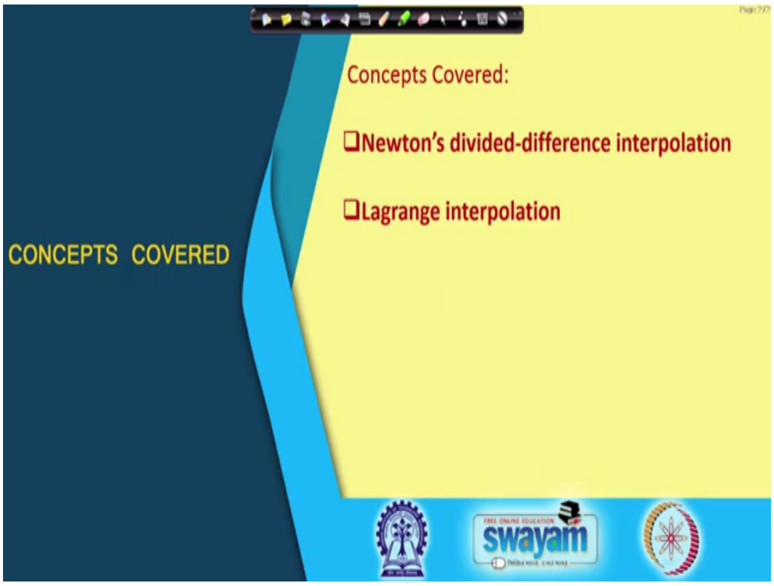
**Mass, Momentum and Energy Balances in Engineering Analysis**
**Prof. Pavitra Sandilya**
**Department of Cryogenics Engineering Center**
**Indian Institute of Technology, Kharagpur**

**Lecture - 39**
**Interpolation**

Welcome. In our earlier lecture, we learnt about regression and there I told you that why we need to do regression to correlate the data. And in those cases, we also find that we can use the regression to get the values of some dependent variables at some points which are not given to us. In similar fashion we also do interpolation in case of interpolation we do not need to do any kind of curve fitting, we do not need to know the exact nature. But in this case we deal with still a very limited range of data to find out the values of the dependent variables at some points, at some values of the independent variables which are not given to us ok. So, that is the difference between the regression and the interpolation.

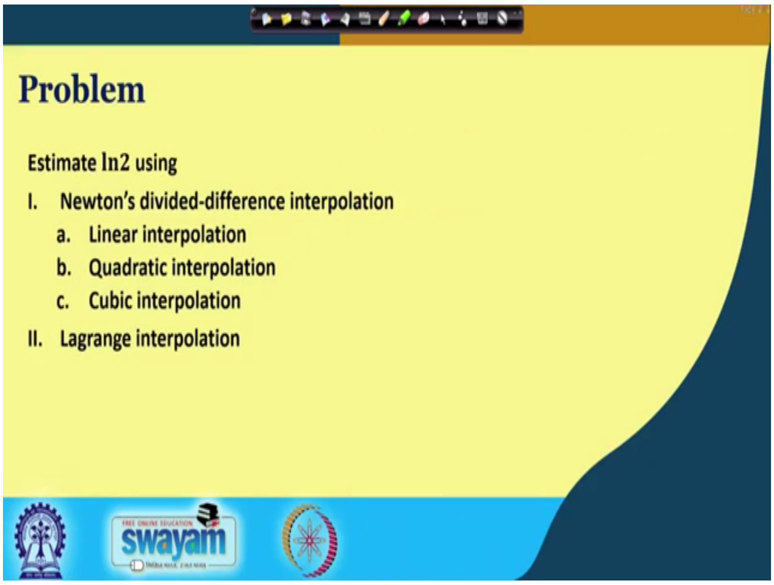
So, in this particular lecture we shall be looking into the some very commonly used regression methods which is again not very exhaustive because in separate courses on the numerical techniques you get to learn more about this interpolation and you are given many many interpolation methods.

(Refer Slide Time: 01:34)



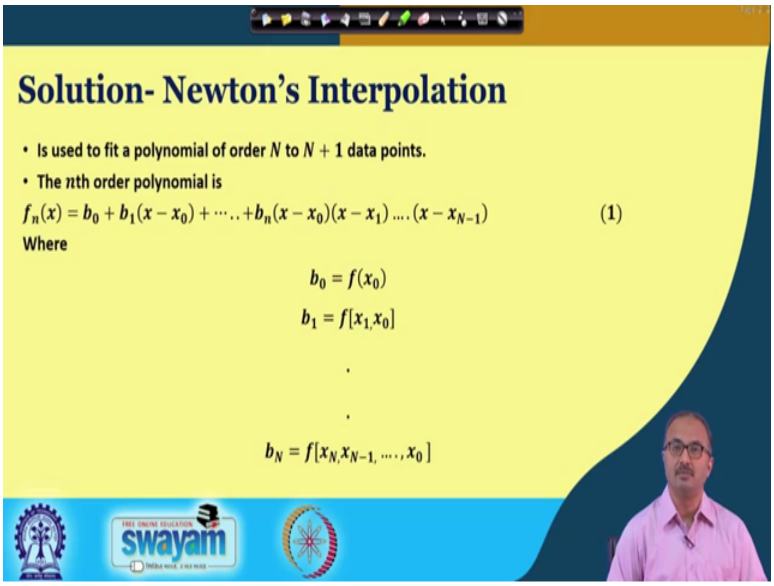

So, here I shall be restricting myself only to 2 types of interpolation methods.

(Refer Slide Time: 01:38)




So, first we shall go by the examples and in that only I shall be explaining to you how those formula have been obtained. So, now let us see that it is very common problem that we have to find the value of logarithm of 2, that is natural logarithm of 2 using Newton's difference formula and again you see there are many variations of that and Lagrange. Now, you see that this ln 2 you can find the value easily from any kind of logarithmic chart. Now, we shall see that how do we get to interpolate this values ok.

