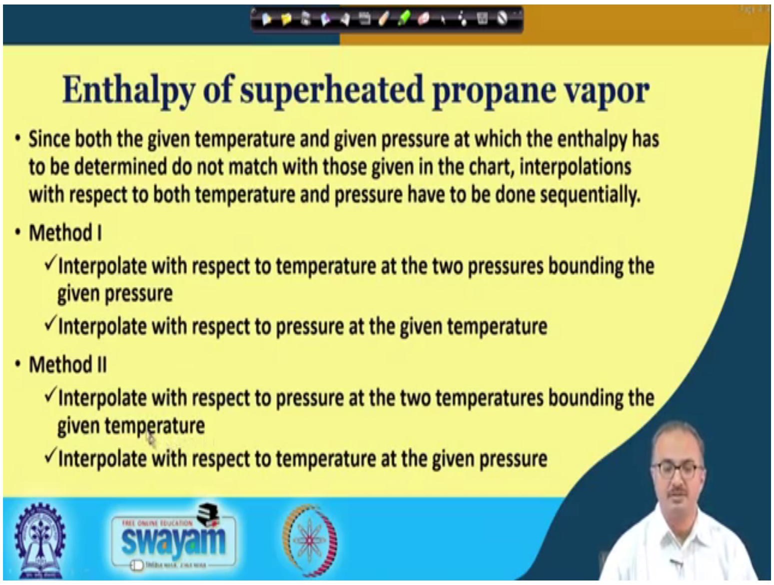
Properties of Superheated Propane Vapor

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	p = 0.05 bar = 0.005 MPa (T_{sat} = −93.28°C)				p = 0.1 bar = 0.01 MPa (T_{sat} = −83.87°C)			
Sat.	6.752	326.0	359.8	2.081	3.542	367.3	370.8	2.011
−90	6.877	329.4	363.8	2.103				
−80	7.258	339.8	376.1	2.169	3.617	339.5	375.7	2.037
−70	7.639	350.6	388.8	2.233	[illegible].808	350.3	388.4	2.101
−60	8.018	361.8	401.9	2.296	3.999	361.5	401.5	2.164
−50	8.397	373.3	415.3	2.357	4.190	373.1	415.0	2.226
−40	8.776	385.1	429.0	2.418	4.380	385.0	428.8	2.286
−30	9.155	397.4	443.2	2.477	4.570	397.3	443.0	2.346
−20	9.533	410.1	457.8	2.536	4.760	410.0	457.6	2.405
−10	9.911	423.2	472.8	2.594	4.950	423.1	472.6	2.463
0	10.29	436.8	488.2	2.652	5.139	436.7	488.1	2.520
10	10.67	450.8	504.1	2.709	5.329	450.6	503.9	2.578
20	11.05	270.6	520.4	2.765	5.518	465.1	520.3	2.634

Now, let us go to another problem. Now, in this problem the thing is this that we are now going to deal with two independent variables at which we have to interpolate. Now, earlier problem we found that we were interpolating only with respect to the temperature, in this case we have to interpolate with respect to both temperature and pressure. So, here the problem is that we have to find the enthalpy of superheated propane vapour at minus 5 degree centigrade and 0.07 bar.

Now, here you can see that this minus 5 degree centigrade does not exist in this temperature and this is 0.07 bar is also not there. So, it is 0.05 bar and 0.1 bar that means, for that the enthalpy has to be interpolated walls with respect the temperature and then with respect to the pressure, ok.

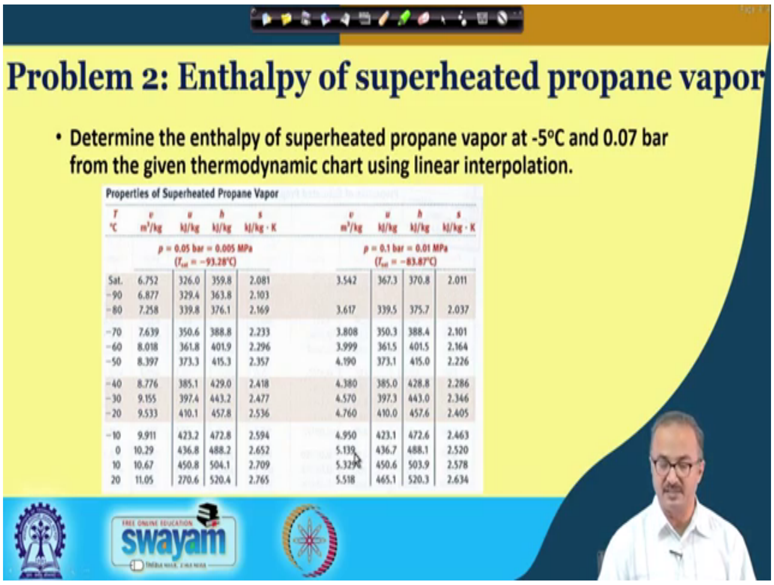
(Refer Slide Time: 14:15)



