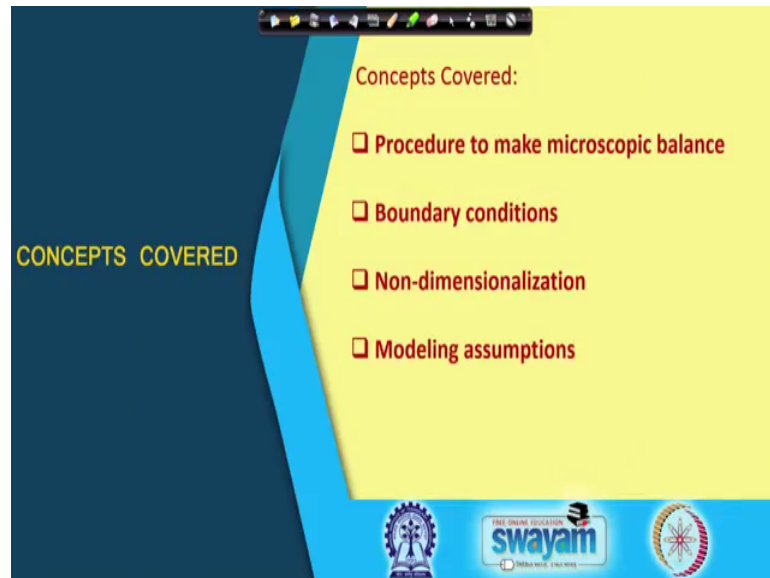


Mass, Momentum and Energy Balances in Engineering Analysis
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Lecture – 29
Microscopic Balance – VI

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Welcome. Today, we shall look into some of the examples of the Microscopic Balances. So, in this particular to lecture what we shall be looking into that how we can apply the model equations for the heat exchanger.

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Example

- Heat transfer using a double pipe heat-exchanger

A liquid stream is heated by the condensation of steam at constant temperature T_s . The liquid enters with temperature $T_{l,in}$ at $z = 0$ and exits at $z = L$ (the length of the tube) with temperature $T_{l,out}$.

Heat transfer from steam to the liquid involves heat transfer resistances between steam and the wall of the inner pipe, and between wall and the liquid.

Establish the balance equations to determine the temperature profiles in the flowing liquid and the wall of the inner pipe

Handwritten notes: *Cocurrent parallel flow*, *Cross flow*, *Counter-current*

Now, here we have a case of heat transfer using a double pipe heat-exchanger. Before, I start with the problem, let us first understand what we mean by heat-exchanger. Heat-exchanger is a device, which is used to cool or heat up a fluid using another fluid. The fluid which is to be heated up or cooled down is the main fluid, which is going to be used in the process or is already in the use in the process. So, this fluid is called the process fluid. And the fluid which is there to help in the heating or cooling of the process fluid is called the utility fluid.

So, we have two types of fluids here. For example, if we have say water which is to be heated up, so to heat up the water we may use some other hotter water or steam or some other kind of fluid. So, this cold water will be considered the process fluid, and the other water will be considered the utility fluid ok. So, now these fluids may be gas or may be liquid.

In the industries, you will find that in at various places, we are many a times doing this heating or cooling very very frequently, and everywhere you will find that there are heat-exchangers being used. We do heat exchange also at our home in a different manner, and there are various kind of configurations of these heat exchange. Now, let us take some example from our home. Many a times you have found that you are trying to cool down say hot milk or say hot tea or some hot water, and what you do for that is that you take a

glass of the hot milk or hot water, and you dip this glass in a bowl, which is containing some tap water, which is at a lower temperature than the hotter milk or the hotter water.

So, in this way you find that through the wall of the glass, heat is exchanged, and due to which you will find that the hotter liquid gets cooled down, at the same time, you will find that the liquid that is the tap water, which was there initially in the bowl is also getting a bit warmed up. So that way you see that there is a heat exchange between the two liquids, and you are able to get the cooling of the hotter milk or the hotter water.

So, in this case, you can see the hotter milk or the hotter water is the process fluid, and the tap water which was used to cool down is the utility fluid. Another way you can see that many a times you try to cool down the hot water, which we get from the geyser to take shower. Now, in that case what you do, you mix up the water from the tap directly into the hot water obtaining from the geyser, and you try to cool it down.

