Mass, Momentum and Energy Balances in Engineering Analysis
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Lecture – 33
Microscopic Balances Illustrations – III

Welcome, today we shall look into some more illustrations on the microscopic mass balances.

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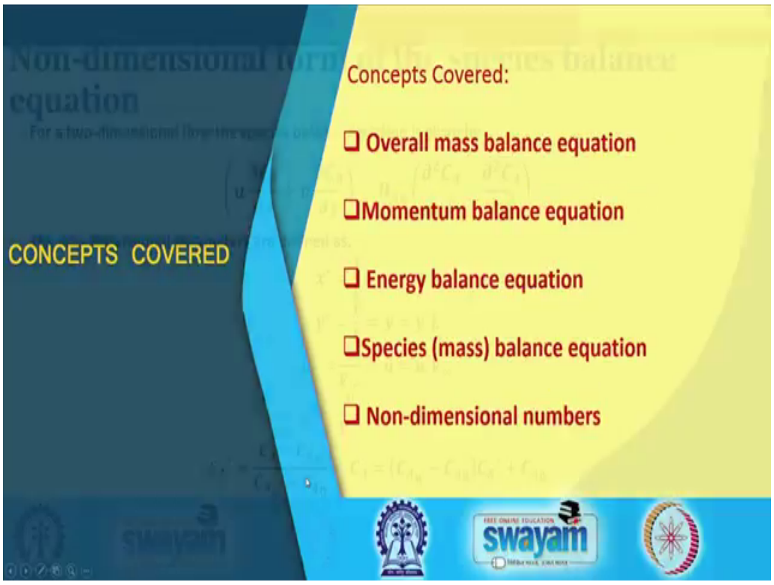

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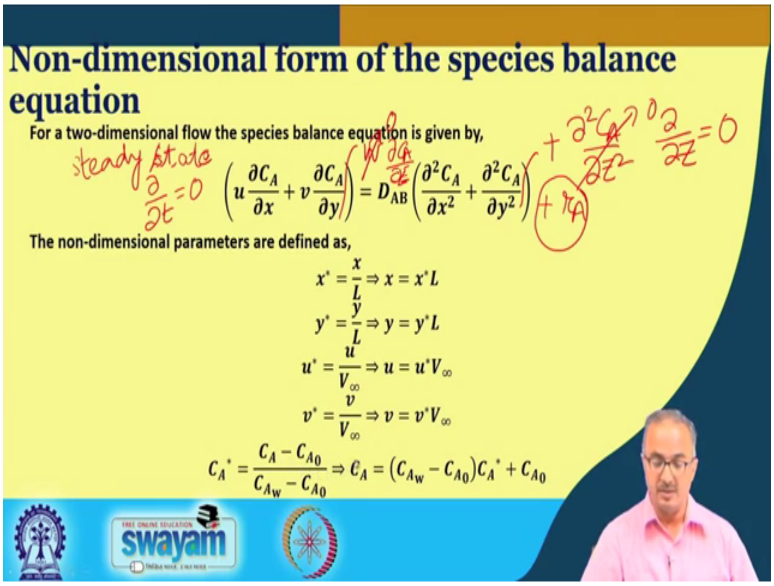

As we have done in our earlier lecture, we took the steady state two dimensional momentum balance and the energy balance. Now, we are now going to deal with the species balance or the component material balance. So, here we have a steady state two dimensional species balance in the flowing system.

So, for this we can reduce the, this is a steady state problem we have steady state problem here. So, because of steady state we know that our all the rho partial derivatives with respect to time will be 0 and again we are assuming the all the w components to here at will be A w component also and we are taking to be two dimensional flow.

So, we are taking the z to be 0 this w to be 0. So, it is not appearing and similarly here we will be another term that is dou 2 C A by dou Z square ok. So, this term is also taken to be 0 by taking that dou by dou Z is equal to 0 ok. So, in this case we are also assuming that there is no reaction so, that there is no generation or consumption of the species A this A is signifying the species here. So, there is no consumption and all the generation of species A otherwise, if there is any kind of reaction by which the species A is getting generated or consumed then we can write that kind of a term over here ok. So, this will signify that there is a generation or consumption.

Now, with this assumptions now again as we have done earlier that we are first non dimensionalizing the equation. So, we define the non dimensional x and y coordinate like this way and you can see that these are also normalized. And here we are defining the u component and v component of the velocity in a non dimensional fashion in this way. So, we are getting this non dimensional x, y, u, v, and for the concentration again we are putting this kind of a non dimensional equation. Now depending on whether is C A is bound between C A naught and C A w or naught we shall be having either normalized or not a normalized dimensionless concentration.