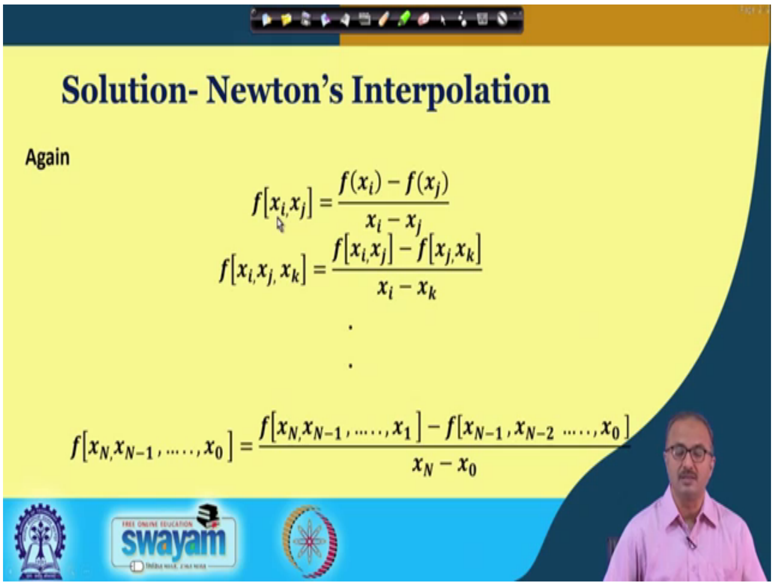
(Refer Slide Time: 02:12)




So, first before we go to the solution, let us first understand a bit of this interpolation principle. In this we have say N plus 1 data points and in through this N plus 1 data points we can fit a curve of degree N ok. And that this polynomial fit will be looking something like this, that if you have a polynomial of degree n then it will look like this.

Now, you can see that this all these b 0, b 1 as such of b N values are can be given like this, again I am not going into the derivation of this equations. Things is this that this particular thing this in third bracket what you see this is not representing that it is some unknown function it is representing some kind of difference function and how these difference functions obtain we shall see later.

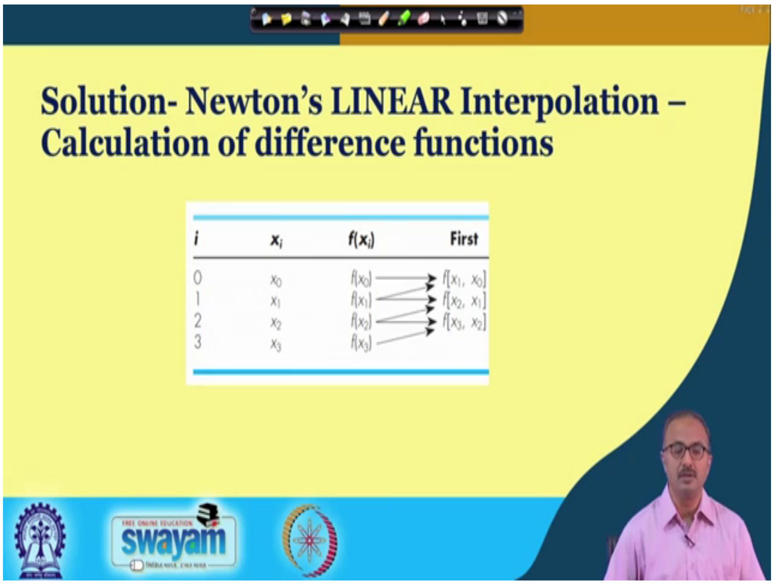
(Refer Slide Time: 03:13)




Now, here we have the explanations that b x i, x j, f x that means, that f x i means if f xi minus f x j divided by x i x j. Now, this f x i, f x j, x i x j they will be given to you and from that you can find this value of this f third bracket x 1 comma x j. Then we have x i, x j and x k and this what it means that first you take f x ij and then you take f jk. So, this is the way you are going to find out this value. And again you see that this f x i x j can be found from this formula, and this also can be found from this formula.

Similarly, if you keep proceeding with more and more number of points you see that the value of this f, this left hand side will depend on the values of the f on the one we have found in just in the previous step and this is how we continue.

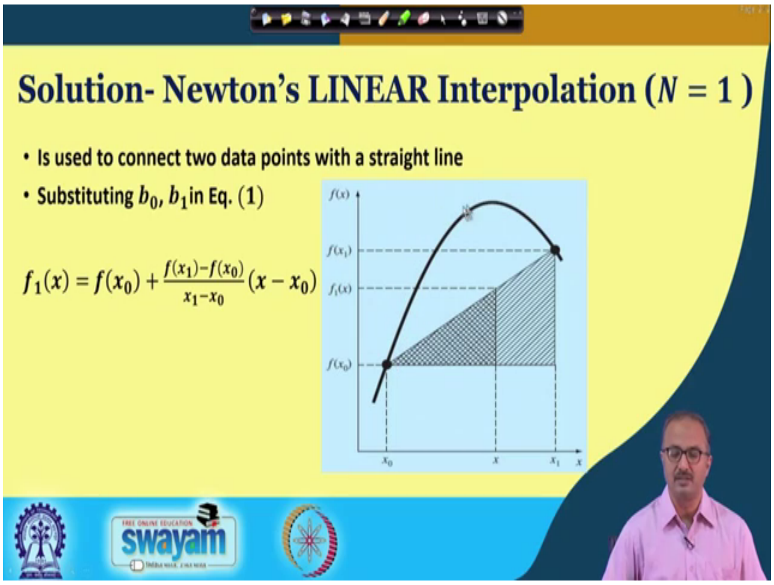
(Refer Slide Time: 04:16)




Now, here if you put in a tabular form you can see that how it will look like. Suppose you are given 4 points, one is x 0, x 1, x 2, x 3 like there are 4 point are given and at this 4 points you are given some function values. Now, if you go with the linear interpolation then you need to find only one difference points ok; that means, if you arrange this f x 0, f x 1, f x 2, f x 2 like this and then by taking the difference between that means, this value minus this value will be this, then this value minus this value will be this and this value minus this value will be this ok.

Now, you see that this choice of the f x 0, x 1, x 2, x 3 is generally arbitrary, but when you are doing interpolation the you cannot you should not do it arbitrarily otherwise what happens that your accuracy of the result will change. So, generally what we do that we generally put this x 0, x 1, x 2, x 3 values either increasing order or decreasing order, the instead of making it arbitrary even though this formula can be applied in an arbitrary arrangement of this x 0, x 1, x 2 and x 3 ok.

