Mass, Momentum and Energy Balances in Engineering Analysis
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Lecture - 35
Microscopic Balance Illustrations – V

Welcome. Today in this lecture we shall be covering a few more illustrations on the Microscopic Balances.

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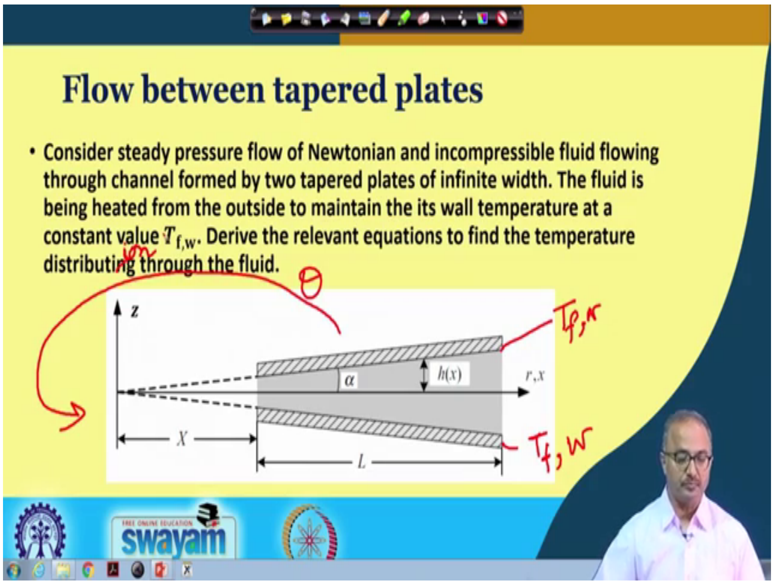

So, first problem we shall be considering will be flow between tapered plates. Now you see that in many applications industrially or in our research, we find that many times the flow does not always happen over a conduit or over a free or a unconfined surfaces. A sometimes these flows are also confined within some plates. Like for example, we may have a flow through 2 concentric circular plates, through which some flow is going and this kind of flow configuration in a similar fashion you find that is used for constructing some heat exchangers or some mass exchangers, and there we get some kind of change in the heat transfer and mass transfer characteristics.

Also you find that this kind of flow between 2 channels are also found in some viscometers, where we are having some kind of fluid and we are trying to rotate one particular plate over another. And during that we measure the stress that is obtained and we find the from some formula, we obtain the viscosity. So, in that we will find that there are many applications, where the flow happens through 2 plates and these plates may be parallel or they may not be parallel. So, they may be sometimes tapered. If you look through the flow through some kind of nozzle or this diffuser for example, if you can this nozzles or diffuser are used in the space shuttles to get the thrust, to lift the particular space shuttle. Also you find that to find the velocity or flow rate you use the venturi meters which is also kind of converging diverging sections.

In that case also you find that you have the particular flow going through this kind of tapered surfaces ok. So, in consideration of this, the this particular problem has been taken to figure out that how we are going to model the particular process with if the process involves some kind of heat transfer or mass transfer, how we are going to model the process. And so far in our earlier lecture, if you have mostly confined ourselves with the Cartesian coordinate. Now in this particular case we shall be looking into the cylindrical coordinate. So, this problem is like this that, considered steady pressure flow of Newtonian and incompressible fluid flowing through channel formed by 2 tapered plates of in finite width.

What it means that, a pressure flow means this the flow is occurring due to some kind of pressure gradient. As we have seen earlier like quiet flow in quiet flow. The flow occurs not by any pressure gradient by, but by the relative motion of the 2 surfaces. So, it is mentioned in this particular problem that it is due to some kind of pressure gradient. And what we mean by infinite width is like this that, if I draw these plates a kind of 3D it would look like this. So, you can imagine that we have 2 plates of like this. So, this particular width is taken to be infinite ok. So, this is this. So, we have these 2 plates. So, we have 2 plates like this and through which this particular thing is going out.