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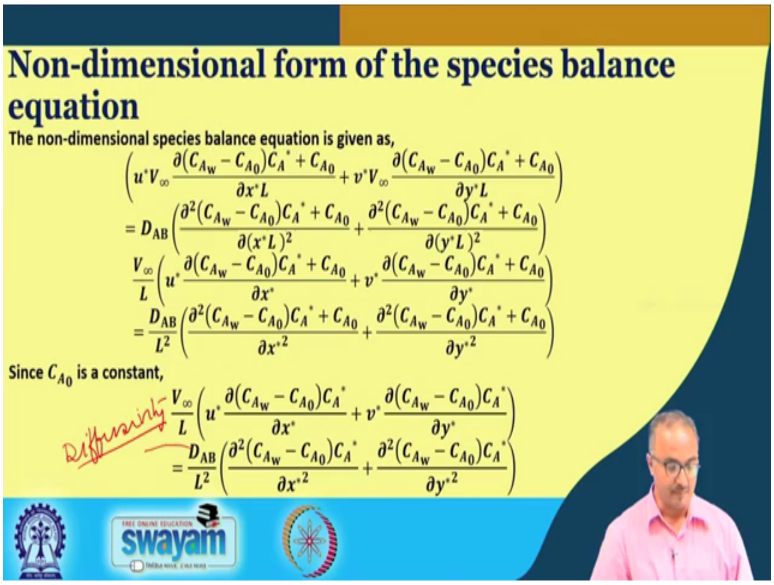

So, whatever it may be, so we find that we with all this kind of transformations what we now do, that we put all these non dimensional numbers in the given equation and we do the mathematical manipulations in the equation. So, this you can do very easily and you take out all the diffusivity term outside.

We are assuming that D AB is constant, this the D AB is the diffusivity this is a diffusivity and this here assuming to be constant D AB as we learnt earlier that D AB signifies the diffusivity of A in another species B ok. So, here we are assuming that as if it has only two species it is a binary mixture.

So, with that assumption we are saying that it is a D AB ok. Now if there is a multicomponent mixture also then also we can talk of diffusivity, but then we do not write like a D AB and multicomponent situations are not being built in this particular equation and they have a different kind of formulation for the diffusivity coefficient ok. So none the less whatever formulations we are deriving here they are applicable for either binary or for non dimension multi component systems.

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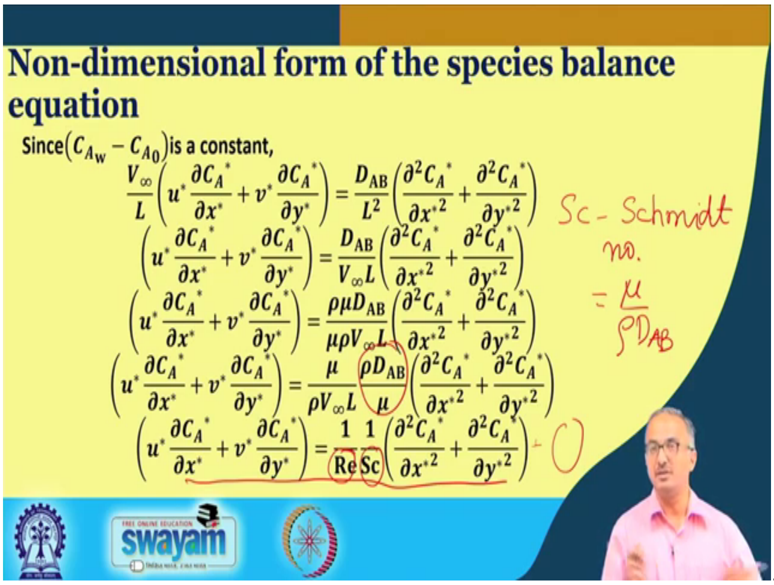

After this what we find that here we have the after rearranging the equations we get this particular equation, this is the final equation we are obtaining ok. And here you can again see that we are having two dimensional less numbers, one is the Reynolds number and S c is the Schmidt numbers.

So, Sc is the Schmidt number and this Sc is equal to from here you can see it is mu by rho D AB ok. So, this is the Schmidt number and you can see this Schmidt number is a property of the fluid because this we have viscosity, density, the diffusivity are the property of the fluid. So, now, this is the equation we are going to solve and please understand that if we have any kind of source term they will get added over here. And here also you will find that you can you will get some kind of non dimensional numbers which will be involving the reaction rate constant along with this density and the diffusivity coefficients ok.

So, all these things will be also coming here and another kind of source term may come in even if it is not reacting there could be transfer of species between two phases. Here we are talking about single phase only. If we have suppose two phases, it could be that I have a gas and a liquid and some species is getting from the gas side the liquid side or vice versa. In that case this flux of the species from one phase to the other will add to a source term for this equation and to account for that kind of source term we have to write a flux equation for the particular species between the two phases and this also we have covered in our earlier lecture. So, in this species balance equation we may have the reaction term and the term which will accounting for some either the introduction of a species from a different phase or outgoing species from the given phase.

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Now, after having learnt all the non dimensional way of the momentum equation the energy equation and the species balance equation along with the continuity equation we found that there are many non dimensional numbers appearing in this equations. Now non dimensional numbers there are plenty of them. So, what I am going to do now I will be taking a few of them which we encountered very frequently. So, I will be telling one telling about their significance and taking one by one.