(Refer Slide Time: 05:35)

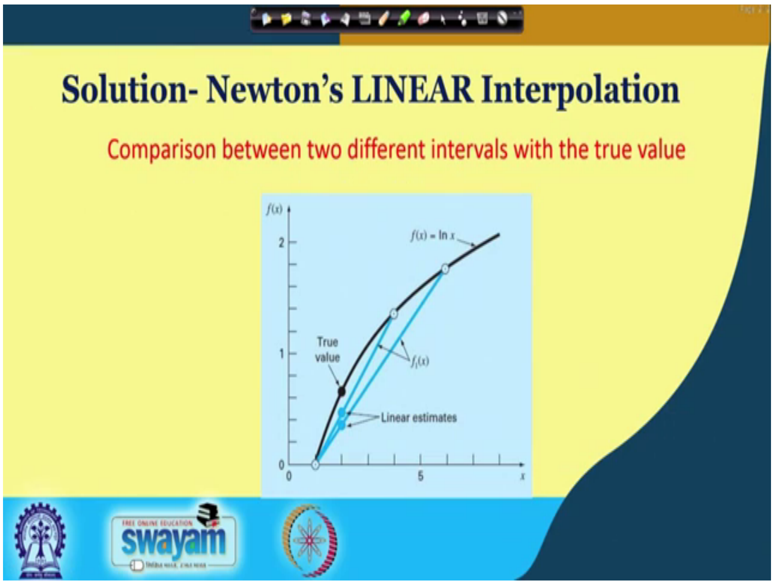



Now, let us see that let us suppose that this we have the given curve like this ok. Now, please understand this given curve may be the actual curve which we are getting from some experimental data points or some theory or this may be regressed curve also ok. That now you see that how the directions come in to picture, that I can generally this curve either from theory or from regression from experimental data.

And now, what we have to next is that suppose, we are we have to find the value at some point x here given the points x 1 and x 0 ok. Now, you can see if you are doing a linear interpolation, you can see that the value which will be given by this interpolation may be somewhere here because we are linearly interpolating it, we are assuming a straight line between this x 0 and this x 1 point straight line. So, this is the value which we shall be obtaining at some unknown value which is lying between x 0 and x 1. However, you see the exact value at this point is this one.

So, you can see that depending on the linearity or non-linearity of the given function this is a given function the accuracy of the order of the interpolation will differ. So, in this case you can clearly see that linear interpolation is not giving us a very good result, but anyhow we shall see that how we are going to go with this linear interpolation first. So, this is the formula you can easily find out from the linear interpolation.

(Refer Slide Time: 07:13)

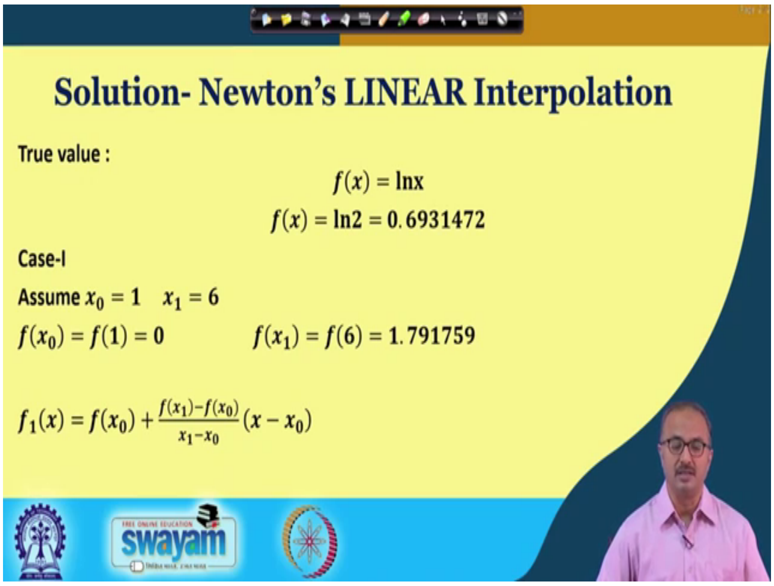



And let us go about and we will see that depending on that the range we take for the interpolation. Suppose you can take suppose this is a given functions say log x. So, either you can take a very large range or you can take very small range and this is the value you have to find the value of the function.

So, if you take a very large thing you see that this is the value you find ok. So, this is this is where you want to find the value of the function, but if you take this larger range you get this particular value. And now, if you make the range smaller you find that you have improved the value from these value to these value why because the error between the actual value and the estimated value is coming down ok. So, that is what I was telling that what kind of range you are choosing and what kind of functionality you have these two will determine the accuracy of your result. So, you have to be very cautious whenever you are going to interpolate that what range you are going to adopt.

If you have some understanding of the nature of the behaviour of the dependent variable on the independent variable a priori that means, before you start doing the interpolation sometimes we get this kind of things from the theory itself, theory tells us that how the a particular dependent variable will change with the independent variable. For example, if you want to know that how enthalpy will change with temperature or pressure, how the entropy or the internal energy or density. So, all these things how they will change with the temperature pressure if you have some kind of understanding a priori from theory then you can judiciously select the range for interpolation.

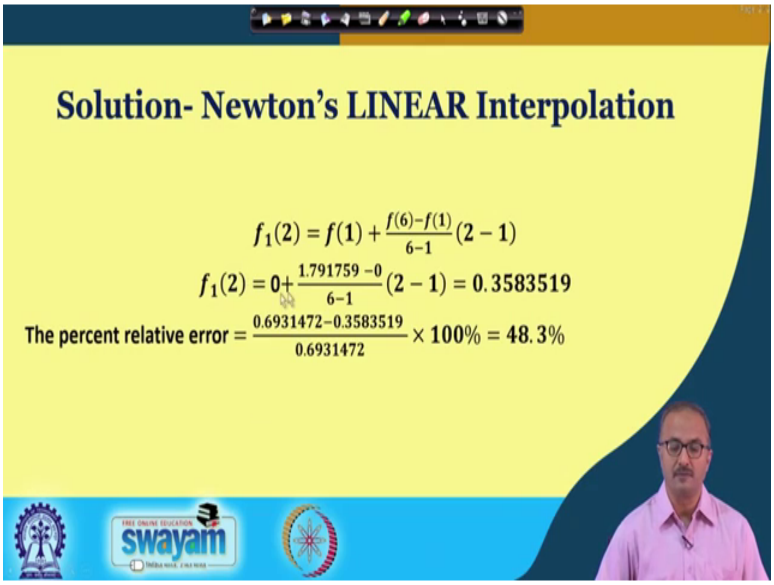
(Refer Slide Time: 09:03)




Now, let us see. Now, this we know that if our value is f x equal to ln 2 that is given to us. So, this is the exact value of the log 2. Now, let us first take that we are going for linear interpolation. So, let us arbitrarily choose two values of x and one we choose that x 0 is 1 and x 1 is 6. Now, you see that we have chosen a very large range and our value has to be found at 2 ok, but nonetheless let us do that. So, we find that from the given value given log x we first put 1 here and we find f x 0 equal to 0.

