Mass, Momentum and Energy Balances in Engineering Analysis

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Lecture – 32
Microscopic Balance Illustrations – II

Welcome, we started with some illustrations on the Microscopic Balances. Today we shall be taking some more of such illustrations and in this we shall be looking into the various transfer equations momentum energy, mass energy, etcetera.

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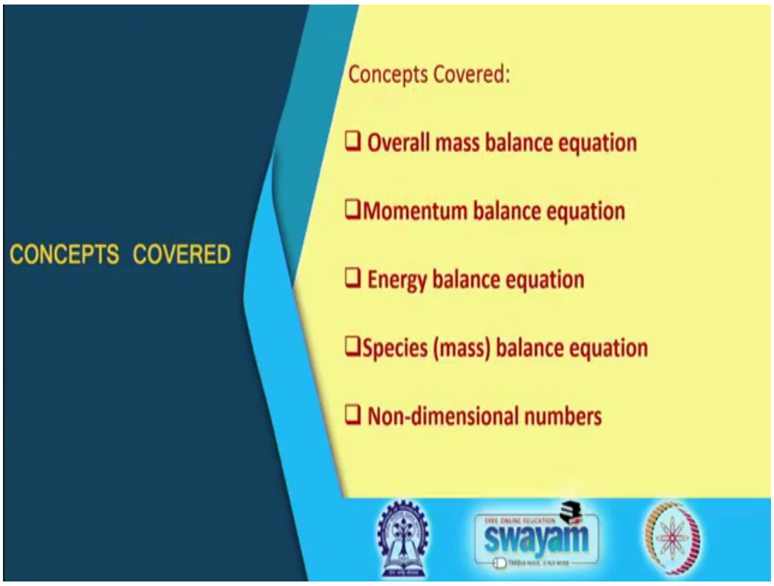

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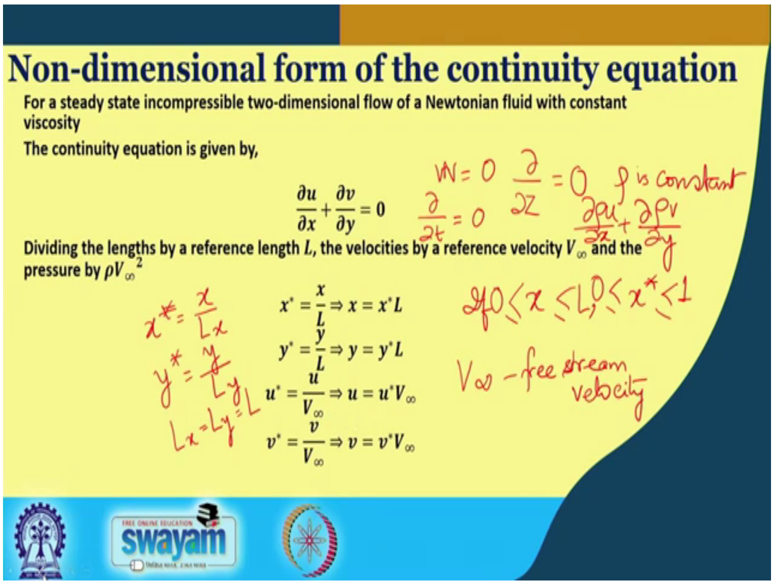

So, first let us go with the continuity equation as you know that continuity equation gives us the overall mass balance for a system. And continuity is overall mass balance whereas, separately we will do the mass balance for the species.

So, here we assume that we have a steady state incompressible two dimensional flow of a Newtonian fluid with constant viscosity. Now these, all these phrases should be noted very carefully, because depending on what we assume our balance equations will change. So, because of the steady state assumption, we will not be having any time differential in the equations. Then because of incompatibility, we shall be having the density to be constant. So, we can take out the density from any of the differentials.

Then we have two dimensional; that means, in space we when we have x y z or r theta z or r theta phi, then we can drop one of the dimensions. And Newtonian fluid means the stress in a fluid can be given in terms of the Newton's Law of viscosity that is stress equal to mu into dou v by dou y.