So, here you can see that since both the given temperature and pressure at which the enthalpy has to be determined do not match with those given in the chart interpolations with respect to both temperature and pressure have to be done sequentially.

Now, point is this what sequence we should adopt, whether we should first go with the temperature interpolation and then pressure interpolation or first pressure interpolation then temperature interpolation? Now, this can be arbitrarily done. And we shall see that it really does not matter that which sequence you follow. So, that we can say that in method one may be that we interpolate with respect to temperature first at two pressures bounding the given pressure and then we interpolate with respect to pressure at the given temperature.

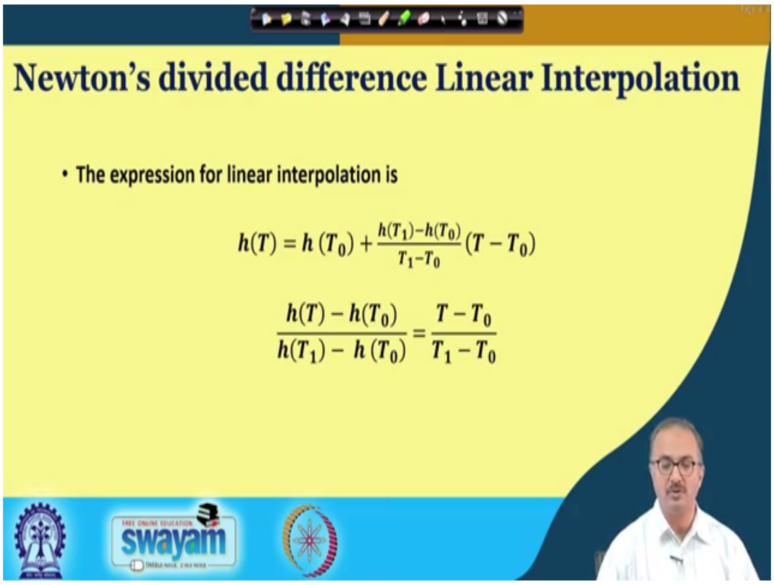
(Refer Slide Time: 15:05)



What it means is this suppose what we do that we go to the temple, ok. Now, first we interpolate with respect to temperature that means what? That we have 0 and minus 10. So, we get the value at minus 5 degree centigrade at 0.05 bar. Similarly, we can get the value of enthalpy at minus 5 degree centigrade at 0.1 bar by interpolation. Now, with these two values we again interpolate to get the value of the enthalpy at 0.07 bar and at minus 5 degree centigrade so this method 1, ok.

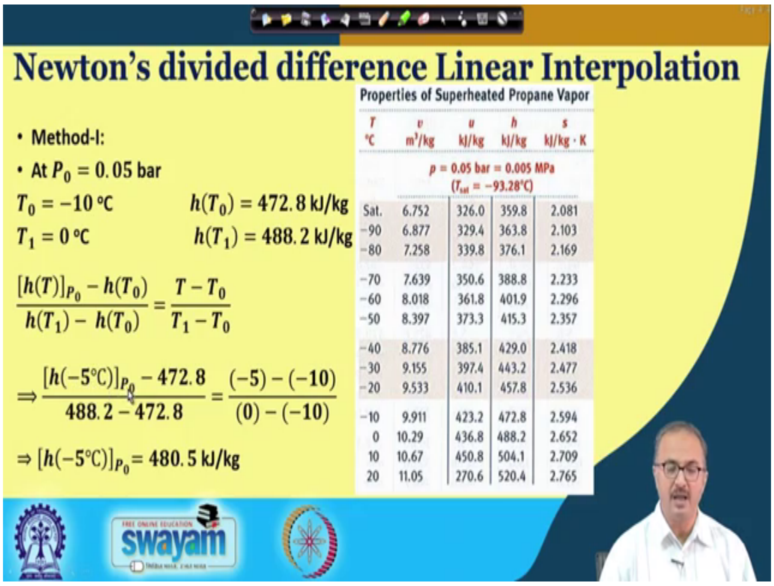
And in the method 2 what we were saying, that interpolate with respect to pressure at two temperatures bounding the given temperature and then interpolate with respect to temperature at the given pressure. That means what we are doing? That first with respect to the that means, you find the value at 0.07 bar first and then and at minus minus 10 degree centigrade and 0 degree centigrade, and then what to do with those two values you find the value of the enthalpy at minus 5 degree centigrade and 0.07 bar.