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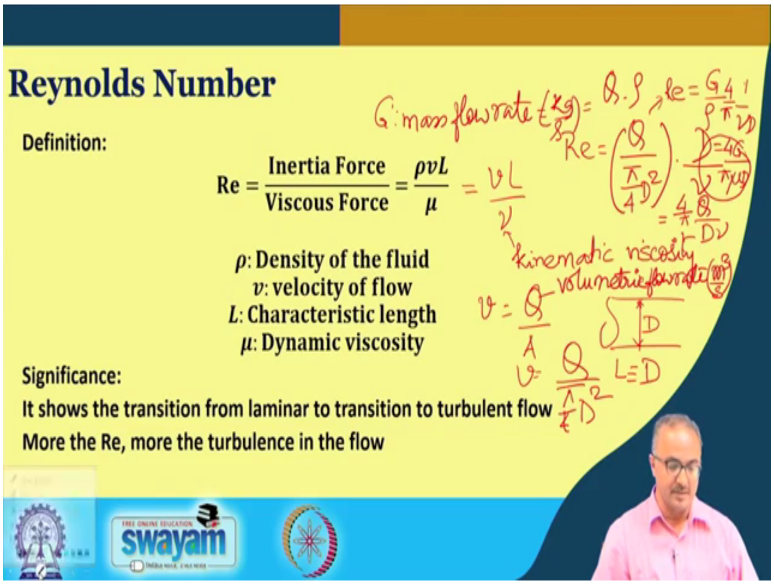

So, first we shall take this Reynolds number and this Reynolds number we have seen that it is defined as the ratio of the inertial force to the viscous force ok, and I will not be going into the detailing and the derivation of this Reynolds number.

So, I will be giving only the final term final, expression of the Reynolds number and here you find that this is the rho v L by mu. Here rho is the density and mu is the viscosity of the particular fluid and Reynolds number is associated with only fluids that is gases or liquids and not with any solids ok, that you must remember. So, here this is we can also sometimes we write it like v into L and divided by nu, this nu is the kinematics viscosity, this is the kinematic viscosity and you can also find this Reynolds number manlier times is given in terms of the volumetric flow rate, sometimes it is given in terms of the mass flow rate.

So, if you want to put the Reynolds number in terms of volumetric flow rate it is very easy you can see that these v is equal to Q by say A ok. There is volumetric fluid divided by the area of cross section through which the fluid is flowing. So, you can put this like for example, in a pipe of say diameter D ok. So, this v will be equal to Q by pi by 4 D square and in that case the L is taken to be the D the diameter of the pipe. So, if you put these values over here what you find the Reynolds number is coming out to be Q by pi by 4 D square into D divided by nu ok.

So, and what is this you find that this will be 4 by pi Q by D nu. So, manlier times you will find such an expression for the Reynolds number and sometimes it is given in terms of the mass flow rate. So, mass flow rate is suppose G is the mass flow rate is the mass flow rate or Q is the volumetric flow rate, volumetric flow rate, then in SI unit it will be meter cube per second and SI unit it will be kg per second ok. Now you can see that this mass flow rate is nothing, but the volumetric flow rate into the density ok. So, you can again put this instead of Q you can put in terms of mass flow rate and you will get this same Reynolds number.

Now, it will be Re equal to G by rho into 4 by pi by nu and because nu equal to mu by rho you can see it can be written as 4 G by pi into mu into D. So, sometimes you will also find Reynolds number in these terms. So, there could be many many different representations of the Reynolds number in terms of velocity, in terms of volumetric flow rate and in terms of mass flow rate, whatever it may be, whatever it may be you will find the ultimately this is the ratio of the inertia force to the viscous force; and how is it important? It is important because this relative values of these two forces tell us about the nature of the flow.

Nature means whether the flow is laminar or whether the flow is turbulent. Whenever the viscous effect is less the fluid tends to have it turbulent flow; that means, whenever the viscosity is low then the with the fluid is going towards a turbulent regime and you know that the nature of the flow dictates the heat transfer and the mass transfer rates.

So, the heat transfer rate and the mass transfer rate increases as the flow becomes more and more turbulent ok, and you know that the plug assumptions which we are making in that we are making there is a very good mixing of this. So, there in that case we were talking about that it is a plug flow, because there is a fully mixing full mixing in that particular flow. The flow rate also dictates the pressure drop, pressure drop because as we increase the flow rate

We find that the pressure drop through any particular element will also keep increasing and many times you have found that in the fluid mechanics that the pressure drop is given as a function of the Reynolds number ok. So, we call sometimes all these things as thermo hydraulic characteristics means hydro thermos, means it is the thermal characteristics that the heat transfer rates and hydraulic means how the pressure drop is getting affected by this flow of the particular fluid. So, as we have written here that its this Reynolds number gives us the transition from the laminar to the turbulent flow through between there will be some kind of transition flow and the more the Reynolds number the more the tendency towards turbulence.