So, with all these assumptions let, let us see how the continuity equation looks like. So, here we are assuming that the velocity has three components u v w and we are taking the w to be 0 and two dimensional so, we are never getting all the z differential; that means, we are taking w to be 0 and we are taking all the dou by dou z term to be 0, ok. So, with this assumptions we are writing the continuity equation like this and also because it is incompressible. So, rho is also constant so, because rho is constant so, we are able to take out the rho from the differential. So, generally otherwise this expressions look like dou u by dou x plus dou rho v by dou y.

So, they look like this and because of incompressibility, constant density we are able to take this rho out of the differential. And, because of steady state we are saying that dou by dou t is equal to 0, ok. So, with this understanding we are able to get this particular equation. Now our intention is to gets a non dimensional form of this particular equation. Now for non dimensionality first let us see what are the variables involved here. We have x y u and v.

So, for each of them let us see that how we do it. Now let us suppose that for the x direction, we have some characteristic length. So, first we have to define some characteristic value with respect to each of the variables. So, for x direction, we are saying that let, let L be the characteristic dimension. Now L can be anything, L should not be interpreted as a kind of length ah, it can be breadth, it can be width anything.

So, let us assume that this L is that that kind of length in the x direction. So, we are having this x star as the non dimensional x. So, this x star is defined as x by L. Now in this case as I mentioned you earlier that sometimes by doing the non dimensionalization, we also land up with some normalization. Now if the x varies between 0 and L, we find that the x star will also vary between 0 and 1.

So, that way x star is not only non dimensional, but it is also a normalized non dimensional number, ok. So, in if x varies between 0 and L then means x star will vary between 0 and 1, ok. So, this is normalized. So, in this fashion, we can also define another y star that is non dimensional y in terms of the same length. Now please understand that this characteristic length need not be the same in the x and the y direction. So, I can have, I can also say like x star I can put something like this that x by L x and y star I can put something like y by L y; that means, why ; that means, this Lx is the characteristic length in the x direction y is the characteristic length in the y direction, ok.

So, in this fashion you should be understanding or you should be making this kind of non dimensional numbers, ok. So, for simplistic way here we are taking this two is; that means, in this case we are taking L x equal L y equal to L, ok. So, this is the understanding now next coming to the velocity. Here again we can define some kind of characteristic velocity. Now in this case you see that u and v, we have this two velocities and generally what happens that whenever if fluid is flowing, we take that away from some surface, we take that it is going with a free stream velocity.

So, we say that V infinity is the free stream velocity, ok. When you just free stream velocity, we can take it to be same in any direction. So, that is why you see that when we are non dimensionalizing u and v, we are taking the same characteristic velocity, ok. So, with this understanding, we find that these are the four ways, we are able to do the non dimensionalization, only one more thing is to be noted is this that u star and v star they maybe or may not be norm normalized, ok.

Generally, you will find that whether u will be restricted to V infinity or V will be restricted to V infinity not that will depend on the physics of the problem, ok. So, with that understanding we say that they may be or may not be normalized.

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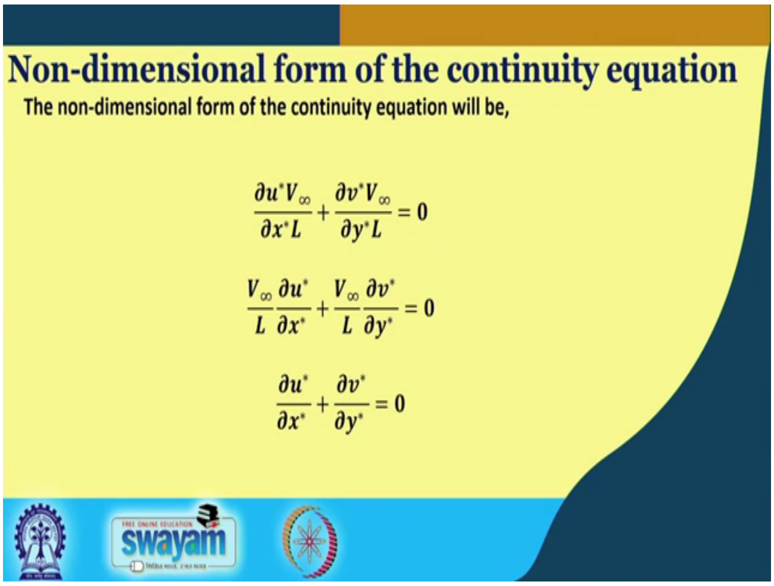

Now let us go ahead here we have this non dimensional form of the continuity, what we have done with those u and v, we have substituted in terms of u star and v star and x and y, we are substituted in terms of x star and y star.