(Refer Slide Time: 16:16)



So, let us see these two. So, here we have we adopt arbitrarily again a Newton's divided difference linear interpolation and this is the interpolation formula.

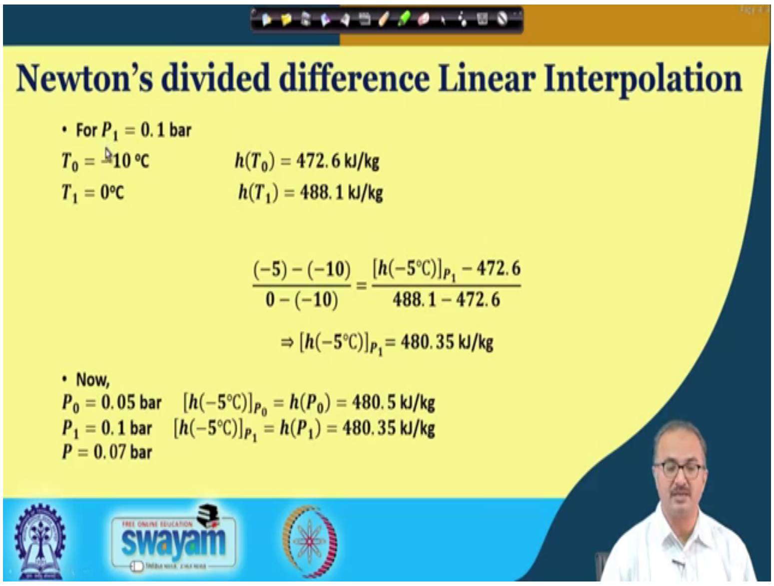
(Refer Slide Time: 16:25)



So, in the method one what we do that let us take the P 0 to be 0.05 bar and let us choose the temperatures as 0 degree centigrade and minus 10 degree centigrade. So, here are the enthalpy values at these two temperatures and at this particular pressure.

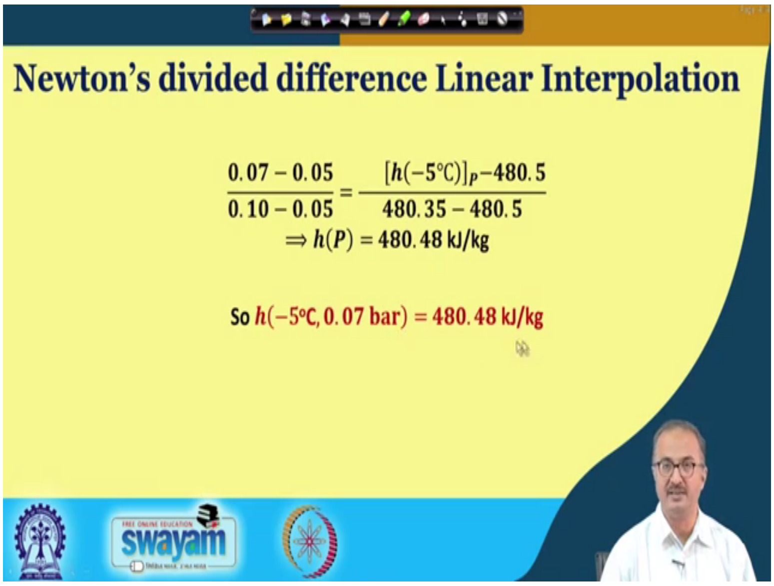
Now, what we do? We find the temperature, the given temperature is minus 5 degree centigrade. So, at this temperature, at this P 0 pressure, we find the value of the enthalpy from the Newton's linear interpolation formula.