Now, in this case you will see that again you are getting some cooling, but now you get some mixing also. And in this case the geyser water is the process fluid, and the utility fluid is the water from the tap. But, in this case the heat exchange is occurring by the direct contact between these two fluids ok. So, in this you see that we are having two types of heat exchange one is the indirect cooling that is the 1st example I gave you, and the second one is the direct cooling the second example I gave you. So, this is one two configurations.

Then there could be in the industries, you will find that only one contact may not be sufficient, you need to have sufficient contact to cool down or heat up a particular process fluid. So, in that case you need some more contact time or sometimes you need to give more space to get this kind of cooling or heating. So, you will find that in the industrial applications, there are various types of heat-exchangers possible.

And moreover the more and more work are being done on this heat-exchanger innovations why, because we are trying also to reduce the size of the equipment, so that we can get some advantage in terms of the capital cost also in terms of the operational, and maintenance cost of the equipment. So, this kind of research work is going on, so that we can also bring down the size ok.

So, in the industries today you will find that there are some conventional heat-exchangers, and there are some non-conventional heat exchangers. Now, the double pipe heat-exchangers, which we are talking about in this particular problem is belonging to the conventional type of heat-exchanger. Now, in the double pipe the name signifies that there are two pipes, and these two pipes are containing the two fluids.

So, let us let me first tell you how these two look like. So, let us say there is a pipe of a some diameter, and into this pipe we put another pipe of smaller diameter ok. So, these are these are two pipes, so I can say this is another pipe ok. And these two pipes are generally put in a coaxial manner, so that they have the same axis my drawing is not showing that same axis, but it is generally put coaxially their axis. So, along this axis, this is symmetric you can see.

Now, you see that in these two pipes you are having two fluids. So, suppose I am putting fluid one over here ok, and this is the annular space between the inner pipe and the outer pipe. And the other fluid may be going through this the inner tube ok. Now, in you see that again in this case there could be variations in the configurations, I can put this particular fluid in the same direction as the outer fluid or I can also put it in this direction ok.

Now, if it is in this direction, then we call them parallel flow, this is called parallel flow heat-exchanger. And in this case if we are talking of these two, then in this case we have what we call the counter current flow. This parallel flow is also sometimes called the cocurrent flow. So, cocurrent and counter current of course the counter current is also parallel to these two, so it is better to call them concurrent and counter current. Other than these two configurations there could be configuration, where you find that the outer fluid is coming like this that is it is coming at a 90 degree angle to the inner fluid.

So, in this case suppose I talk of this one or this one, both of these will be called the crossflow. So, we have these various configurations in a heat-exchangers ok. Now, with these various configurations we also have some other configuration in the sense that we may have multiple number of the inner tubes ok. And only one outer tube and multiple of inner tubes, in that case we call it a shell and tube, tube heat-exchanger ok. So, I will show a few diagrams of a shell and tube and this and other heat exchangers shortly.

So, whatever it is we understand this that these heat-exchangers can have different configurations, and the choice of the configuration would depend on the objective that how much heat we need to take out or we need to give in, and there are many other considerations in terms of the pressure drop in the heat-exchanger, in terms of the size of the heat exchanger also changes with the change in the configuration of the flow of this two fluids.

So, there are various considerations need to be taken into account to decide the type of the heat exchanger to be used. And these issues are covered separately in dedicated courses on heat-exchanger and heat-transfer. So, I will not be going into the detail of those issues in this lecture. So, here we have the heat transfer using a double-pipe heat exchanger. A liquid stream is heated by the condensation of steam at constant temperature T_s .

Now, you see that here the colder liquid is to be heated up using steam. And here we are saying that steam is at a constant temperature, now when can that happen? Generally, you know whenever the heat is been transferred from a given fluid, the fluid is supposed to get cooled down. But, when we are talking of a saturated vapour, then you see that if the saturated vapour is being condensed, and it is going to a saturated liquid state, then during these process what we find that the temperature would remain constant, if the vapour is of a pure component ok.