So, now, you can see that you can take out the V infinity L from the differential and we get this particular equation. And you see this non dimensional equation in this case has a same form has as the dimensional form. Only thing it is seen that it is replaced by the non dimensional velocity and non dimensional length parameters, ok. So, this is very straightforward way of finding the non dimensional form for the continuity equation.

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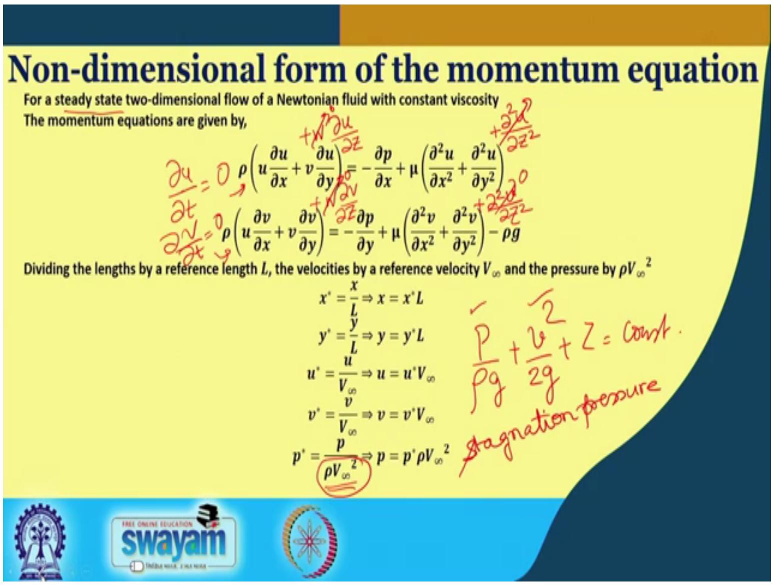

Next we come to the momentum equation. Now again in the momentum equation, we are assuming that in the steady state two dimensional and if flow of a Newtonian fluid with constant viscosity.

Now, please understand, when we say a constant viscosity, it means we are neglecting any kind of variation of the viscosity with respect to temperature and pressure. Now if it is with respect to temperature, there is some variation then understand this that we have to consider also the energy balance, ok. So, to for the time being to avoid the use of the energy balance, we are not going to take the viscosity to be a function of temperature.

For pressure generally we do not have any explicit equation when we talk of this continuity equation. Generally pressure is taken indirectly through the continuity equation about which I shall not be talking in other courses like on in cfd ah, it is known talked that how we can take the pressure variation into account. Now let us look at with this assumption, let us look at this particular situation this is the equation for the x direction momentum equation and this is for the y direction momentum equation.

Now, if you look at the full a moment equation for the Newtonian fluid you will find with this is added w dou u by dou z. So, this is term is added here and when you go for this term you will find it is coming like dou 2 u by dou z square, ok. Now please understand in this x direction equation, we do not have the body force here. Now body force as I said that body force can be many things, ok.

Generally the most often body force is a gravity and gravity is generally vertical ok. So, that is why you find. And then in the x direction we are not having any gravity effect. On the other hand in the y direction, we have the gravity effect, ok. Thus, please understand, it is just not gravity, it maybe some other body forces which may be there in the shorts term.

Now on the because it is steady state so, we are having dou u by dou t equal to 0 for this and dou v by dou t equal to 0 for this, ok. So, that is how we are finding that we are able to neglect this term. So, similarly here also we will be having a w dou v by dou z in here also we shall be having a term dou 2 v by dou z square. So, because of our assumption of independence with respect to z direction so, these two terms goes to 0, ok. And because we are assuming that is two dimensional velocities, it is w is 0, ok.

So, that is how we are reducing this whole 3D equation into this 2D and with the steady state assumption. So, similar to the one what we have done for the continuity equation, we are also defining all these equations, all these non dimensions equations like this that x star equal to x by L y star equal to y by L u star equal to u by V infinity v star equal to u v by V infinity. And here comes the thing next is the pressure. Now in this case the pressure has to be somehow obtained in a non dimensionalization of that have to be done through some other kind of combination of the now existing parameters.