(Refer Slide Time: 16:58)



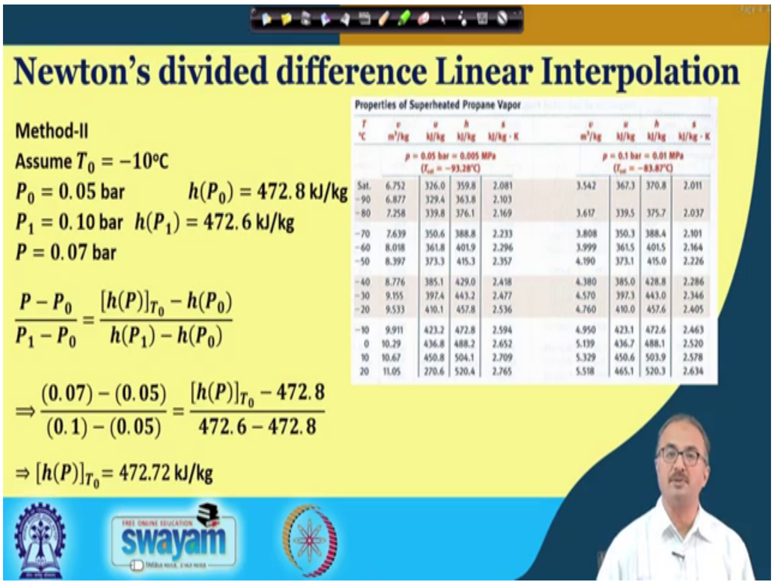
Next what we do? We choose the next pressure that is P 1 that is 0.1 bar because this pressure is bounding the given pressure of 0.07 bar. So, again we at this pressure we take the two temperatures 0 degree centigrade and minus 10 degree centigrade and we get the enthalpies and we again get the value of the enthalpy at minus 5 degree centigrade. And with this, this value and this value what we now do? We again interpolate with respect to pressure and we get the value of the enthalpy like this.

(Refer Slide Time: 17:35)



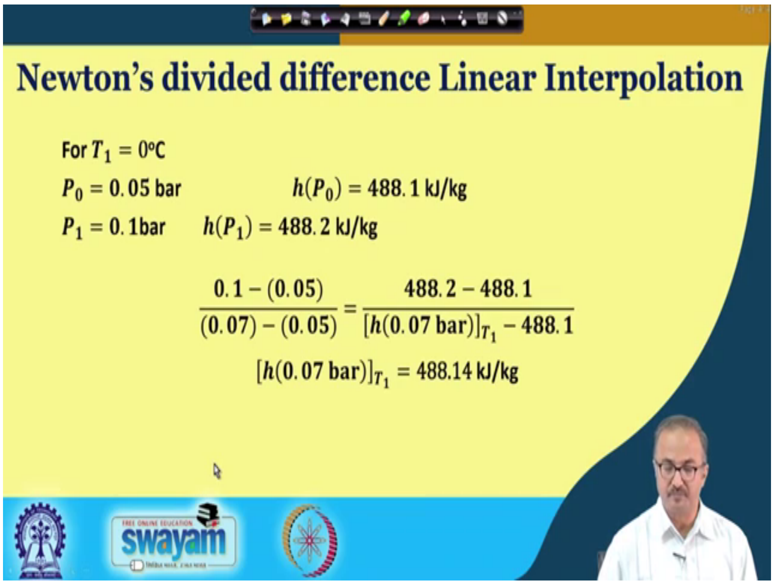
Now, you see that there could be some questions like do we need to really stick to this same range for this pressure and this pressure? Well, perhaps it is advisable to stick to same range assuming that the behaviour of the variation of the enthalpy with respect to temperature should be maintained at both the pressures. It might so happen with some properties that you may you may find that this linearity may be destroyed if you go for some other value of pressure, ok. So, it is better to stick to the same range, but there is no hard and fast rule mathematically to go with the same range of this bounding values.