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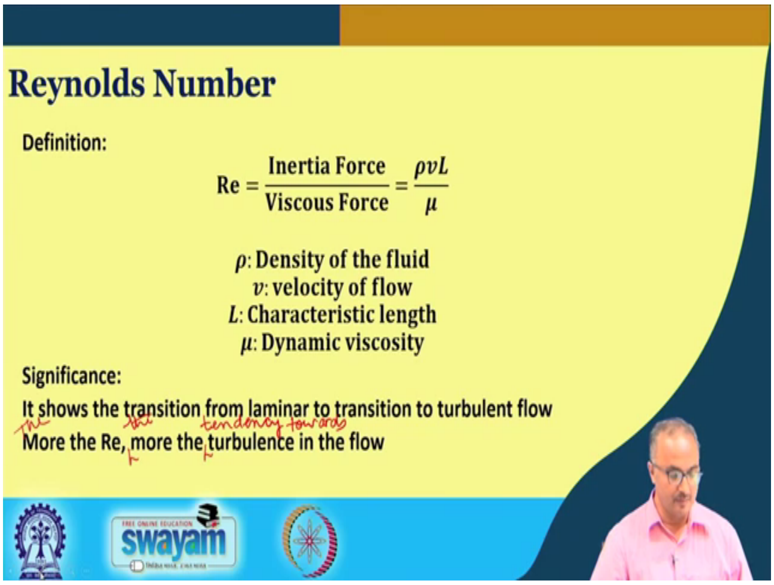

So, the more the Reynolds number the more the tendency towards turbulence towards turbulence. So, it is not that we will always get turbulent flow, it I can within I can stay within the laminar flow, but I can still have a high Reynolds number. And you know that there is something called the critical Reynolds number below which we say assume that there is a there is laminar flow and above which we assume that there is a turbulent flow and you will find that in the heat transfer and the mass transfer, you will find the various correlations proposed and these correlations are also valid for some given range of the Reynolds number. Some correlations are valid for the laminar flow and some correlations are valid for the turbulent flow.

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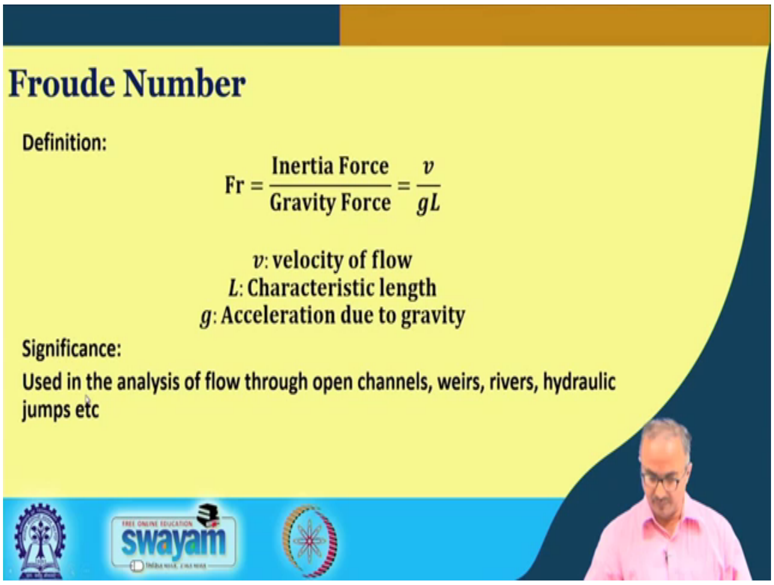

Next we go to another number this is the Froude number or Fluid number manlier. So, many times we have also, we have found it in our derivation when we are deriving the, when we non dimensionalized the momentum balance equation which included the effect of the gravity. We came across this particular number the Froude number and this is the ratio of the inertia force to the gravity force. And as I explained you earlier gravity force comes whenever there is a free surface not an enclosed surface, free surface for example, a river, a canal. So, there we have a free surface that it is unbound on the surface and above that we have air.

So, because air has very very less viscosity, so we sometimes neglect the effect of the air although the air is passing at a very very high flow rate so, that the shear at the interface between the liquid and gas ok. So, whenever there is a free boundary we talk of the Froude number or Froude number and in this we have the effect of the gravity, because you find that in the unbound liquids only there will be wave formation ok.

The wave formation does not come in the conduced flow, when whenever there is a say pipeline enclosed flow there is no wave formation unless otherwise you have two phase flow. In two phase flow only you find that there could be some kind of wavy flow, but that is having a two phases, not single phase and two phase flow comes when a liquid while it is flowing getting evaporated or a gas as its flowing is getting condensed.

So, in that case we get two phase flow or sometimes we also have one liquid flow and one vapor flow and they are going together. In those cases also we have this effect of the gravity due to the difference in the densities of the two fluids ok. Otherwise in an open channel flow we have the fluid number and we have to account for this effect of the gravity.