So, you can see that if you look at the Bernoulli then you can see that a pressure is taken the pressure heat we have like this P by rho g plus v square by 2 g plus Z equal to constant. So, you can see that we can correlate the pressure with the velocity, ok. So, you can see that that kind of inferences are taken to find this kind of characteristic pressure, because it characteristic pressure, ok. And so, gave get this particular non dimensional pressure. Now what we understand by this characteristic pressure let us try to understand this a bit more.

Manlier times you find that if slope comes and suddenly it kind of gets stopped by some kind of hindrance, ok. For example, if you want to find out the point by point velocities in picot tube. Then you find that in picot tube which is taught in the fluid mechanics, you find that we take the vertical stagnation pressure, ok. And in that it is assume that when a fluid is brought to rest suddenly the whole pressure energy gets converted to it is kinetic energy. So, that you can assume that this particular non dimensional pressure is something like the stagnation pressure in which a flowing fluid is brought to rest, ok.

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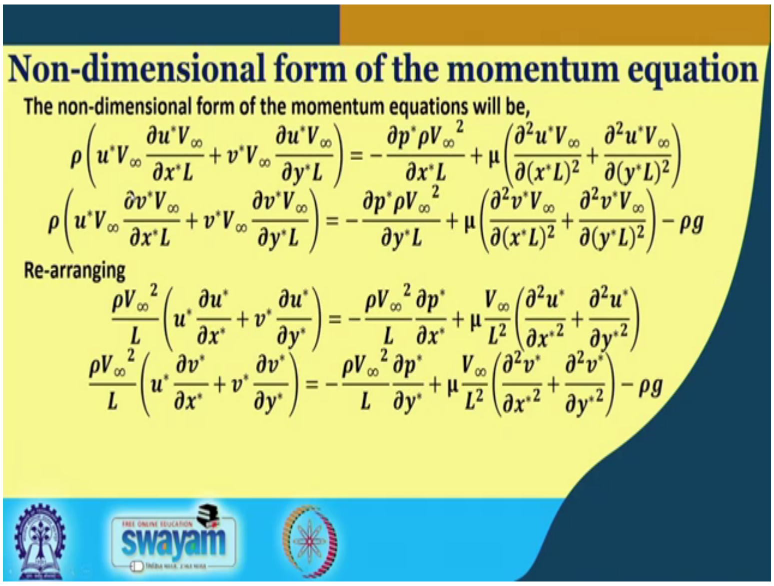

So, that way we take this non dimensional form of the pressure. So, after getting this non dimensional numbers, we plug in all those non dimensional parameters in the momentum balance equations in the x direction and y direction and we get this particular equations. Now after again you can take out all the constant values from the differential and you will get this equation.

And again you can see that these whatever you see in the parentheses they look similar to the one which is there in the, our original dimension of form of the equation. Only thing you see that some kind of terms get included with the convective terms. This is the convective term inertial terms, the pressure term and this particular term for the viscosity term.

Now, understand this, this particular term here we still dimension, it is not non dimensional, because each of these terms are dimensional forms.

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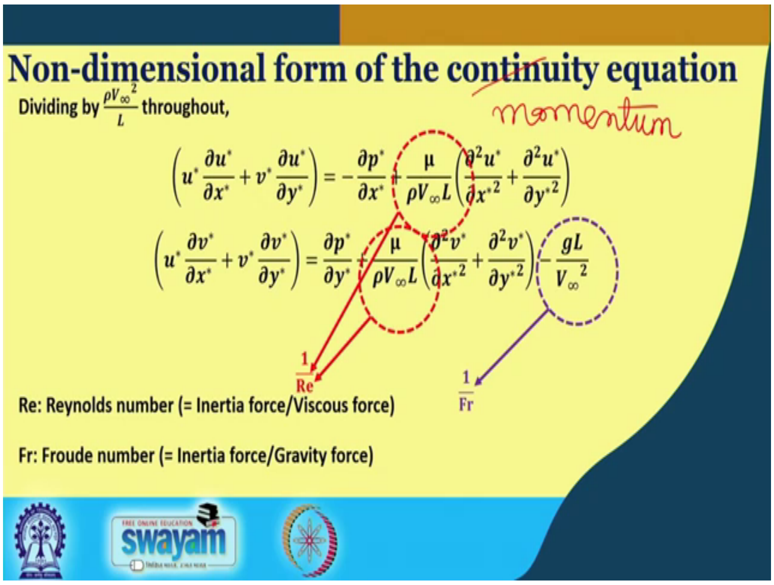