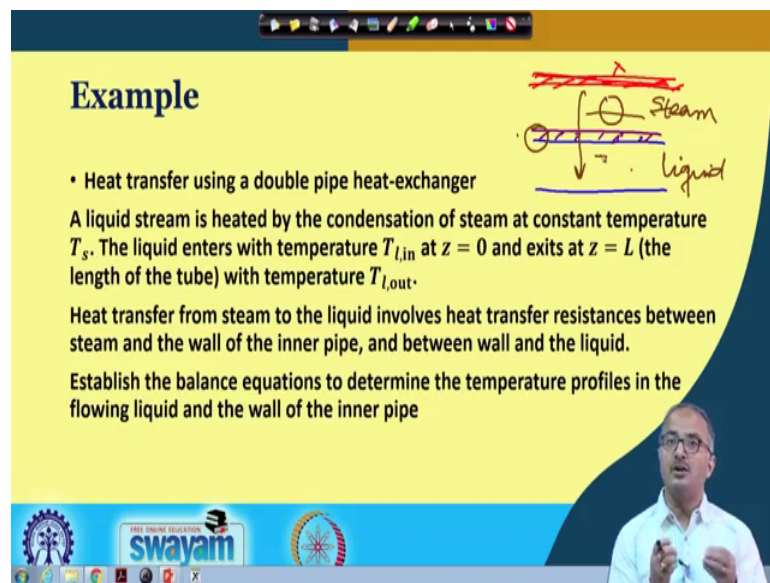
So, steam when it condenses to saturated water, it will be having it will be releasing the latent heat of vaporization which is quite substantial, and that will be enough to heat up a colder liquid ok. So, this is why we are saying that the steam is at a constant temperature T_s . On the other hand, you see that if in reality you find that after the condensation, the condensed water may also get sub-cooled from the saturated state to the sub-cooled state at the given pressure.

In that case also you can say that some amount of heat would be released, and this heat will be coming from the sensible heat transfer from the saturated water to the sub-cooled water. However, you will find that generally for some finite or reasonable temperature difference, you see that the contribution of the latent heat of vaporization is generally much more than the sensible heat contribution. And that is why you see that we many a times neglect the contribution of the sensible heat in comparison with the latent heat of

vaporization, unless otherwise we have a very large ΔT ok. So, in this problem we are assuming that the sub-cooling effect is negligible in comparison to the effect of the condensation of the saturated vapour of the steam.

Now, the liquid enters at in the inner tube at a temperature of $T_{l,in}$ and it is going out at a temperature of $T_{l,out}$ ok. And we are demarcating the inlet and outlet of the inner tube with this Z . The Z signifies the direction of the flow, at the inlet it is having a value giving a value of 0, and at the outlet we are giving it a value of L that means, the length of the inner tube is L . Heat transfer from the steam to the liquid involves heat transfer resistances from steam and the wall of the inner pipe, and the between the wall and the liquid. Now, you see that whenever there is some heat transfer occurring there will be some resistance to heat transfer.

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Example

- Heat transfer using a double pipe heat-exchanger

A liquid stream is heated by the condensation of steam at constant temperature T_s . The liquid enters with temperature $T_{l,in}$ at $z = 0$ and exits at $z = L$ (the length of the tube) with temperature $T_{l,out}$.

Heat transfer from steam to the liquid involves heat transfer resistances between steam and the wall of the inner pipe, and between wall and the liquid.

Establish the balance equations to determine the temperature profiles in the flowing liquid and the wall of the inner pipe

The diagram shows a cross-section of a double pipe heat exchanger. An outer tube contains steam, indicated by a red circle and the word 'Steam'. Inside this is an inner tube containing liquid, indicated by a blue circle and the word 'liquid'. Arrows show the flow direction. The lecturer, a man with glasses, is visible in the bottom right corner of the slide.

Now, when you stock of now if I draw here the inner tube first ok, and then what I draw? I draw with this I draw the pipe, this is a suppose a pipe the pipe wall. So, I am drawing on one side the same thing will be repeated on the lower side also. And here I am drawing the outer tube ok. And outer tube will also have some thickness wall thickness ok. So, here you can we can also put one wall thickness over here, this is the this shape portion is the wall.