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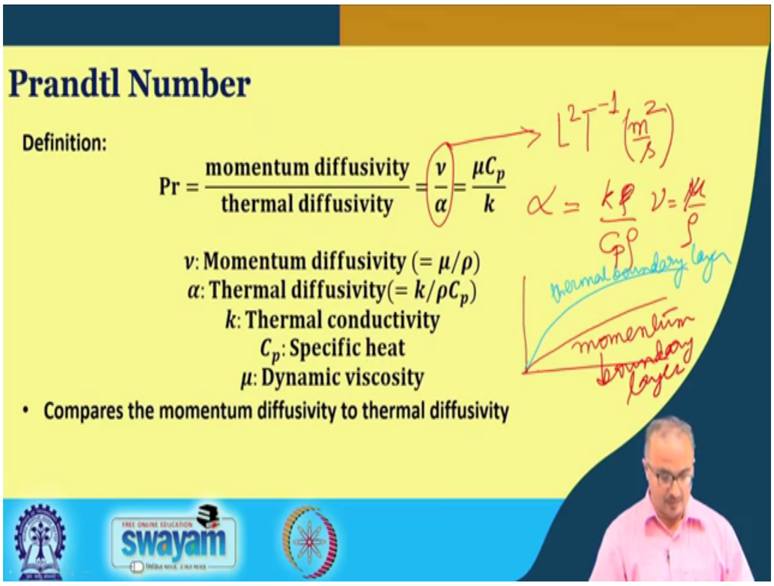

Next we come to the Prandtl number. This Prandtl number we have seen in the heat transfer analysis when we try to non dimensionalize the energy balance equation and here we can see that Prandtl number is given as the ratio of the momentum diffusivity and the thermal diffusivity. Please note that all this diffusivity terms whether its momentum diffusivity or thermal diffusivity or mass diffusivity all of them have the same units.

So, you will see all of them in if I write the dimension it will be L square T minus 1 L length and time ah, but unit time length square per unit time and in case of in your si unit it will be say meter square per second. So, all the three diffusivities will have the same unit and the same dimension ok. So, now we write the momentum diffusivity as the same as the kinematics viscosity that is mu by rho and this alpha is the thermal diffusivity that is k by rho C p.

So, if you put those values over here you will get this particular thing that mu C p by ok. So, with this you can find that this coming this comes to mu C p by k. So, you can see that this Prandtl number depends purely on the type of the fluid and as the temperature changes you find that.

So, there is a mistake over here, it will be k by rho C p. So, you can see that as the temperature of pressure changes there will be some effect on this properties also, but generally what happens that when you take the ratio of this diffusivities the ratio does not change much with the temperature and the pressure. So, and this particular Prandtl number comes in the heat transfer analysis and it dictate that how the momentum boundary layer changes with respect to the thermal boundary layer and you know that boundary layer means that within which the flow is developing. So, many a times you find that whenever there is a flow is developing you find that for some time there is a boundary layer formation and then it reaches a particular constant value. So, this bounded layer maybe for the momentum, maybe for the species, maybe for the energy.

So, when you have this kind of situation that I may have two types of boundary layers, say this is the thermal boundary layer and I have the say momentum boundary layer. Now you can see that whichever is growing faster; that means, it is going to reach the ultimate thickness faster in that case you can say that that is the other one that slower one is going to dictate the particular phenomenon when we have a combination of the flow and the heat transfer and when you find that this thermal diffusivity is equal to the momentum diffusivity you find that these two boundaries are would merged there will be only a single boundary layer. So, they tell us about the relative rate at which the heat transfer and the fluid flow are occurring.

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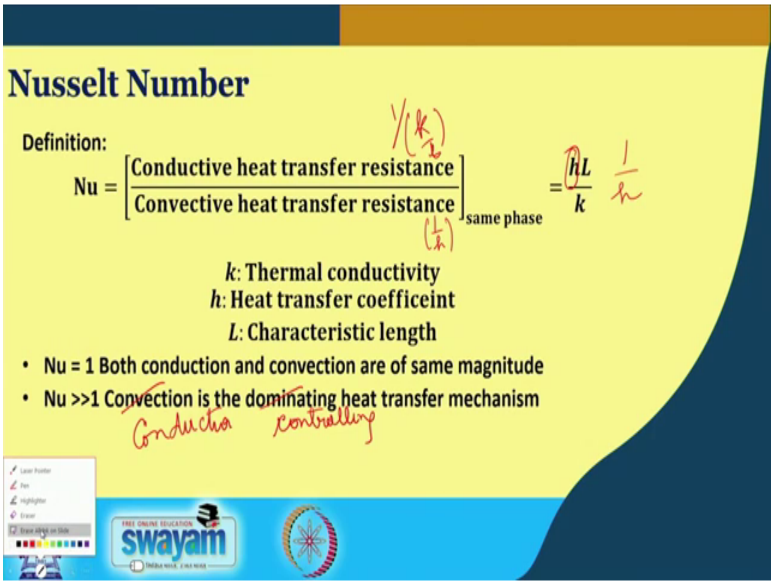

Now, next we go to the other dimensions less number which is associated with an Nusselt number. This will be with the heat transfer that that is the Nusselt number this we also have encountered whenever non dimensionalizing the energy balance equation and here it is defined as the ratio of the conductive heat transfer resistance to the convective heat transfer resistance and the resistance you know it is the inverse of the conductance ok.

And this heat transfer coefficient or the k by L this they this heat transfer this 1 by h is the convective heat transfer resistance and this k by L is the conductive heat transfer resistance 1 by k by L ok. So, when you put plug in this when you find that you get this is an Nusselt number and this is the within the given fluid that whether if the you find that the fluid is very very viscous in that case you will find that generally with the increasing viscosity also you find that the thermal conductivity is coming down even though there is there independent properties.Bbut we can say that the convective tran resistance becomes more and more when the viscosity increases ok.