So, we rewrite the equations by dividing by rho v square by V infinity square by L and we get this particular equation for this and this. Now you can see here that we get some non dimensional numbers and this particular thing is the, what we call inverse of the Reynolds number, ok. And this particular thing is the inverse of the fluid number. Now what is Reynolds number? We know that Reynolds number denotes the relative importance of the inertial force to the viscous force. Now what is the meaning of this? Now suppose a fluid is flowing over a surface, ok.

So, because of the viscosity of the fluid there will be a drag offered by the surface which will try to retired the motion of the fluid, though that is what we got the viscous force which is generated due to the viscosity of the fluid. And you know that there are something called the ideal fluid and the ideal fluid, we say that the viscosity effect is 0; that means, ideal fluid are non viscous, ok. In that case, there will not be any kind of viscous force, ok. And in that case, we can easily see that Reynolds number will tend to infinity, ok. On the other hand even if it is not an ideal fluid, in that case what we say that the inertial force means which is coming due to the motion of the fluid.

Now, as the motion of the fluid is increasing, it will have it will be kind of over dominating the viscous drag. And due to this, what will happen that the fluid, nature of the flow will change. And in this manner you perhaps also know that when the viscosity is very high compared to the inertial force or the velocity then we get what we call the laminar flow, ok. And when the viscous effect is negligible in comparison to the inertial force then we get the turbulent flow, ok.

And this Reynolds number is the deciding factor, we look at the Reynolds number value to decide that whether we are having the turbulent flow or the laminar flow. And perhaps, you also know that there will be something called the critical Reynolds number for a given situation. For example, in a circular pipe the critical Reynolds number is taken to be about 2100, what it means is this that any Reynolds numbers below this 2100 will represents laminar flow ; that means, even if you disturb the flow, if you disturb the flow be.

If the Reynolds number is less than 2100 the flow will again go back to the laminar region. On the other hand above 2100 there will be a transition zone, that there will be both laminar and turbulent coexisting, but generally for our simplicity sake what we do that above this 2100, we take that flow to be turbulent.And in that case what it means that even if there is some kind of disturbance is there in the flow, it will never come back to laminar, it will stay turbulent, ok. And you know that turbulent flow when you want to kind of maintain the turbulent flow; you need to supply energy from outside to keep the turbulence because turbulence always try to dissipate the energy, ok.

And if dissipate the energy it has a tendency to come back to laminar region, but when this Reynolds number is quite high then it will not come back to laminar region, it will stay in the turbulent region. Similarly, we have this another number, we called the Fluid number and this Fluid number is the ratio of the inertial force to the gravity force.

Now gravity force when if it happens that for example, when you have the waves on a river or in some kind of channel this waves are formed. Due to this gravity effect, you can see the waves come and then they can fall, ok. So, this is due to this buoyancy and then gravity.

So, this buoyancy effect or gravity effect, they are also coming into picture in dictating the motion of the fluid other than the inertial force, because when you look at the ocean or some river at the surface of the ocean, about near the surface of the river you do not find any solid surface, ok.

But you still find that the fluid motion is restricted, it is not just going away it is just, because due to this gravity effect, ok. So, whenever we have this gravity effect then we talk of the Fluid number and gravity versus the inertial force, inertial that is a motion of the Fluid, ok. So, you will find that when we non-dimensionalize the momentum balance equation, it will be not continuity, it will be momentum equation, there is a mistake here this is the momentum equation.

So, so, this is the way we can visualize the momentum balance equation you will find that I have taken some very simple examples of this, but you will find that in a reality there could be many other situations. And as we have discussed in my earlier lecture, I took the example of a flow over an inclined surface or a flow between two concentric cylinders. In all these cases, you will also find that you can carry out such kind of analysis. Now in terms of all these non dimensional numbers and you will find that when you non dimensionalize any of these equations whether a continuity equation whether momentum equation, you will find that generally you will always land up with some or the other non dimensional numbers. And these non dimensional numbers will account for the various effects in a given system.