(Refer Slide Time: 18:27)



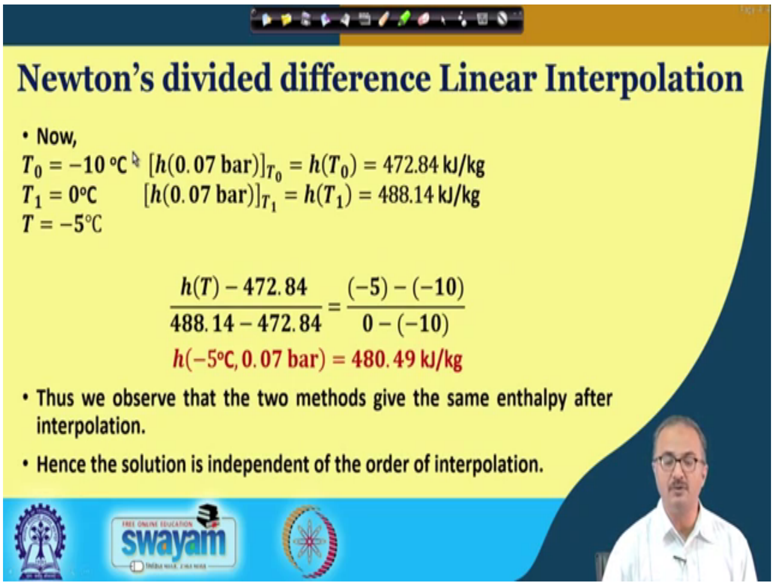
Now, what we do? We adopt the second method. In the second method what we do we first choose another one limit that is minus 10 degree centigrade and at this temperature for 0.05 bar and 0.1 bar we readout the values of the enthalpies. Now, with these two values of the enthalpies what we now do? We find out the value of the enthalpy at this particular pressure that is 0.07 bar, ok.

(Refer Slide Time: 18:55)



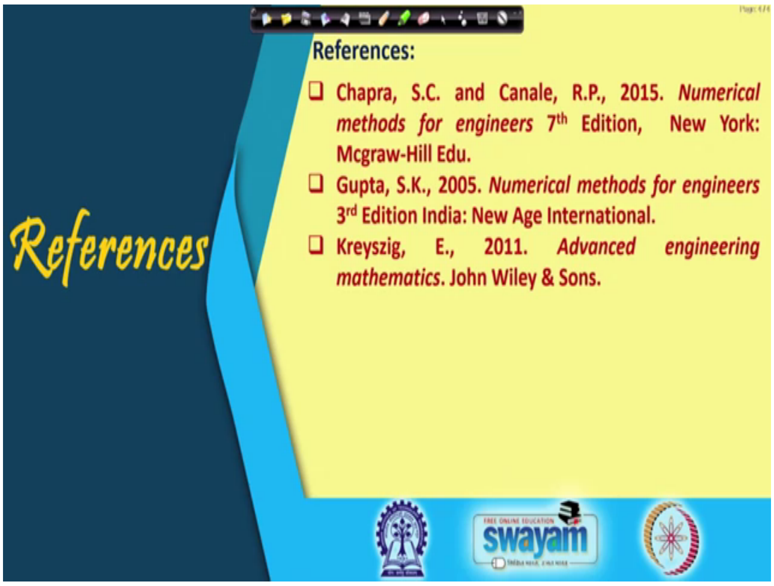
So, this is after getting this 07 bar we are getting this enthalpy value. Now, what we do? We go for the 0 degree centigrade, and again we at of the same method and we get the value of the enthalpy at this 0.07 bar at 0 degree centigrade. Now, with these two values we now interpolate with respect to temperature to get the value of the enthalpy at minus 5 degree centigrade.

(Refer Slide Time: 19:14)



And now, you can see that this particular value which is the value we wanted to have this value and this value are almost the same. So, what we conclude from here? We conclude that this two methods give the same enthalpy after interpolation and the solution is independent of the order of interpolation that means, the sequence which whatever sequence you might maintain whether first temperature then pressure or first pressure then temperature it really does not matter and we are going to have the same interpolated value.

(Refer Slide Time: 19:59)



So, these are some of the references which will give you some more detail about the methods we have talked about.

Thank you.