And then what we do? We put this 6 here and find the value of this f 6 as this, Now, we apply this formula for the interpolation ok. Now, we put plug in this values of f x 1, f x 0, x 1, x 0 etcetera and this x we are putting the value of 2 ok.

(Refer Slide Time: 10:03)

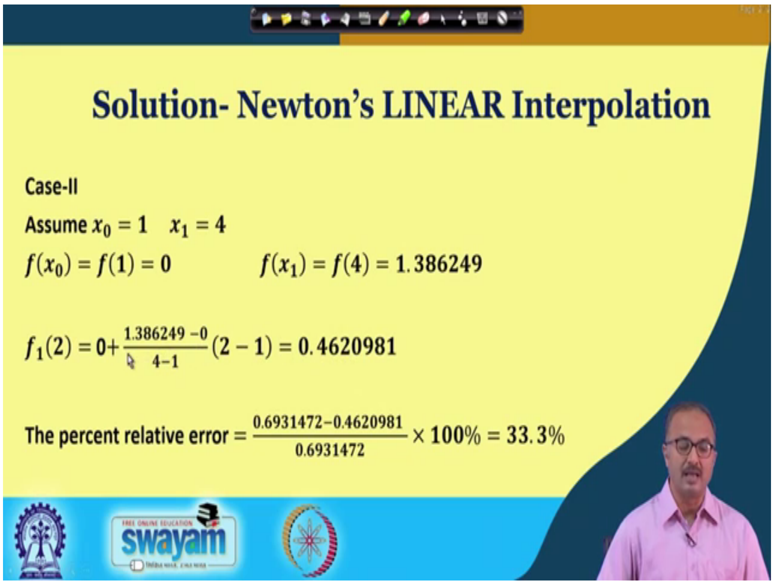



And now, what you find? That after solving you get that this is the value of to log 2 by interpolation. And now, you see the exact value was given like this and this is the estimated value from interpolation and see that we are getting such a large deviation this is a percentage error we are getting ok.

Now, let us go try out another one. Every time this is this can be known only if we know the exact value. Now, suppose you do not know the exact value what you should do? In that case another way is this you choose some arbitrary range initially, you find the value of the particular function at the given independent value. And then what you do? You again choose another range and you decrease the range by say half again you find the value of this particular function ok.

Now, by doing again you now, what you do that you just compare the values you obtain for the function for the consecutive ranges. If you find this is coming almost same then you can say perhaps the any of this ranges would do and you can take that to be the right value. But if you find there is a big deviation between these two values again you go for another smaller range may be. So, in that way slowly and slowly you keep on changing the range and keep figuring out the relative deviation between the values at two consecutive ranges and that way you sort of go to the I will not say right value, but the some conversed value ok.

(Refer Slide Time: 11:53)

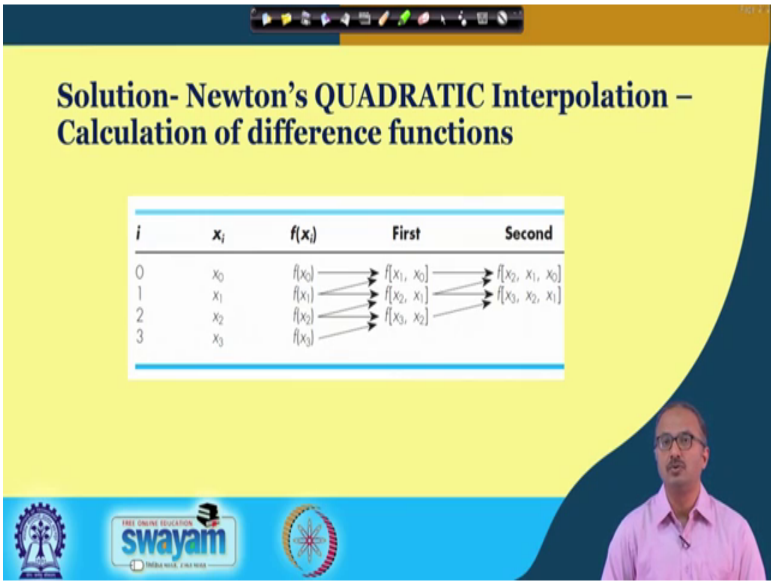



So, now what we do that we go for another range. Here we choose x 0 same as previously what we have done as 1 and now, we take reshrink the range to 4. Now, again we find the value of f 4 that is log of 4 and this is the value. We plug in for this in this formula to find the value of f 1 2 and we find this value.

Now, please understand here that as I told you that x 0 x 1 an arbitrary, so that means, you can put this as x 0 and this as x 1 2 ok. So, if you put this as x 0 and this as x 1 you have to make the appropriate changes in your formula ok. So, this choice of the x 0, x 1 is arbitrary just you have to modify the formula, but whatever you choose you will find that the values of the interpolated values will be coming almost the same ok. So, here we have chosen this values of x 0, x 1 and we get the value of this log two at by interpolation here.

Now, we see if you compare this value with the exact value we find now, we have brought down the relative error to this, but it is still quite high ok, it is still quite high 48 to 33 it is still quite high. So, if you want to still make it more and more accurate you just keep on shrinking this range and you will find that after a few iterations you will be able to figure out that what range is suitable to make the interpolation.

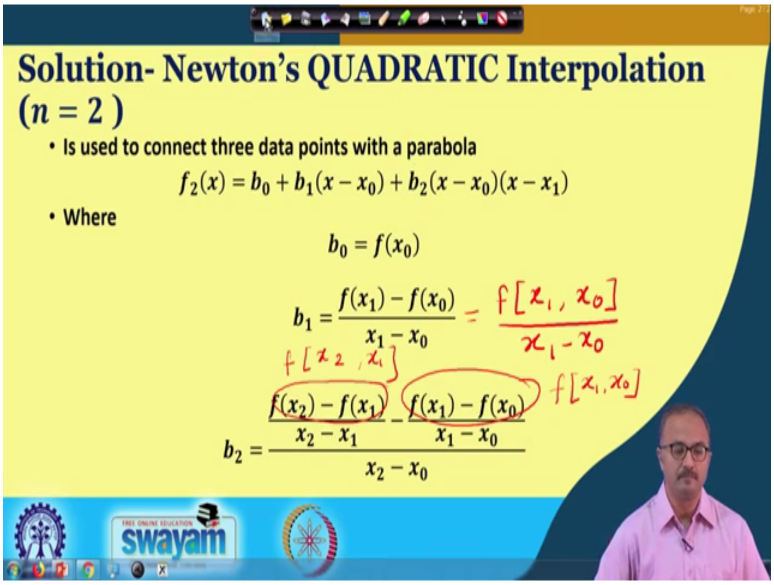
(Refer Slide Time: 13:31)




Now, we go to a quadratic interpolation. Quadratic means we shall be now fitting a polynomial of degree 2 to the given values. Now, you see that when you talk of a putting a fitting a polynomial of degree 2 that means, you would need to plus 1 that is 3 points, there from that only you can get a polynomial of degree 2.