Now, when this fluid is moving you can see that, there could be because you this tapering is there it may be assumed that there could be some kind of if I there is axis here this here you can see that the axis over which this particular thing is there. So, these plates have some inclination of alpha over here ok. So, this is just giving that the angle of the tapering and this you can see that the fluid height is changing as it is moving there. So, whatever coordinate system is being used, the origin is taken on this axis ok. The or this should cannot show me confuse that we are not taking this as the origin, but we are putting some axis around which as if this particular thing is as if it is rotating.

So, with this consideration we are saying that, here we have shown the z axis and r axis. So, r is corresponding to the radial distance from this particular axis and theta is this direction. Theta is this direction around the axis though these are theta direction and this is the r direction is the z direction. So, in this fashion we can see that, if we the question asked is that the fluid is being heated from the outside to maintain its wall temperature at a constant value T f w; that means, as if you are having here we have a value of T f that is fluid at the wall ok. So, similarly on this side also you have T f wall temperature ok.

So, this is; that means, this is a glitch that kind of condition. So, it is assumed that the fluid is at a different temperature from the wall. So, there will be some kind of heat transfer from the wall to the fluid. So, derive the relevant equations to find a temperature distribution into the distribution through the fluid. Now because the fluid is moving so, we know that the in even if you want to take the temperature distribution, it is that you have to consider the energy balance equation. And in the energy balance equation we have the inertial term that is due to the flow of the fluid. So, if you want to find out the temperature distribution, we should also know the velocity profile that will dictate the temperature distribution.

So, what we shall do now we shall see that how we are going to make the balance equation.

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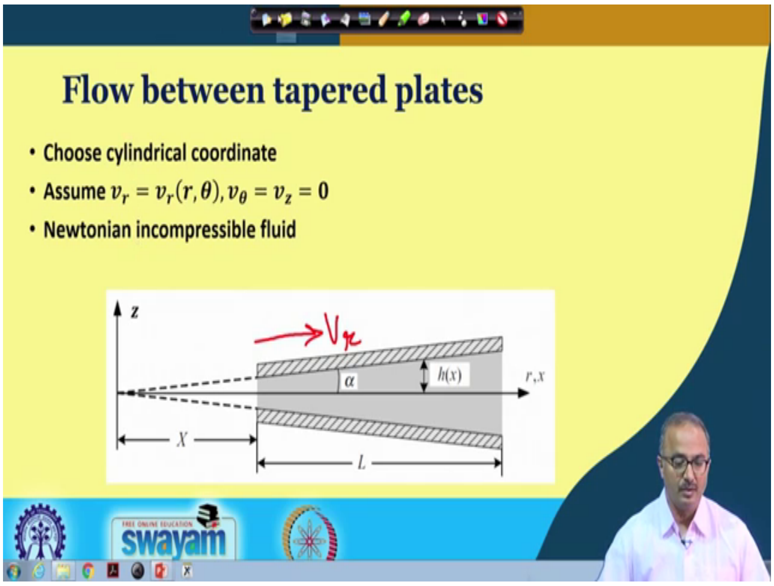

So, first we shall go with the coordinate axis that we have chosen cylindrical coordinate and here we are assuming that v r that is the radial velocity, radial velocity is in this direction these are radial velocity direction. So, this is v r. So, this v r is taken to be a function of r and theta and v theta and v z are taken to be 0; that means, we are considering a one dimensional flow. Also we are considering the liquid to be Newtonian and incompressible; that means we are assuming that the density is remaining constant or also we make one assumptions because nothing is given in the particular problem. So, if it is a gas you may take the viscosity to be a function of the temperature or for a liquid for some small range of temperature, you may consider the viscosity to be a constant, but these are not very restrictive assumptions.

So, let us now go further.