Now, you see that now we are considering only the heat transfer between the outer tube and the inner tube. So, we are neglecting the heat transfer over here. So, we are not

concerned with that heat transfer. So, we are not concerned with the wall of the outer tube. Now, when the heat is getting transferred, so let me out with some other colour. So, when a heat is getting transferred from here to here, it is encountering three resistances. One resistance will be within this steam, so this is steam and this is the liquid.

So, we shall be having three resistances. One resistance will be within the steam ok, one resistance will be within the wall of the inner tube, and another resistance will be within the liquid. Now, you see that in the steam and in the liquid, we neglect we have generally we have three types of heat transfer. One is conduction, convection, and radiation.

So, in the for any kind of fluid generally the fluids have very very low conductivity. So, in case of steam and liquid, we neglect the conductive contribution, we talk of the conductive contribution. Moreover, for the radiative heat transfer to be appreciable, we need to have very very large temperature difference, because you know that for the radiative heat flux the heat transfer depends on the T raise to the power 4 ok.

And you can easily figure out that this radiative heat transfer will be effective, only when the ΔT will be quite large ok. So, we are also neglecting the contribution of radiative heat transfer for the kind of applications, we are talking about ok. But, I am this is not a mandatory assumption, you can first you have to first check the difference between the temperatures of the hotter and the colder fluid based on that only you should account for or neglect the contribution of the radiative heat transfer.

Now, you see that for the steam and the liquid, we are going to consider only the convective heat transfer, so that there are only convective resistances within the steam and in the liquid. On the other hand for any kind of solid, there is no contribution of the convective heat transfer. So, it is only the conductive heat transfer. And as I have just said that we are neglecting the radiative contribution.

So, for the conductive heat transfer to be appreciable, we have to have a high thermal conductivity of the particular solid. So, many a times we assume that the thermal conductivity of the solid is quite high, so that within the wall of the tube there would not be any resistance due to the conduction, so that what it means is this that within the wall of the tube there will not be any temperature gradient ok.

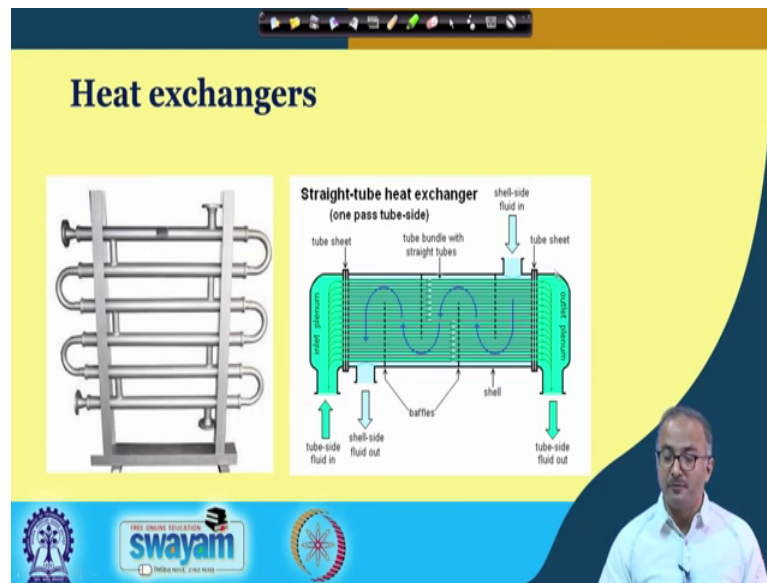
The wall temperature will be constant throughout the thickness of the wall. And this kind of assumption does not incorporate much of error in the final result. And also we have to understand that we choose the material of construction of these tubes in such a manner that the wall has very very low resistance, because we want the heat exchange to be effective.

If the wall also offers a large resistance heat transfer, then what you would land up with you would land up with a bigger heat exchanger. So, from that point also we have to see to it that the material of construction of the tube wall should be such that it does not offer any kind of resistance to heat transfer. So, with these considerations, we can neglect the heat transfer resistance in the wall of the tube.

So, now that is why we are saying that the resistances are lying from the steam to the wall, and then from the wall to the liquid that is within the steam and the liquid. And now we have been asked to establish the balanced equations to determine the temperature profiles in the flowing liquid, and the wall of the inner pipe.