So, here we again we construct same thing that this is the first difference we have found out and the first difference now we find the second difference, so that we can fit a polynomial of degree 2. So, again we find this x 2, x 1, x 0 like this that we are taking the difference between this and this and x 3, x 2, x 1 we takes difference between this and this values ok. So, you can see that it is very easy to do these determination of this difference function if you put them in this tabular form ok.

(Refer Slide Time: 14:42)

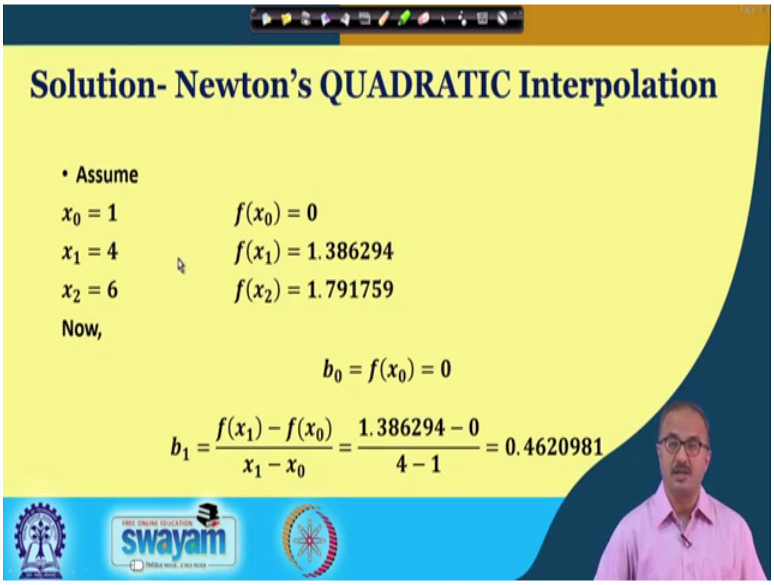



Now, after finding this second difference what we now do? That you go to this formula again and here you find the we have this value of b 0, b 1 and b 2 you have to find out. Now, you can find out the b 0 value easily; if you put x equal to x 0 here, what to find that f 2 x 0 then this is b 0 and is also 0 that means, b 0 is equal to f x 0 ok.

Now, what you do to find the value of say b 1? What you do? You put the value over x 1 here ok. Also put x 1 here you find that this will be 0, this goes. Now, what we left with? That f 2 x 1 equal to b 0 plus b 1 into x 1 minus x 0 and from that you can find the value of b 1 ok. So, you get the b 1 value as f x 1 minus f x 0 divided by f x 1 minus x 0. And what f x 1 minus f x 0? This is nothing but f or bracket x 1 x 0. So, this particular value is nothing but you can write this is that, if x 1 x 0 divided by x 1 minus x 0 ok. So, this is how you see that you can use the difference values right from that particular table.

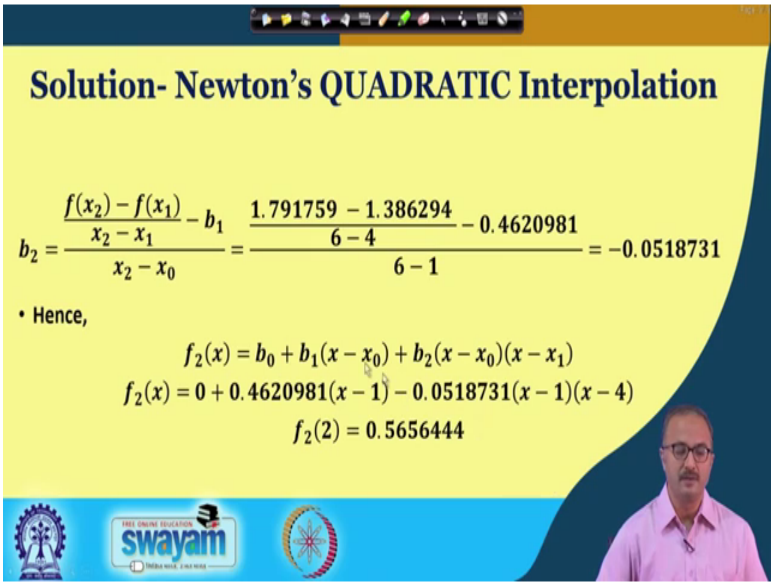
And then you go for determination of this b 2, for that what you do? You put x equal to x 2. Now, if you put x 2 what you know that this x 2 minus x 0 you know the value of b 1, you know the value of b 0 f x 0 and this is the x 2. Now, you can easily, now manipulate this equation to get the value of b 2 like this that f x 2 minus f x 1, where x 2 minus x 1 minus f x 1 minus f x 0 divided x 1 minus x 0 divided by x 2 minus x 0. What it basically means is this you are getting this value like this here this particular thing is nothing but f x 2 x 1 and this is f x 1 x 0 ok. So, you are able to get these values.