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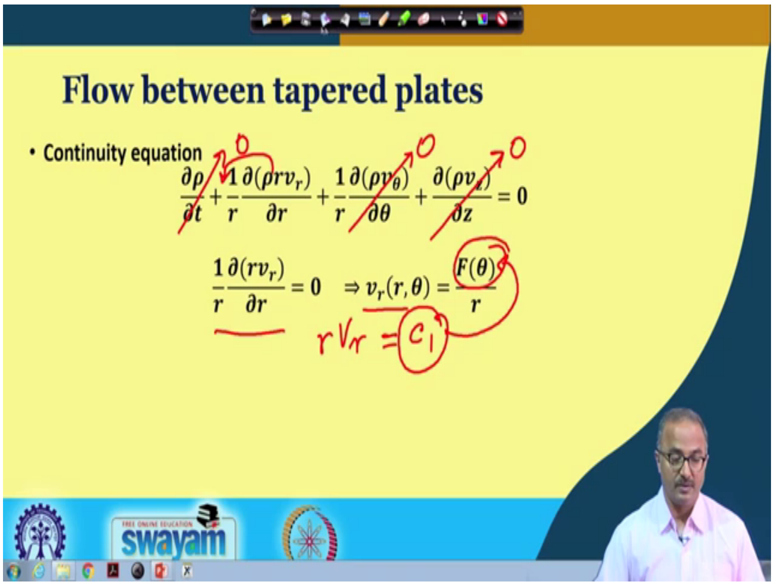

So, here we first go with the continuity equation. Because we always know that mass balance has to be maintained; so, we go with the continuity equation. Now you see that in the rest of this particular problem, we shall be sticking to the cylindrical coordinate system. So, you see that we have taken this cylindrical coordinate system the continuity equation, and here we are assuming that there is no variation with respect to theta and the z. So, that these 2 terms are taken to be 0 and because it is incompressible. So, we are taking this to be 0 and this rho is taken out of the differential.

So, with this we find that we are obtaining this particular equation. And from this we find that it means that r v r is some kind of a constant and this constant is a function of the theta ok. So, this is this f theta signifies this particular constant this is a function of the theta. That is why we are finding that v r theta is equal to F theta by r; that means, the v r is going to decrease with an increase in the velocity sorry increase the radius ok. Now after writing this particular thing for the see these will be generally used later on when we go for the solution of these equations we will be needing this particular relationship. Now we go for the momentum balance equation.

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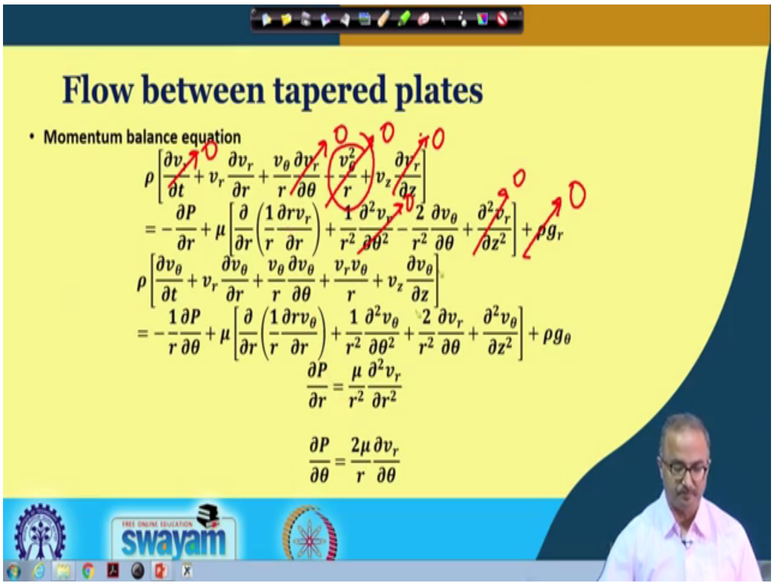

So, this is the momentum balance equation for the r direction, this is for the theta direction and we have another for the z direction.

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So, we shall come to that later.

So, here we have the r direction, now again you see that in this we are assuming as per the problem we are assuming it to be at steady state. So, this particular term will go to 0 and then we are assuming that it is independent of the theta or it is axis symmetric. So, this particular thing goes to 0 ok. And this is also going to 0 because there is no variation with respect to z ok. And then this is also going to 0 in similar fashion, this is also going to 0 and here we have this particular thing you see that this in the problem we are I mean saying that the v theta term is 0; that means, there is no rotation.