Now, you see that that due to the heat transfer, the wall of the tube will be having a gradient of temperature not in a radial direction, but in the axial direction, because the these two fluids the steam and the inner liquid both of them are exchanging the heat, and also due to which this wall is the medium, so wall is also getting affected. So, there will be a gradient of the temperature in the wall in the axial direction ok. So, with this now we go to setup the model equation.

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Now, you see that because we have been asked out to find the temperature gradient, we will be looking into the energy balance equation before this, let me just put you how a typical double-tube or double-pipe heat-exchanger look like is sometime called double-pipe heat exchanger or sometimes it is also called tube-in-tube heat exchanger.

So, you can see that there are in this particular thing, it is a outside from outside how it looks like you see that there is a tube inside this one is showing inner tube, and this outer one is a here, and there is the inlet of the outer tube ok. So, in that the liquid you can say, it will come through this tube which is going like this ok, so we call it passes. So, this is one pass, this is 2nd pass, this is 3rd pass, this 4th pass, 5th pass, and 6th pass. So, there are six passes of the inner liquid in this case.

And this is the flow of the steam, the steam will come like this ok. And it will come like this, it will come like this, so steam is flowing like this. So, in case of steam also you will find there are various passes this is one pass, this is 2nd pass, then 3rd pass, 4th pass, 5th pass, and 6th pass. So, there are passes. And these passes are made why, because we do not want to have a very long pipelines.

If we do not put passes, then we will end up with some. So, long pipe then by calculation it shall come to kilometres, even that is unreasonable ok. So, we have to put a design in such a manner that we can make it very compact the reasonable size, and that is why we

to give more contact time between the two fluids, we make passes. So, this is a tube end tube configuration.

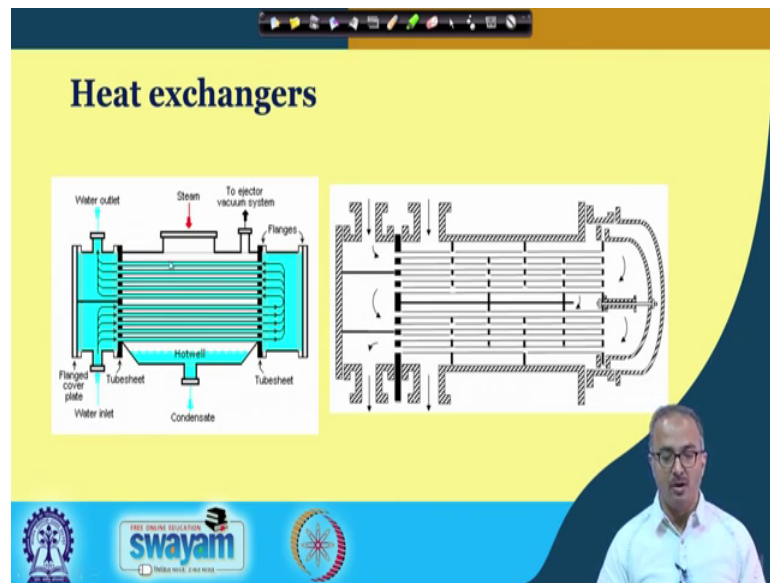
And as I told you there could be shell end tube, in a shell end tube it is something like tube end tube only thing is this the number of inner tubes are now increasing, and the outer tube you can say that is the shell. So, here you can see that from one side the tube side liquid is moving through this shell these tubes, and here you see that the shell side fluid is flowing and it is coming out like this.

And here you can see the baffles are being (Refer Time: 21:25) the baffles have various reasons. One is to hold the tubes in a places, and also to see to it that baffles also kind of affects the heat transfer between the two fluids ok. So, I am not again going into detail of the baffles, because the baffles have various types of designs also, so that is not under the purview of this particular course ok.

And these tubes are held at the two sides with some tube sheets ok. And so you can see that the steam this you can imagine that the steam is going like this, and the liquid is going like this. Now, you had see here that overall it looks that as if the hotter fluid is say coming from this, shell fluid is coming from this side, and going out this side, and the tube side going from this side, and coming out this side. So, overall it looks like a counter current flow.