So, when the convective resistance increases you will find the Nusselt number will keep decreasing and this particular number from this particular number you will find that many correlations are given for the Nusselt number in terms of the say Reynolds number and Prandtl number. So, from that we can find out the estimate the value of this heat transfer coefficient. So, most of the correlations given in the literature to find out the heat transfer coefficient are given in terms of the Nusselt number and you can see that if Nusselt number is equal to 1; that means, both convection and conduction conduct conduction convection are have the same significance and.

As the Nusselt number keeps rising it goes much much higher than one or to two find that that; that means, that this convective resistance keeps coming down and conductance keep going up ok; that means, that you will find that that the particular it is the conduction is a dominating heat transfer is the is the controlling; that means, we need not consider the conviction then we have to only focus on the conduction part.

So, if it is very very less than one then we talk of the convective heat transfer resistance more than the conduction; that means, as if the particular solid, the particular fluid is highly conductive ok.

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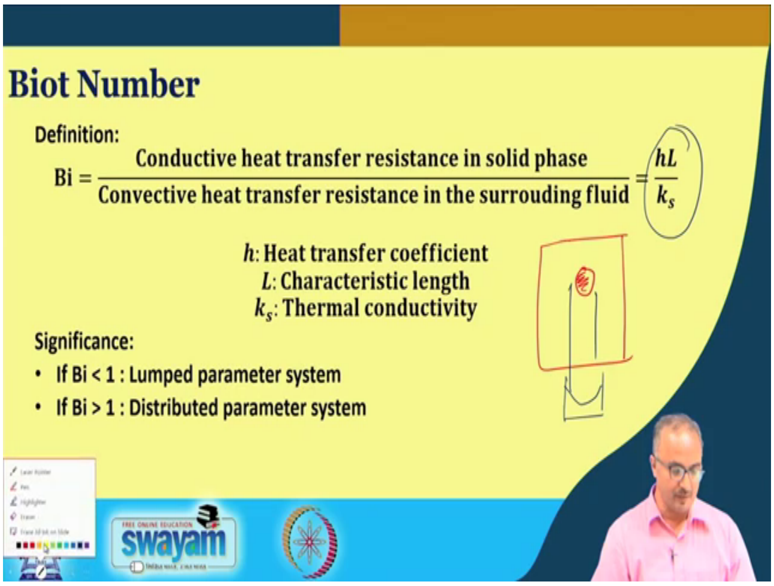

Similar to this Nusselt number we have a Biot number. Again you see that Biot number is a defined as the resistance by the conductive heat transfer, in the solid phase and convective heat transfer within a surrounding fluid. Now difference between Biot number and the Nusselt number is this, with Biot number has two different phases for example, I have a solid and which is emerged in some kind of a fluid ok.

So, now, you see that whenever heat transfer taking place now what happens that from the boundary of the particular fluid we find that now this is a this is the solid and you see that in the solid what we have that there could be in this solid there could be a thermal gradient within this solid, there could be a thermal gradient.

Now, if the solid is highly conductive that means the conductive resistance within the solid is very very less than the whole solid may be assumed to attend a single temperature and the resistance to the heat transfer may be taken to be only within the fluid surrounding the solid ok. On the other hand if the solid has some finite conductivity effect and the fluid is going at a very high flow rate.

So, we may neglect the heat transfer resistance in the fluid and we can say that the fluid is at constant temperature whereas, within the solid there will be a temperature gradient. So, these two effects within a solid and the surrounding fluid, this two effects are taken care of by the Biot number, even though expression wise this look similar.

The difference is this in case of Nusselt number both h and k pertain to the given fluid whereas, in case of the Biot number h pertains to the fluid and that k pertains to the solid which is within the particular fluid. So, this is the difference.

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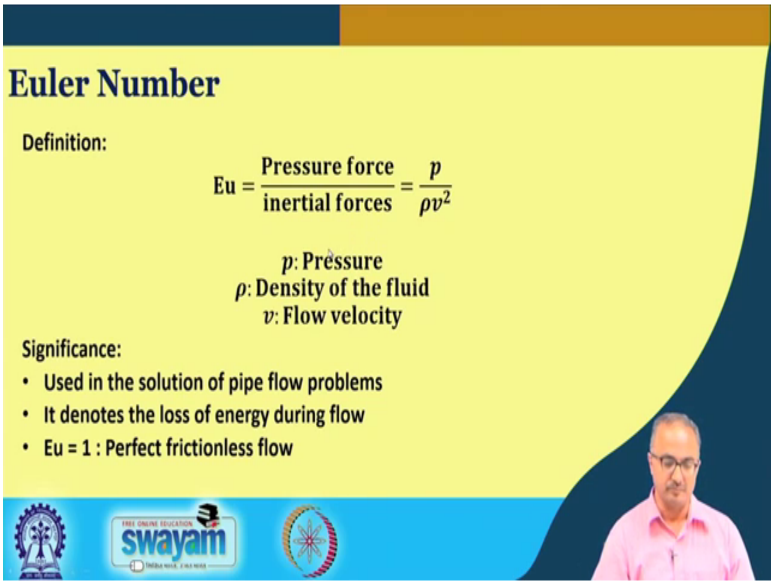

Next we come to Euler number, this is also related to the flow situation in this case we have the pressure force versus the inertial forces this also we found in one of our derivations earlier in solution and we can see that when we have Euler number equal to 1, we have a perfectly frictionless flow. That means, both the pressure force is equal to inertial force; that means, we are not there no contribution because pressure loss is due to the both inertial flow as well as the viscous loss.