Now, you can look looking at this particular equation understand that, this is something to do with the centrifugal force that is acting on the fluid in then and you know that centrifugal force always acts radially ok. So, that is why you find that whenever you are writing the radial momentum balance in the cylindrical coordinate, you always have this v theta square by r term that to take care of the centrifugal force. But in this case this is absent because as per the question this v theta component is 0 ok. So, and we are assuming that there is no body force. So, this particular thing in the radial direction is taken to be 0 ok. And so, we are left with what? We are left with well this particular thing v r dou v r by dou r, then dou p by dou r and this mu this thing ok.

Now, you can see that this dou v r by dou r if you go back to this continuity equation you go to continuity equation.

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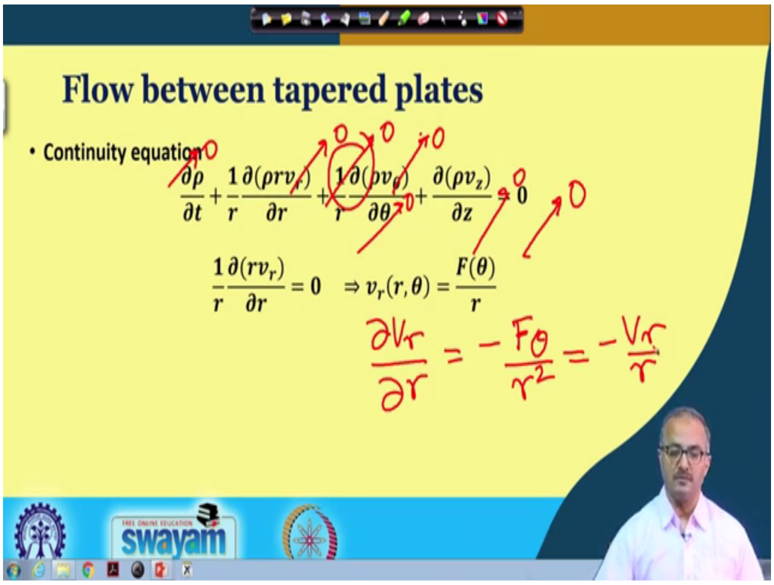

What you find that, dou v r is something like minus F theta by r square ok. So, this is dou v r by dou r and then you find that, F theta by r is nothing, but. So, this is dou v r by dou r is nothing, but minus v r by r. So, with all this we find that this dou v r by dou r will give you minus v r square by r and here we have this particular term. So, you can take put this you can expand this term and you will get this particular type of equation now you see that what is happening that in your this pressure drop is due to the viscous force ok.

And next we come to the theta direction. Now even though there is no theta component of the velocity, but you will find that in the theta direction there could be in this case you see that this is going to 0 here, this is going to 0 here, this particular thing going to 0 here.

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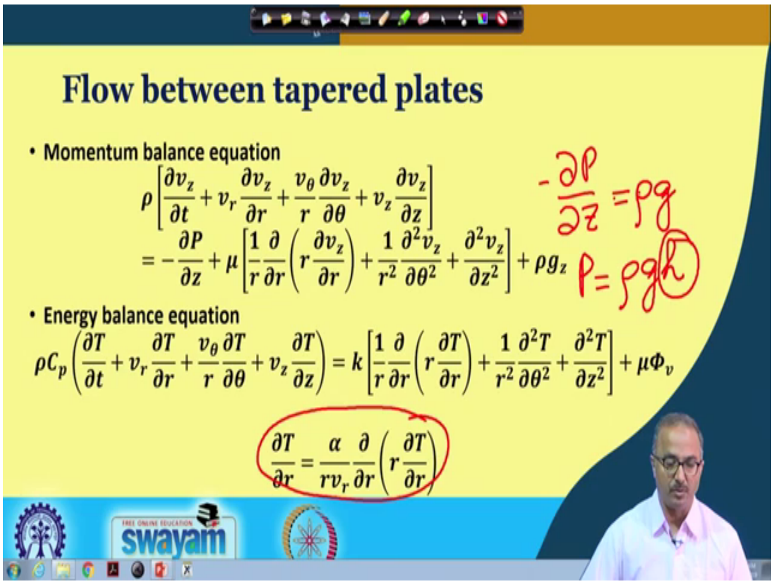