But, if you look inside, you can see that here this tube is going like this, and the fluid is passing like this. So, if you follow this particular path, you can see this is something like a cross flow. So, overall it is counter current flow, but within this you can see something like a cross flow happening ok. And this is we have we call it one pass on the shell side, and one pass on the tube side. We may have multiple passes on the both the sides.

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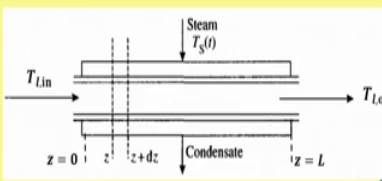
So, here you can see a configuration, where the tube side thing is passing through this the tubes, and it is taking a U-turn and going through these tubes. So, there are two passes on the tube side. One is this pass and this is the 2nd pass. So, there are two passes on the tube side. On the shell side, we have only one pass. And is also you can see it is a cross flow. Here in this configuration you can see that the tube side fluid is going from here, it is first going like this a first pass, then it is taking U-turn, this 2nd pass then taking 3rd turn 3rd pass, and then it is going like this with the 4th pass. So, there are four passes on the tube side.

On the other hand when you look at shell side, you can see the shell side fluid is flowing like this, it is moving like this, and of course it is moving like this taking a configuration like this is a configuration, and then it is taking 2nd turn over here that means, this is one pass for the shell side is taking a second turn over here, again it is moving like this from here like this, it is moving, it is going out. So, you can see here that we have two passes on the shell side and four passes on the tube side. So, this is how we count the passes on the shell and tube heat exchanges though these are just some examples to give you some feeling, how these heat-exchangers work.

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Analysis

- Consider the control volume between z and $z + dz$ for both the liquid and the wall between steam and liquid
- Assumptions:
 - Liquid is in plug flow $\Rightarrow$ one-dimensional flow (in z direction)
 - Heat transfer in the axial flow is only by convection and conductive heat transfer within the liquid is negligible (that is, no diffusive term)
 - Steam temperature is constant
 - No wall resistance to heat transfer
 - Liquid density and specific heat are constant



swayam

Coming to our problem; so, this is how we have drawn the particular figure, so here we have the outer tube, this is the inner tube, the liquid is flowing through this inner tube, the steam is coming through the outer tube, it is getting condensed as a function of given temperature, and this temperature may be a function of time. May be a function of time because; the saturated temperature is a function of the pressure.

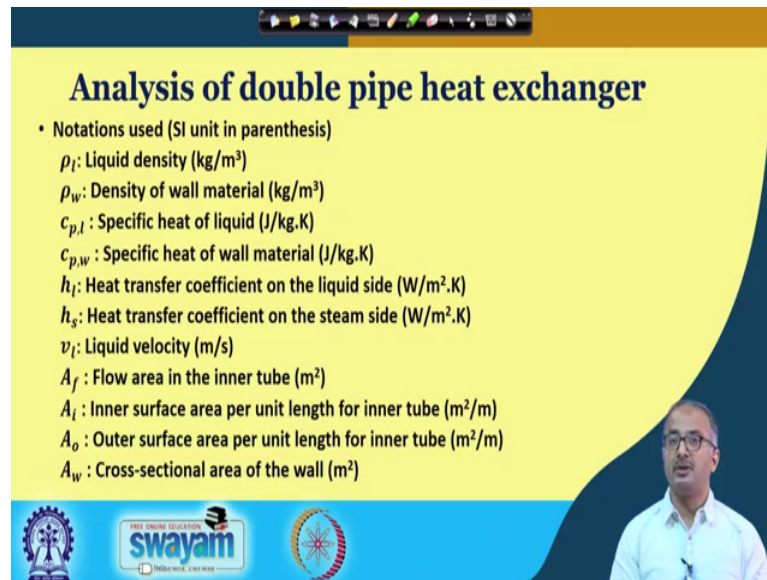
So, depending on the pressure if the pressure is varying, then the T_S will also vary that is why, the T_S has been shown to be a function of time ok. And now we are demarcating the two ends of the tube like Z equal to 0, and Z equal to L . And you can see the steam is condensing, and we are getting the condensate over here, this is we are assuming it to be saturated liquid.