So, if the viscosity effect will be neglected then you find the pressure loss is purely due to the inertial effect and that is coming from the Bernoulli equation.

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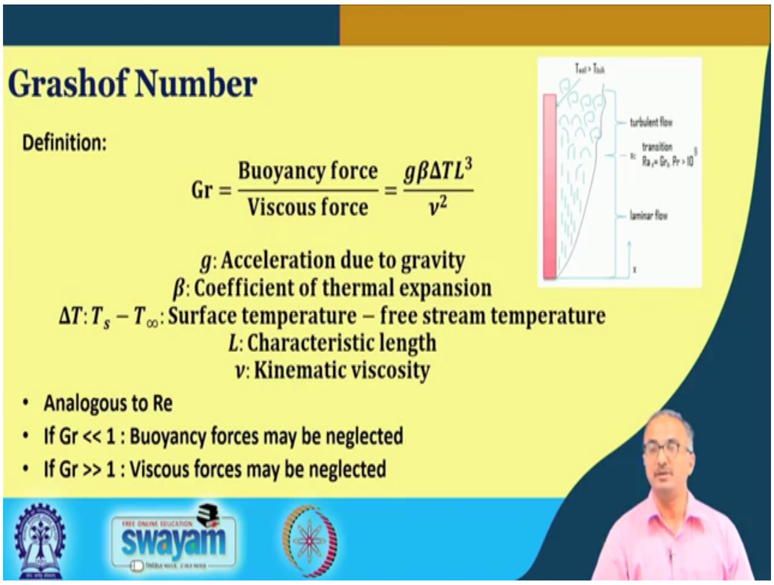

Then we have the Grashof number. This Grashof number comes whenever we have a natural convection ok; that means, the fluid movement is happening due to a difference in the density and these difference in the density is caused because of a temperature gradient and what happens that when you have a temperature gradient within a liquid the say lower part has a higher temperature and upper part has a lower temperature. So, what happens the lower part will get less densified and then it will tend to move up into the liquid, into the top level. This is very common when you find in our kitchen whenever we are trying to heat up some liquid say water or milk.

You find that if you put this particular thing in the transparent jar transparent pot, you find that you can figure out the there is, there must be some kind of a movement of the liquid. What happens is this; the particular liquid layer which is in touch with the oven is getting heated up earlier then the fluid liquid that is at the top. So, the bottom liquid is getting less densified and it is going to the top and from the top which have the more dense liquid, because of the lower temperature that will trying to come down and this way you find that there will be a convective current set within the particular liquid.

And this particular effect of the buoyancy force and viscous force is taken care of by the Grashof number and it is given by this particular expression here. You can see the g is coming for the gravity effect and the beta is coming due to the expansion effect of the liquid, delta T is the driving force for the Grashof number. And then you can see that if Grashof number is very very less than one then it is buoyancy effect, may be neglected and if it is much much more than 1 then you have to take it of the viscous effect ok, then sorry you can neglect the viscous effect.

And it is whenever you find this Nusselt number correlation for natural convection, you will find the Nusselt number correlations is given in terms of the Grashof number. In case of forced convection you will find the Nusselt number is given in terms of the Reynolds number, and we have relay number because manlier times we find that Grashof energy and Prandtl are coming together ok. So, that way you find that we have the relay number.

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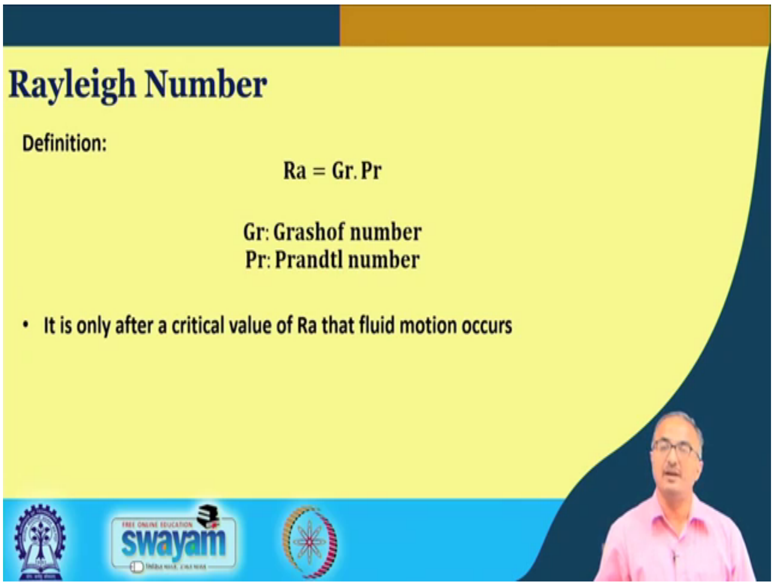