So, all these things will go to 0 and here you find another term is coming which I am explaining. So, you will find that this particular term is signifying the coriolis force. Now coriolis force is something which happens whenever there are 2 missions combined one is radial motion and one is the theta direction motion means. So, when you go to a merry go round suppose you know merry go round and we are walking on the merry go round, you will find that you will be experiencing a force that will try to not just throw you away, but also kind of either take you along the circumference direction. That means, if a liquid is flowing over a rotating surface radially, then a liquid will you may find that as if it is getting spread over the particular surface ok.

And this particular force is called the coriolis force. So, this is the centrifugal force, centrifugal force and this is the coriolis force ok. So, these 2 forces come into picture whenever you are having a motion in the theta direction and of course, in this case all these v theta components are 0. So, there will not be any kind of coriolis force and we see that only thing what is remaining is the this particular term ok. And if we take that v r is also constant with respect to theta, then this will also go to 0 so, that there will not be any kind of pressure drop along the theta direction ok. But in our problem we are taking that v r is a function of both r and theta. So, this will be a nonzero value. So, this will not be equal to 0 that is why we are we are have to even though v theta is 0, but due to this particular component we find that we there will be a pressure drop along the theta direction.

Now, let us come to the other direction that is a v z term. Now in this case v z you see that there is no term in this which is nonzero because see if you look at v z component in any way 0. So, all these things will be 0 and here you will also find that there is no v r term ok. So, there is no v r term. So, actually everything is going to be 0 and there is no we are also considering that there is no body force. Even though in the z direction there could be a hydrostatic gradient due to this gravity, but looking at the particular problem, we may say that the m the height of the liquid with confined within these 2 plates may not be substantial to exert enough hydrostatic gradient ok. With that logic we are saying that dou p by dou z equal to 0, but again you minded that this hydrostatic pressure drop will depend on the height of the fluid.

So, if you have substantial height of the fluid, then you can consider that particular thing and then it will be something like this it will be minus dou p by dou z equal to rho into g and from this you can get P equal to rho g h. So, if this h is very very small, then you can neglect the effect of the hydrostatic head. So, this is all about the momentum balance equation and now you can see from these equations you can find the velocity. Now this velocity will be required to solve for the temperature distribution and here you have it that you have now write down energy balance equation. So, on this left hand side you have the inertial terms and you see that again v theta is 0, v z is 0 the steady state. So, the that is this dou t by dou t will be also 0, then we are assuming that there is no gradient with respect to theta and z. So, these 2 terms are also 0 and in this case we are neglecting any kind of viscous dissipation. So, we are taking this particular term to be also 0.

And in also there are no reactions in this particular system. So, there is no other source term associated with this particular case. So, with these assumptions we find that the dou t by dou r is coming like this; that means, the gradient of temperature along the radial direction will be dictated by this and here you see that this alpha is the thermal diffusivity that is the k by rho c p ok. So, this is the equation which we wanted to derive to find out the temperature distribution within the fluid. So, you see that from this particular problem even though our main concern was to get the temperature distribution and which was to be obtained from the energy balance equation; however, due to the involvement of the velocities or the inertial forces in the energy balance equation, we need to go to the momentum balance and also the continuity equation.

So, that is how you find that, that or the whole problem becomes a bigger problem even though the concern is only for the temperature. So, if you can assume these velocities to be constant there is no gradient, then we need not solve the momentum balance we can just go directly to the energy balance equation.

So, I will be stopping here and as you can see that as for the given problem, the temperature to solve this problem we need 2 temperature boundary conditions. So, what we can do that, we can take one boundary condition near the inlet and 1 boundary condition at the outlet and because we are maintaining the temperature constant at the wall. So, we may say that the whole at the at the at any axis, we can assume that there is a good amount of mixing along the axis. So, the hole temperature is constant along the radial sorry the z direction.