(Refer Slide Time: 17:07)



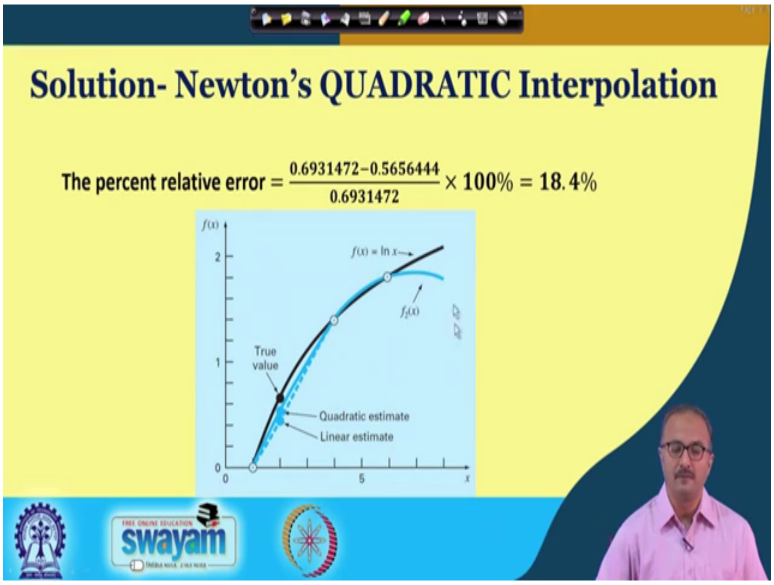

Now, let us assume when we go back to the problem does assume that x 0 equal to 1, x 1 equal to 4 and x 2 equal to 6. Now, we are taking 3 points to get the value of lon 2. So, as usual that f x 0 equal to 0 and f x 1 that is lon 4 equal to this and f x 2 that is lon 6 is equal to this ok. Now, as we have found earlier that b 0 equal to f x 0 that is 0. So, b 1 we get the value of b 1 like this ok. And we get the b 2 like this ok.

(Refer Slide Time: 17:39)




And you we find that if you plug in the values of this is the value of the f 2.

(Refer Slide Time: 17:50)

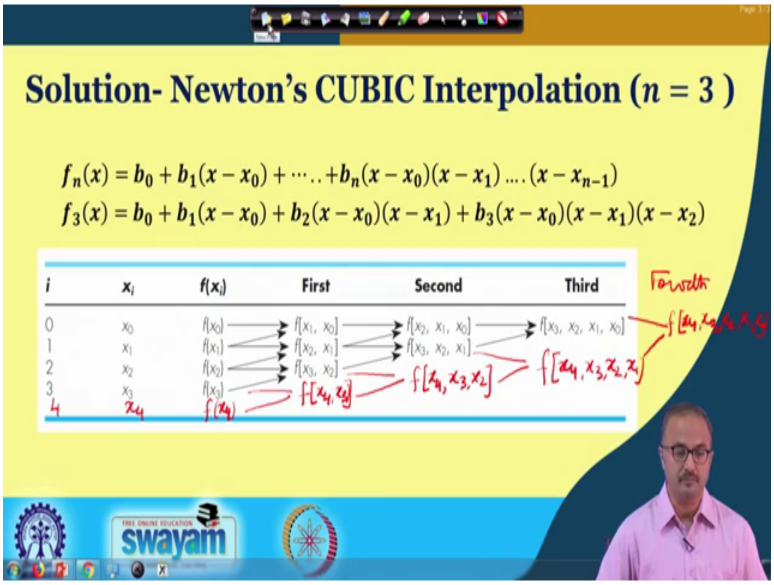



Now, you see that if you compare this value with the earlier exact value you find. Now, by doing a quadratic interpolation we have been able to reduce the percentage error, but still it is quite high, it is still 18 percent. So, we can again play with this ranges ok. Now, here we have shown that this is the this black thing is showing you the exact function.

Now, what we are doing that we are taking 3 points this is one point, this is second point and the third point ok. Now, if you at linear interpolation we find that when we do linearly this is the estimate we are getting and when we do a quadratic interpolation we have been able to improve the values ok. Now, that means, for that by increasing the degree of the polynomial we are able to have a more accurate value.

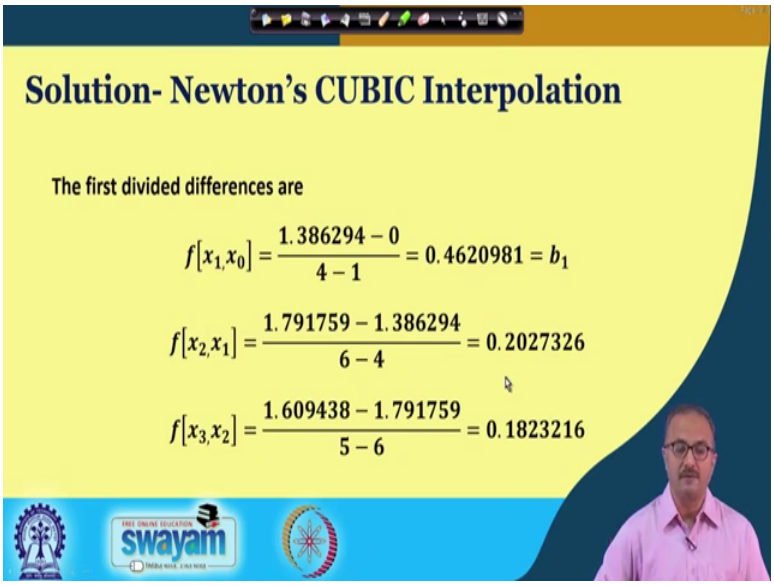
But you must also understand that as you increase the degree of freedom sorry degree of the polynomial for interpolation you also are paying for the calculation ok. So, you have to be very cautious that whether you should go with higher degree or whether you should stay at a lower degree, but keep shrinking the this range for interpolation ok. So, this has to be decided case by case basis.

(Refer Slide Time: 19:09)



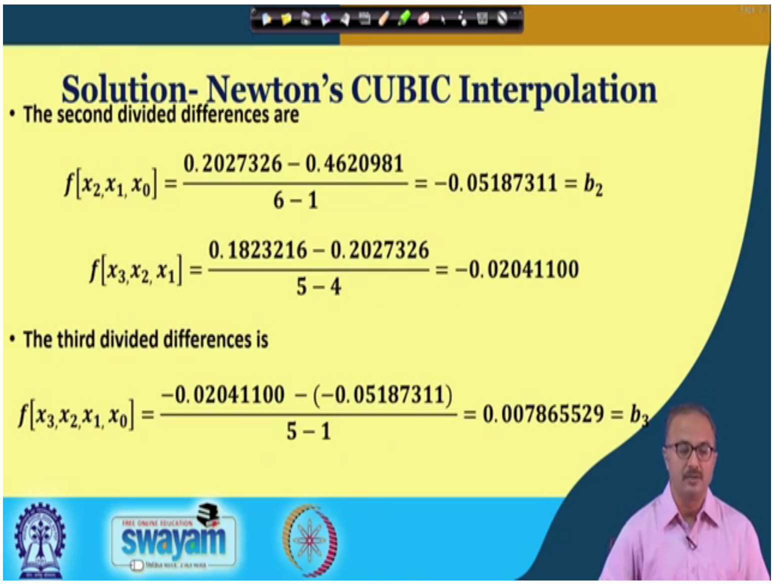

Now, we go for cubic. Now, cubic, now we are going for this third difference. Again, you can see that how this third differences coming ok. Now, you see here are this is a formula over here and here we are getting the difference table we are making. So, here we have the first difference that is between these two and the second difference between these two and third difference between these two.