Now, we are assuming some things like that we assume that the liquid is in plug flow, when it is plug flow that means, there is a good mixing in the radial direction. So, there is only one velocity involved, and we call it a one dimensional flow. And this flow is taken to be in the Z direction. Then we have the heat transfer in the axial flow is only by convection and conduction through within the liquid. As we say that the liquid we are talking only of the convection, and the conductive heat transfer is neglected.

And the steam temperature is taken to be constant. As I said that the wall is not this wall is not offering any resistance to the heat transfer. And we are assuming the liquid density, the liquid specific heat are remain in constant. And this particular assumption is not bad

for the liquids, because you will find generally the within some reasonable temperature difference the properties of a liquid do not vary drastically. But, in case of gases this will not be the cases in that case, you will have to generally take the variations of the properties with the pressure and the temperature.

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Analysis of double pipe heat exchanger

- Notations used (SI unit in parenthesis)
 - ρ_l : Liquid density (kg/m^3)
 - ρ_w : Density of wall material (kg/m^3)
 - $c_{p,l}$: Specific heat of liquid (J/kg.K)
 - $c_{p,w}$: Specific heat of wall material (J/kg.K)
 - h_l : Heat transfer coefficient on the liquid side ($\text{W/m}^2.\text{K}$)
 - h_s : Heat transfer coefficient on the steam side ($\text{W/m}^2.\text{K}$)
 - v_l : Liquid velocity (m/s)
 - A_f : Flow area in the inner tube (m^2)
 - A_i : Inner surface area per unit length for inner tube (m^2/m)
 - A_o : Outer surface area per unit length for inner tube (m^2/m)
 - A_w : Cross-sectional area of the wall (m^2)

So, here I have shown you the various notations, I have used in this particular formulation, and I have also given you for sake of your understanding the SI units for these various notations why? Because, whenever you are writing the balance equations, you should be very very careful about the dimension on the two sides of the equations.

And you know that of there is dimensional inconsistency, then you are having a wrong model equation, and wrong model equation will lead to wrong result. So, I have put all these units to use, so that you can check with these units whether you are getting dimensionally consistent energy balance or not.

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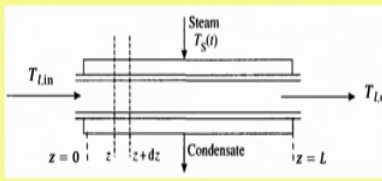
Analysis

- Choose rectangular coordinate system
- Variables involved:
 - ❖ $0 < z < L$
 - ❖ $t \geq 0$
 - ❖ $T_l(z, t)$
 - ❖ $T_w(z, t)$
- Energy balance for the liquid

$$\rho_l c_{p,l} \frac{\partial T_l}{\partial t} = -v_l \rho_l c_{p,l} \frac{\partial T_l}{\partial z} + \frac{h_l (T_w - T_l) A_l}{A_f}$$
- Energy balance for the wall

$$\rho_w c_{p,w} \frac{\partial T_w}{\partial t} = -\frac{h_l (T_w - T_l) A_l}{A_w} + \frac{h_s (T_s - T_w) A_o}{A_w}$$

We get a set of two coupled PDEs



So, here we have the balance equations you can see, the various variables which are involved in this particular system (Refer Time: 27:14) z that is the axial position. Now, you see the z is being varied between 0 and L without including the 0 and L why, because we have been given the rules of the temperatures at the inlet and the outlet. So, we will be solving the energy balance equations only for the region within this 0 and L ok.

Now, this is you can see that this is the situation, when the boundary conditions are of type one or (Refer Time: 27:46) boundary condition. So, if you have normal boundary condition that means, you do not know the outlet temperature, you may specify say the first derivative of the temperature with respect to the axial position to be 0 that means, $\frac{dT}{dz}$ to be 0 that means, you do not know the temperature at z equal to L . In that case, you have to include that z here with L , so you will put less than equal to L . So, for this particular problem, we are neglecting the calculation of the temperature at the position L .

Then time is taken to be more than equal to 0. Now, understand this what we understand by time equal to 0, time is not 0 actually. In any process, whenever we are analysing it, we take the time to be 0 at the start of the process ok. So, we are generally not concerned with the history of the process many a times. In some cases, we have to know the history in but those cases are different like, for example in non-Newtonian flow, a non-

Newtonian fluid dynamics sometimes we are also concerned with the history, because the stress strain relationship depends on the history of the particular fluid, but that is not the case here.