So, for this particular problem solve you can give 2 conditions, one you can specify at the inlet and you can say another boundary condition you can specify at the outlet of the particular channel ok. So, that is how you can solve this problem, Next we go to a problem which is concerning the mass transfer. In this particular problem you see that the we have to consider the evaporation of a volatile component liquid A into a pure B in a tube of infinite length, the liquid level is always maintained at z equal to 0.

It is something like this that suppose this evaporation problem will find many a times like from the ponds, from the river, from the ocean you get the liquid is evaporating and in that case we say that there is infinite means that if I consider the atmospheric to be of a infinite length ok. So, what happens that, this particular liquid goes into the atmosphere and nearby the interface there will be some gradient, but as we move away from the surface of the liquid, we will find that this gradient in this concentration of the particular species which is evaporating will also come to constant.

All for example, you find that when we are trying to dry some kind of material or clothes what we do that, we find that in the we put some fan to make it a dry faster and why we do that? Because when this particular component is evaporating from the weight surface, it forms a mass transfer boundary layer near the surface and that creates a resistance to the mass transfer or evaporation.

Now in the more the humidity in the atmosphere, we find that it takes longer time because this particular boundary layer thickens. Now why we run the fan is that by running the fan we are trying to destroy the formation of the boundary layer over the rate surface. So, that we can maintain the large enough gradient for the solute to or the in this case water to go from the weight substance into the atmosphere.

So, this is the kind of problem we consider and we see those kind of how to model those kind of problems. So, here we have to develop appropriate balance equation to determine the concentration profile of the evaporating component A, and the assumptions taken out that temperature pressure remain thus constant and vapor of A and B from the ideal mixture. It is not a very restrictive assumption, but by putting ideal mixture we just we are just trying to simplify our analysis. Then diffusion coefficient is taken to be constant species B is insoluble in A. It is something like this that suppose when water is evaporating into the atmosphere, we assume that the components in the air that is if majority of them is nitrogen and oxygen, we assume that this nitrogen oxygen are not coming back into the water. So, from practical purposes we assume that the air is insoluble in water ok. So, that is the meaning of this particular assumption the overall concentration of the species remains constant.

Now, let us go.

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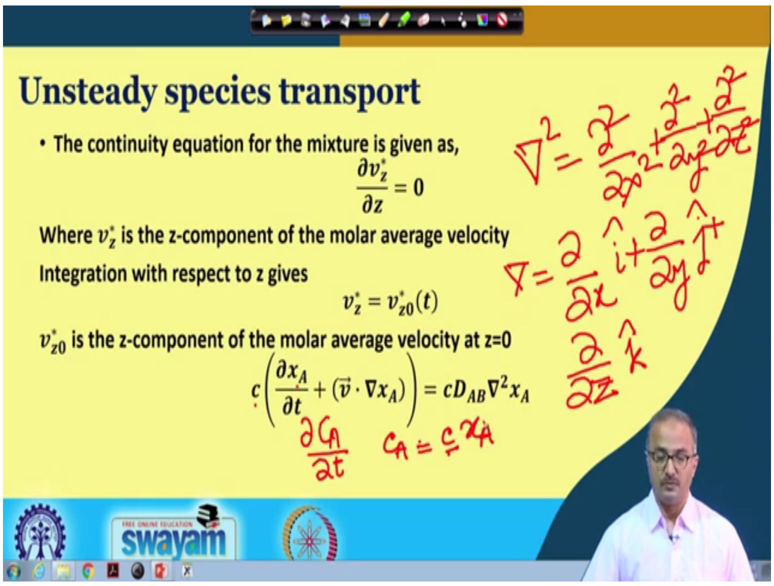

Now, if you look at a continuity equation you find that we are assuming only the movement in the z direction that is in the axial direction. So, we are for a timing we are not considering the spread of the solute or in the theta direction. So, we are find then having that dou v z by dou z equal to 0 ok. And here we are this v z is the z component of the molar average velocity, because molar average because we have both A and B components together because even though A is going into the thing, but once is go into the B phase, then we find that both A and B combine and that the velocity of the A will be dictated also by the presence of the B and these things are covered in more detail in the mass transfer analysis. So, for the timing I am not going into those details.