(Refer Slide Time: 19:34)



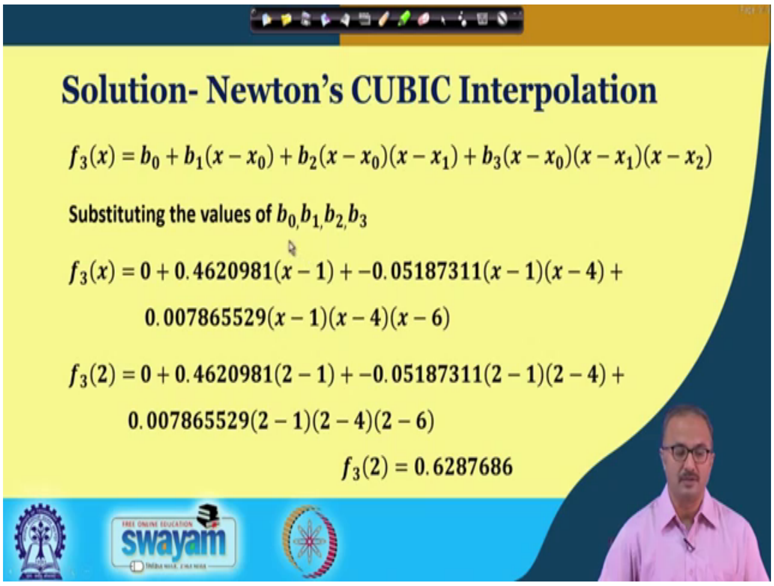

Now, you can see that after making these things you can again find this divide the difference you can find out this b 1 etcetera as I explained to previously ok.

(Refer Slide Time: 19:42)



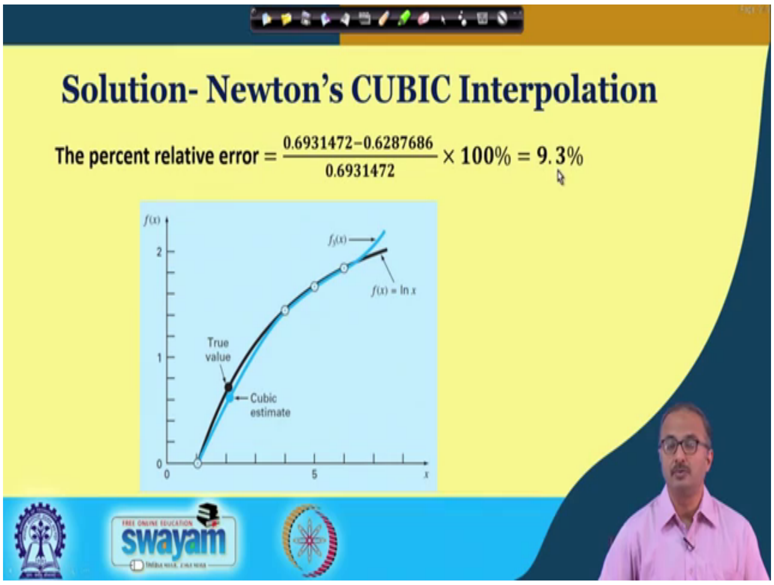

Now, you can find this second divided difference here, then third divided difference here and these are the values of b 2 and b 3 ok.

(Refer Slide Time: 19:54)




So, from this differences what you find that ultimately you plug in the values over here directly and you get the value of the function like this.

(Refer Slide Time: 20:02)



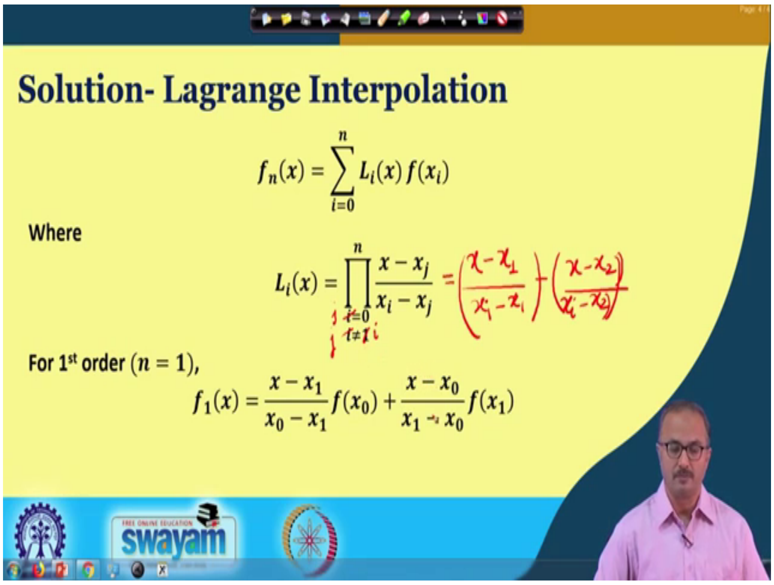

Now, what is the thing you have got? You have been able to reduce the error to 9 percent. Now, again you can see that, now this is the cubic polynomial we have done and here the estimate has coming come to still nearer to the exact value, but it is all coming at the cost of the calculation steps ok. Now, you see that you can, now we can go for a fourth order polynomial. Now, it is a fourth order polynomial you can see that you have to consider another point that is you have to consider 5 points ok.

The same thing will be there. Now, if you have to have that fourth order polynomial, then what you can do that if I go back to this difference table you can see that here you have to have another point over here. Suppose I equal to 4 you put here then it is x 4, then if it is f x 4, and then these two will give you that f x 4, x 3 and then these two will give you f x 4, x 3, x 2 and these two will give you f x 4, x 3, x 2, x 1 and then you have fourth divided difference and this will give you ultimately f x 4, x 3, x 2, x 1 and x 0 ok. So, you can see now you can see that this is the way you can proceed with higher and higher degree polynomial.

Now, how much you should go? That will be decided that what is the difference in the accuracy you are getting and how much you are paying for the calculation. These two will generally dictate that how much means which degree of polynomial or which degree of interpolation we should be applying in a given case ok. Sometimes you find that many a times we do linear interpolation only, it is very popular that we choose a very small range and we get an one iteratively we get the value of the dependent variable at the unknown that some intermediate independent variable value.