So, here we are finding that we have Reynolds number and Fluid number, but this is not restrictive. For other situations, we have many other type of non dimensional numbers and about which we shall be looking in the other lecture.

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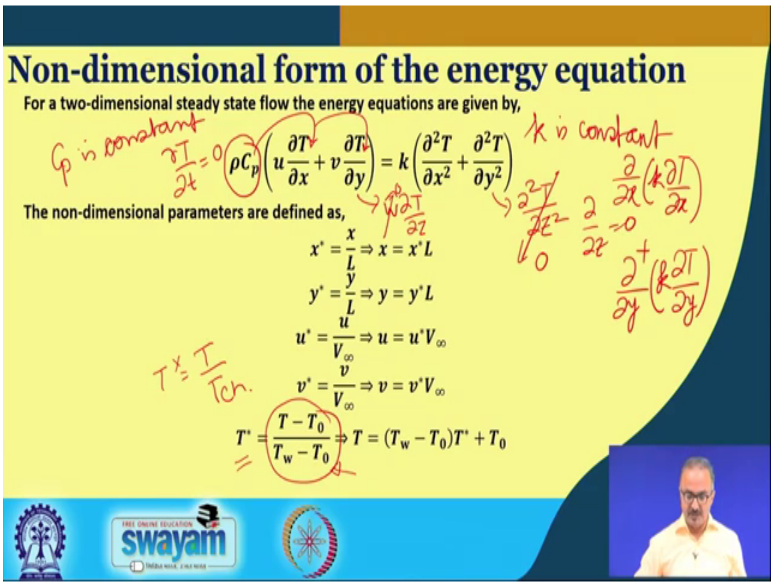

Now, we come to the energy balance equation in the energy balance equation. So, again we have taking the same two dimensional steady state flow of energy equations. And here we again find that in a understand this because energy is given in terms of the temperature which is a scalar. So, we shall be having only one equation unlike the case of the momentum equation.

So, continuity and energy will be only one equation each, but momentum will be three equations, because velocity has three components. So, here again you see, because steady state we are having this dou T by dou T term as 0, ok. And here we shall be also having something like this. So, because of this is 0 so, this is not there. And similarly, here we shall be also having, ok. So, this dou by dou z is taken to be 0, ok. So, that is why it is also 0.

Only thing is this, here we are assuming that there is no source term that is as if it is a non reacting system, ok. And also you please see that we are taking the thermal conductivity to be constant. If it is not constant it is; that means, other than assuming the viscosity to be constant. We are here assuming that k is constant, thermal conductive is constant. If it is not constant then it will look like this, ok. So, if it is not that difficult for you to account for a variable thermal conductivity, ok.

Similarly, we are also assuming this specific heat to be constant, so, it is outside this. If it is not constant, this simply that you have to take this thing inside this in the differential, ok. So, here we are assuming the cp is constant, ok. So, with these assumptions, we are able to write this energy balance equation. Now you see that here we have similar to the 1, we have done previously we have defined all the x y u v. new thing in this case is the temperature. Now in this case of time when you want to non dimensions temperature there can be many ways. In fact, you can also define temperature in terms of some characteristic temperature, some characters temperature.

Now, generally it was happens that that the temp if we have suppose a reference temperature say T 0 and say a wall temperature; that means, is as if the fluid is flowing over a surface and the surfaces has some kind of temperature and T may not can be taken to be some characteristics, may be the may be the free stream temperature, ok. So, in that case we define, we can define like this, but understand one thing this temperature need not be always normalized.

To normalize it we have to make sure that temperature lies between T naught and T double o. Only if we find suppose we are having two surfaces at once a hot surface when this coldest surface and we have say that one surface at T h and the rest T c, ok. T h means hot temperature and T c means cold surface temperature. In that case, we know that if there is no other external sources of energy.

Then temperature must lie be between T h and T c. And in that case, if we apply this particular type of non dimensionalization; that means, you define T star is equal to T minus T c divided by T h minus T c.

So, we know that T will be wearing between T c and T h. So, that T star will be wearing between 0 and 1, it is normalized, ok. Otherwise if we cannot ascertain that the temperature will be bounded by some values then even if we are defining like this as in as is shown it does not ensure that we are going to obtain any normalized non dimensional temperature. So, this should be clearly understood.