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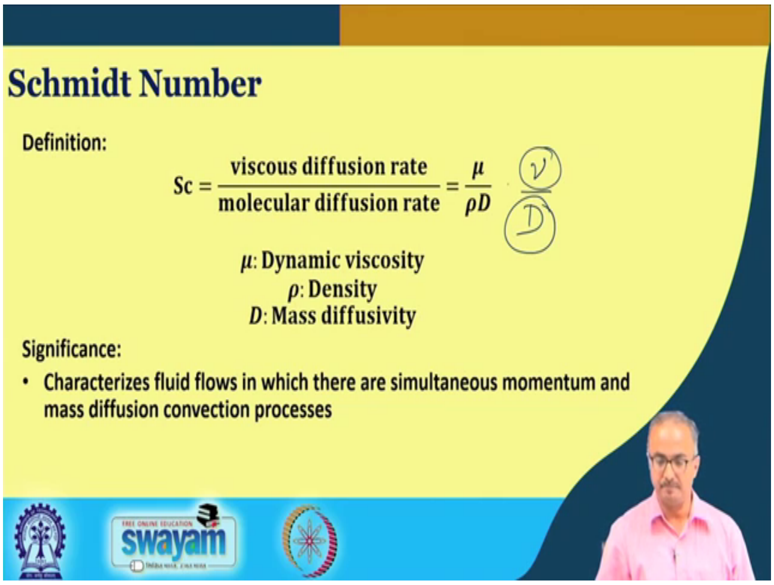

And we have Schmidt number as I described you earlier what it is? It is the viscous diffusion rate to the molecular diffusion rate or the viscous diffusivity to the molecular diffusivity, this you can also write as nu by D ok.

So, this is the momentum diffusivity to the mass diffusivity ok. So, this is similar to the Prandtl number which was the momentum momentum diffusivity to the thermal diffusivity ok. So, it also tells us that how these two boundary layers due to the momentum and due to the species concentration would develop, and when this Schmidt number is equal to 1; that means, the momentum diffusivity and the mass diffusivity are the same. So, that the boundary layer thicknesses are the same.

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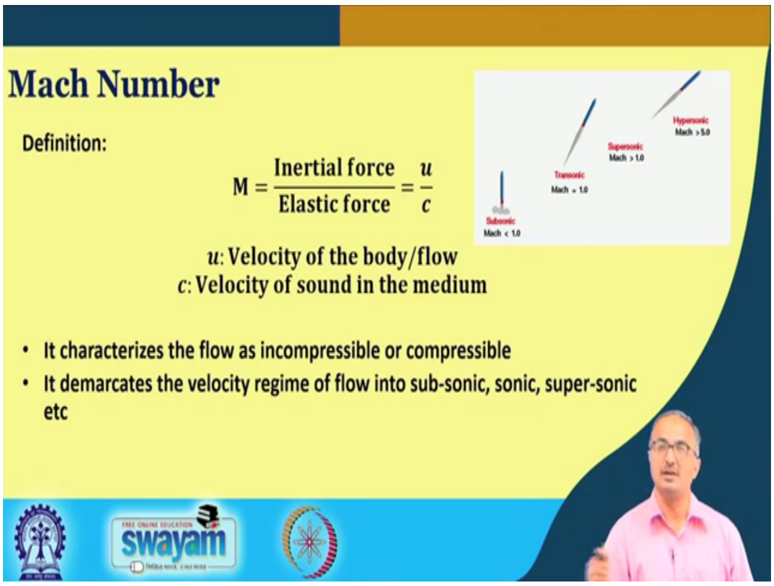

And another number which is very common that is the Mach number, Mach number signifies what kind of flow we have and whether we have compressible flow or incompressible flow. Please understand that compressible incompressible flow is different from the incompressible compressible fluid ok.

A compressible fluid may undergo a incompressible flow ok. So, in this case is Mach number is given as the ratio of the inertial to elastic force, elastic because there is a compression and the rarefaction of the particular fluid medium ok, and this is given in terms of the velocity of the sound. So, this elastic force is pertaining to this compression or rarefaction of the particular fluid medium and it is given in terms of the velocity of the, particular fluid divided by the velocity of sound through that particular fluid ok. And Mach number in array you can see that when Mach number is less than 1, then we have the subsonic flow when we have equal to 1 we have sonic flow.

And when we have more than 1 we have supersonic flow, and when the Mach number is still more than 1 then, but it is still more than 5 then we call it hypersonic flow. Now you see that significance of the Mach number is this that type of the equations to be used for the analysis will also change. So, the equations which we use at a lower Mach number will not be valid, if you go for the higher mach number and this is we have shown it by the movement of a rocket ok. This rocket also undergoes this change in the Mach number as it goes out of the earth orbit.

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So, these are some of the books which you can refer to for more explanation on these topics.

Thank you.