So, from this equation we find that v z star can be written in this particular fashion ok. And then we find that these v z star is the velocity v z at z equal to 0 ok,and then we go for the species balanced equation. So, this is the overall mass balance the species balance equation and a species balance again we take the inertial term that is this inertial term and this is due to the diffusion term and there is no reaction between A and B. So, that is why we are writing that dou x by dou t equal to this and you understand that, there is dou x A by dou t here, it is because we are assumed that the overall constant remains the same. So, this it is essentially dou C A by dou t, but then this C A is taken as the overall concentration into x A ok.

So, this is taken to this constant. So, this is going out of the differential and we are having only x A over here and x A over here. So, this is the basis of writing in terms of the mole fraction. And this nabla x A as you know this nabla is nothing, but the gradient of x A. So, nabla is you know that this is say in the Cartesian coordinate it is dou x then ok. So, this is nabla. So, this nabla into x A gives the gradient of x A ok. And this one is the delta square nabla square meant it will be the scalar. So, this is this is the nabla square is dou square by dou x square plus dou square by dou y square plus dou square by dou z square. Please understand we are not writing in I j k over here because this is a scalar ok.

So, this is how we are able to set up the material balanced equation for this condition. So, here we see that, when I am writing those x that should not be confused with the x A that is being used to represent the mole fraction that you should be careful about. So, this x A is the this x A is the mole fraction this is the mole fraction ok. And when we are writing only x that is the signifying the coordinate direction this should not be confused. Sometimes this mole fraction is also represented in terms of y A ok. So, generally what happens that when we talk of the gaseous phase, we put in terms of y and when we talk about a liquid phase we put in terms of x. So, these are some conventions.

So, now let us see that this particular problem would be completely defined only when we have the initial condition and the boundary condition, because here we find that this particular equation has an unsteady state term and also it has some differential with respect to the coordinate direction. So, let us now see how we shall be specifying theinitial condition boundary conditions.

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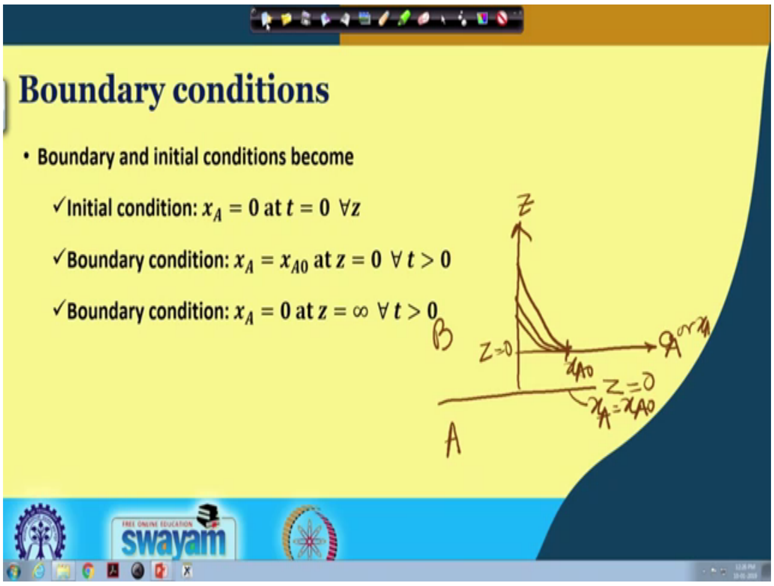

So, here we have the initial condition that as if at in the phase B there is no exit to start with. So, we are putting x A equal to 0 for all z and understand this, this z is being taken from the interface between A and B and if this x z is moving inside the B from the interface of A and B it is moving inside the B ok. So, that is the meaning of that all z and then for the boundary condition we are assuming some kind of interfacial concentration, interfacial concentration of A at the boundary that is given by x A is 0 and that is the meaning of z equal to z 0 and for all t and for z equal to infinity; that means, at a large distance from the interface, we are assuming that there is no x a; that means, it is going to 0 ok.