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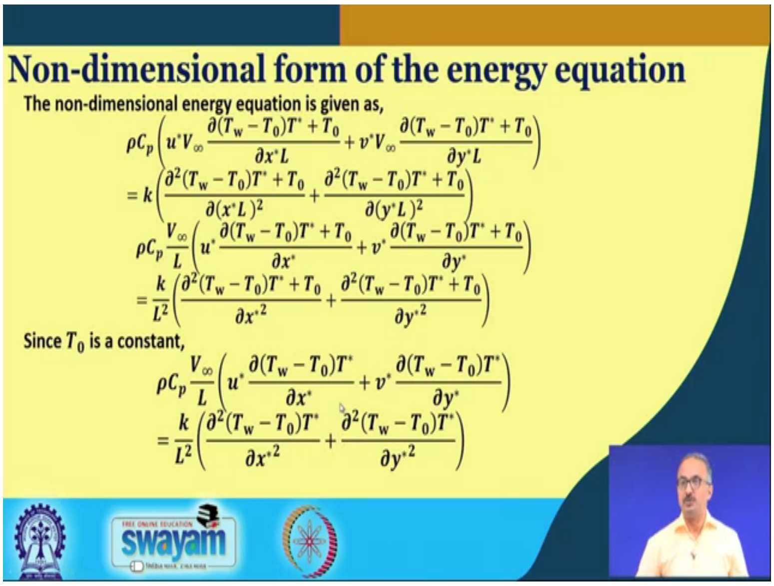

Now after doing this kind of definitions then we just simply put all those non dimensional numbers in the energy equation. And we find we are getting this particular equation, ok. So, these are simple mathematics. So, you can do it yourself. So, we find that we are getting this kind of equations.

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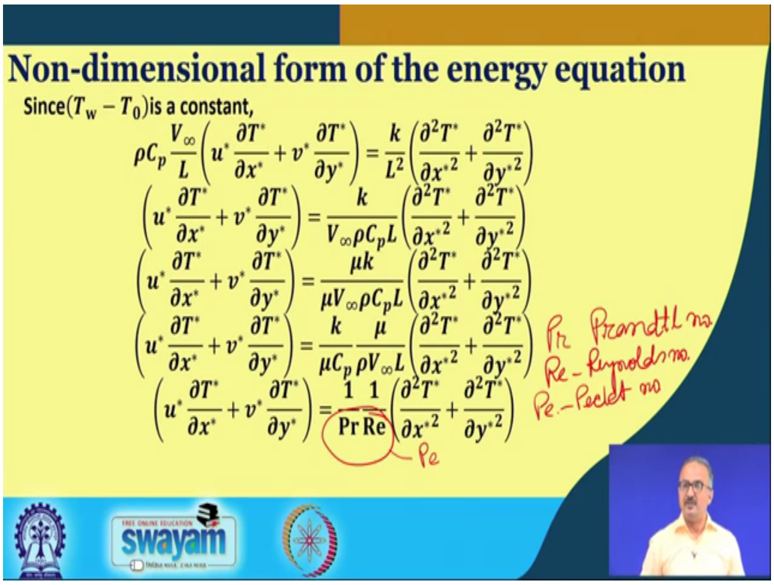

And now we obtain this equation. Again you see the basic form of the equation remains the same in non dimensional form as in case of dimensional form. Only this is this we are getting some other terms in with all these terms, because these are the extra terms which are coming up, ok. And now if you look at these terms again we see that we can break up these term into 2, 1 this particular k by mu c p is what nothing, but the inverse of the Prandtl number and this mu by rho infinity is the inverse of the Reynolds number. So, we are having these two numbers general some manlier times we write in terms of Peclet number.

So, this manlier times this particular product it appears. So, often ok so, sometimes this is called the Peclet number, ok. So, this P r is the Prandtl number, Prandtl number then r is the Reynolds number and P e is the Peclet number, ok. So, you find that this kind of numbers appear as I was telling you whenever we are non dimensional things. And we will learn about these non dimensional numbers in our next lectures in a very brief manner, ok.

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So, these are some of the references you can refer to get some more explanation on the theories today we have learnt.

Thank you.