After this is the Newton's divided difference. In this case what we are seeing that we are taking these differences generally in a very means methodical manner as I told you that we put the things in the decreasing or increasing order. Another way is this the another formula is by Lagrangian interpolation. Here you see that this x 0, x 1, x 2 this can be chosen arbitrarily ok.

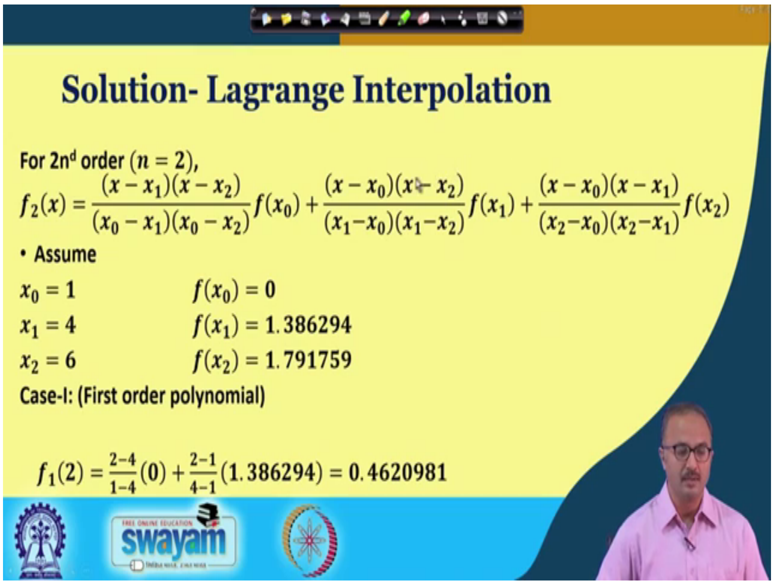
(Refer Slide Time: 22:56)




Again, without going into the detail of the particular derivation just you understand that this is the way we are putting the Lagrange formula. Here again we have a nth order polynomial, here we have write like this and this L is the Lagrange from polynomial, here you see that it looks like this ok. This is a this particular this a capital pi the show the product, so this product means it is if you expand this product it will look like this, that it will be x minus say if x 1 divided by x 1 x i minus x 1 into x minus x 2 divided by x i minus x 2 like this it will go ok. Now, this way it will go on. So, it will be it will be here g over here, g is not equal to y ok. So, this it will go do this sight what we have to do. So, this the way it will go on ok.

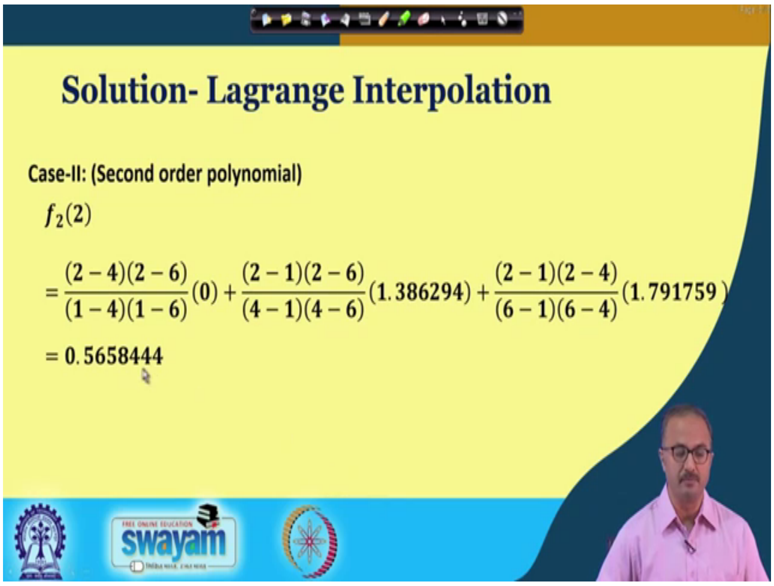
Now, once you do this then you find that you can easily find out the for the first order again you can have several different orders if you take n 1, n 2, etcetera. So, this is the first order thing you can find out and we shall be applying this formula also in our thing.

(Refer Slide Time: 24:46)



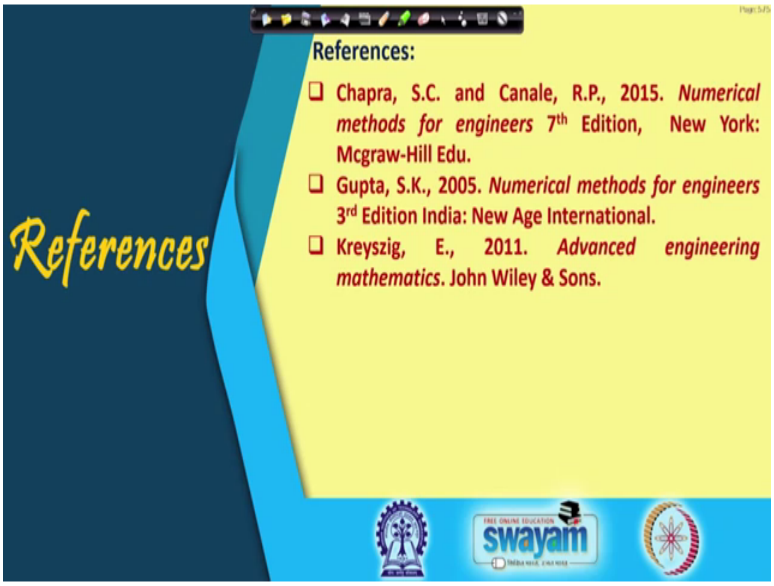

Now, this is the second order formula. Again I am not going to detail of this things. So, let us first go with this thing that we take that if we take a second order first order polynomial if we take if we take it to be, this 1, 4, 6 if we take here, so we find that f x 0 equal to this coming and here we plug in the values over here and we get this value of the thing a lon 2. And this is the first second order polynomial. You can easily see that our value was some 0.61 something and it is we are getting by first order we are getting 0.46 and by second order we have improved to 0.56 ok.

(Refer Slide Time: 25:21)



So that means, we are what we are finding that in this Lagrange also, we are able to get a better value if you are increasing the order of the polynomial. And similarly, we can keep on going for higher and higher order using the Lagrange formula.

(Refer Slide Time: 25:43)




So, these are the books which you can refer to for further explanation on this theories.