So, generally we take the time to be 0 for the starting time of the process, and we always march in future, so that is why we say that time is more than 0 ok. And this not the time of the hour watch ok, this is the time of the starting of the process ok. And then we have the T_l that is the liquid temperature, which is a function of the axial position and the time. And we have the wall temperature of the inner tube that is the function of the axial position and the time.

Now, let us write the energy balance for the liquid and for the wall, and you can see that in the case of the liquid, you will find that this is this term is giving us the how much energy is being changed within the particular liquid ok. So, this is the accumulation term. And then for the liquid, we have the convective term ok. And convective term we are talking about only one-dimensional flow. So, we are having only one component of the velocity. And then we have we are neglecting the conductive heat transfer. So, we do not have any conductive contribution over here.

And what we have here now is a source term. This source term is coming due to the transfer of the heat from the wall to the liquid. And this term is taken in terms of the Newton's law of cooling, and that is why we are taking at $h_l \Delta T$. And all these other A_f and A_i have been explained in the notation, this A_f is the cross sectional area of the inner tube ok. So, you can easily derive this particular equation for the liquid side energy balance.

And then we come to the energy balance for the wall. Again we are having this term, which is signifying that how the heat within the particular solid or the wall is changing, so that is the accumulation term for the wall. And in case of wall we have neglected the convection heat transfer. So, we do not have the convective heat transfer term unlike the one we have in the case of liquid. Here we have and as we say that we have also neglecting the heat transfer resistance due to the conduction in the solid. So, we do not have that term also only what we have the source terms.

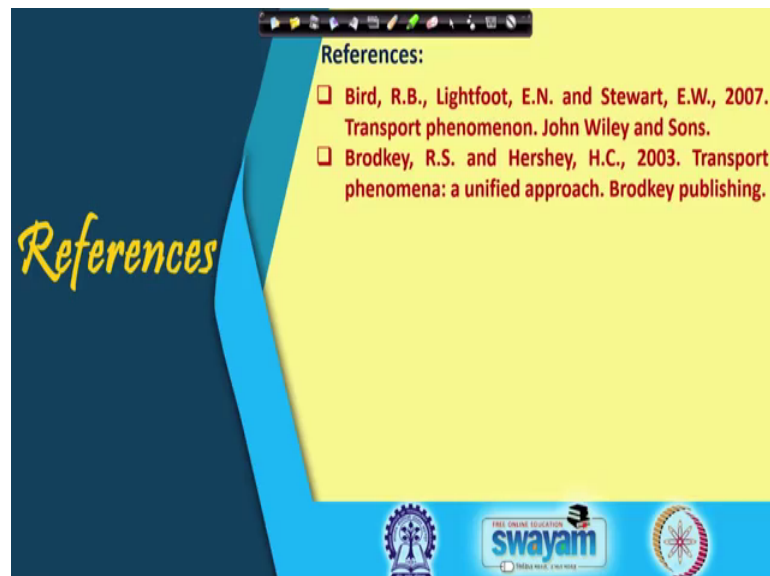
And these source terms are coming why, one is because the heat is transferred from the steam to the wall. So, this is coming within the wall, and this is going out of the wall that

is why it is negative sign ok. So, this is going from the wall to the liquid ok. So, you see this sign signify the direction of the flow of the heat ok. And that is how, we are able to derive the energy balance equation for the wall.

Now, you can see that with these two equations in hand, you see that they will give us the variation of the temperature of the tube of the wall, and the liquid. And in these two equations you find that if I want to solve this equation for T_l , I need the value of T_w . But, I for knowing the value of T_w I have to solve this equation, but this can be solved for if I know the value of T_l . So, what I find that we have two unknowns, and these two unknowns are appearing in both the equations.

So, we are having two coupled set of partially partial differential equations, which have to be solved simultaneously to get the solution ok, but this not the end of the situation, because we can solve it only if we know the initial condition and the boundary conditions for this particular system. So, I will be stopping at this the derivation of this balanced equation. In my next lecture, I will continue this problem to tell you how to specify the initial condition and the boundary conditions, so that we can solve these equations.

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You can know more about this process from these references.

Thank you.