So; that means, if we can without any solving, we can say that if we plot suppose we are plotting. So, this is z equal to 0 and this is z is going to in this is z direction over this is straight line. So, if we are seeing the see the concentration we can check that this is as if going to it is it is some x A x A 0 value here and it is going to a 0 value this is. Now you see that in this particular expression do not confuse this x A with the coordinate direction, this x A with this subscript signifies it is the mole fraction of component a and in the nabla operator also we wrote the x, but in that case the x is the direction of the coordinate and many times in the literature you will find that we are using either x or y to represent the mole fractions, but generally these x and y will be accompanied by a subscript ah. So, that will make it different from the coordinate axis.

Now, this problem can be solved only when we have the appropriate initial and boundary conditions and because it is a unsteady state situation. So, we need some initial condition and in this particular thing we find that the we have the second degree in the derivative, and because it is one dimensional in the z direction. So, we need 2 boundary conditions in the z direction. Now in this case you see the z is taken to be the interface where between A and B, and the positive z direction is taken to be going inside the B phase from the interface ok. So, with this considerations let us go to see how we are going to specify the initial condition and boundary conditions. So, initially at t equal to 0 we say that there is no A present in the B phase or in the gaseous phase. So, with that consideration we take that x A is equal to 0 for all z.

And for the boundary conditions we say that at the interface that is at z equal to 0, we have x A equal to x A naught some specified value and for all t more than 0. And then the another condition we take that at z equal to infinity that is much away from the interface, the amount of the A is taken to be 0; that means, that is it is nothing is present. So, this can be all these things can be represented like this that if I consider that this is the interface this is the z equal to 0, and from here I am count this is the B and this is A is coming from here and this is the z direction ok. So, here we are taking x A at this position we are taking x A equal to some x A naught x A naught.

Now, here you understand this is that when I putting x A naught it means that this A is not a pure component it is in a mixture and we are assuming that other than A no other component in the liquid mixture are volatile. So, only A is the only volatile component which is going into the B ok.

Now, suppose this is this I also take as the suppose this is I take as the coordinate axis for C A sorry or x A C A on x A suppose. So, if I if we make the profile of the x A. So, we will find that from this suppose this is z z equal to 0. So, we find the profile will look something like this ok. So, it is C A is 0 here. So, and this is the value we call it x A x A naught ok. So, this is how and this profile will be a function of time this was a function. So, you will find that you at different times you will have different profiles like this ok. So, this is the way we visualize that this is a way this profile of the A component will be developing with time.

So, this is a qualitative understanding of the concentration distribution of a component which is evaporating in another component. So, with this we can complete the formulation of this particular problem to get the concentration distribution of n evaporating component.

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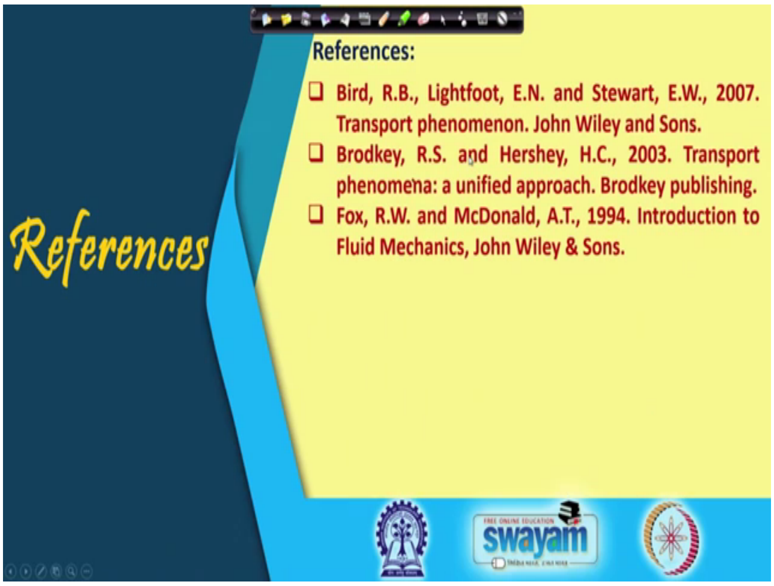

These are the references, which you can consult for more detail of this particular processes.

Thank you.