Mass, Momentum and Energy Balances in Engineering Analysis
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Lecture – 27
Microscpic Balance – IV

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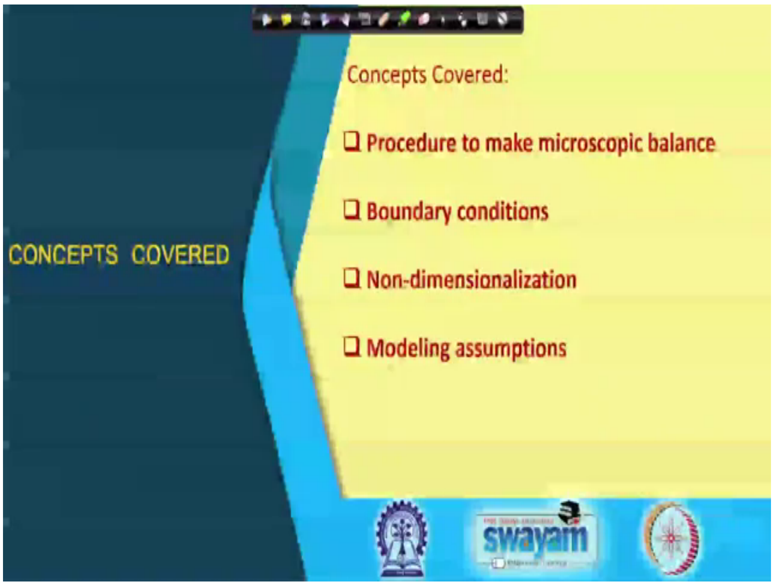

Welcome back today in the continuous series of the Microscopic Balances we shall be looking more into this aspect.

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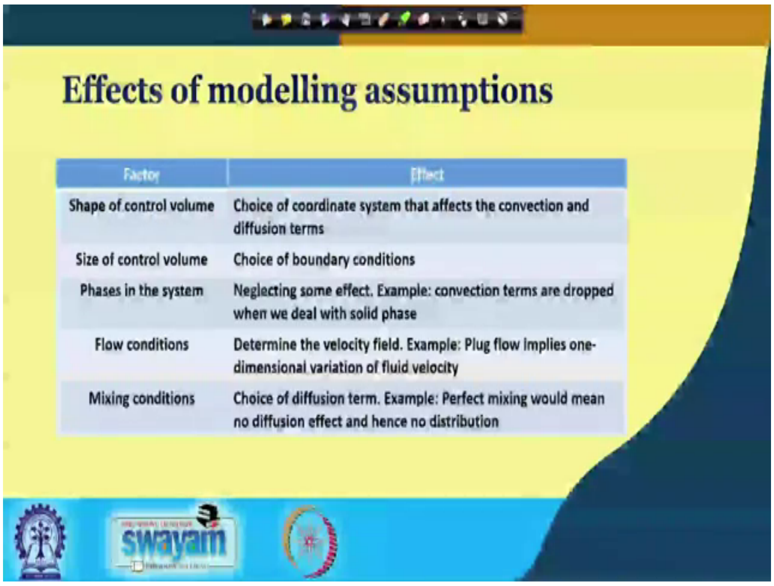

And we shall now see the what are the kinds of modeling assumptions we make and what are the effects on the balance equations. So, first is the shape of the control volume and as we have seen in our earlier lecture that depending on the shape of the control volume we have to choose the appropriate coded system. For example, if we have a rectangular or a cuboid structure then we will choose rectangular coordinates. If we have some cylindrical structure generally we go by syndicate coordinate. And if you have some spherical structure we go by the spherical co ordinate. Now, in this case you must be very careful in understanding that when we talk of the control volume and when we talk of the actual system these two may not be the same.

That means, that whenever you are studying some system you choose the control volume may be within the system, because your focus may on the particular domain of the overall system. So, depending on which domain is our interest the shape of the control volume will depend on that particular domain and not on the physical boundary of the actual system. And this particular point I also explain to you in earlier lecture when I told you that suppose you have a sphere and in the sphere we define generally two boundaries; one at the centre of the sphere and at the periphery of the sphere, but actually there is only one boundary that is the at the periphery of the sphere.

So, for mathematical purpose we define two boundaries that is for the control volume. We define two boundaries one at the centre and one at the periphery. So, similarly you can see that depending on what you want to study in what region you want to mix the studies you have to choose the appropriate shape of the control volume and depending on the shape of the control volume you have to choose the appropriate co ordinate system. Next, we come to the size of the control volume. Now the control volume size will depend on how much you want to study. Like earlier when we are talking about the macroscopic and balances and in that what we found the in a macroscopic balance when we were doing the overall system you are looking into and we also assumed that all the properties all having some average value or some a single value.

So, in that case you have to put the size at the overall system where we were not will be bothered about the distribution or the profile of any of the extensive or intensive properties. On the other hand when we went to the microscopic balance what we did we disintegrated the whole big system into small small pieces. And then we started looking into what is happening in that particular small or what we call the (Refer Time: 03:56) control volume. So, in that you see the depending on the size of the control volume we will be having different types of information.

And in case of macroscopic balances we choose a very small size of the control volume over which we make the balance equations or apply the conservation laws and then look at the whole domain to find out how the particular variable like temperature concentration etcetera are varying over the control volume and this is first. And second thing is this that when we go for the selection of the boundary conditions that is this size is very important to us why. Because some many a times we say that we have infinite boundary. Now how do we realize this infinite boundary? It is not that any physical domain will have some kind of finiteness. Infinite means that the with respect to the region of our study the boundaries of the physical system may be very far away.

In that case what we do we many a times neglect the effect of the boundary which is far away from the control volume of our study on interest ok. In that respect we can say that at the far boundary we sometimes assume that the particular variable is having some constant value or we can also assume the flux of the particular variable is 0. That means we may either assume at reach that boundary condition or we may assume a normal boundary condition. Now 0 I just give you for example, you it may also have some non 0 values as well ok.

So, that means, the infinite volume in a mathematical term is given, will be given in terms of the size of the control volume of our interest. Next comes the phases in the system many times we find that we are during with not similar phase what mean you say this multiple phases. Phases means, we may be having liquid with solid and then gas or vapor. Now you see that for example, if you talk of a liquid evaporating from some phases into the atmosphere. Here we find that there may be in a liquid phase one liquid phase and from that liquid phase and the vapor phase is getting generated ok.

Similarly suppose you are hitting some liquid on some oven then what we find; that from the bottom of the vessel we find that vapor bubbles are formed and which then moved through the liquid. So, in that case what we find that initially which was a single phase system we are generating two phase system. Now these two phase characteristics will be different from the single phase characteristics; that means, when you want to know the temperature distribution within this particular liquid. Now when the liquid is in single phase you will find due to this heat there will be a certain temperature distribution on the other hand when there are two phases.

That means, the bubbles are now getting generated in the liquid you will find that the temperature distribution has got affected, because if there will be now some energy interactions between the bubble or the gas and the liquid due to which the temperature profile will now change ok. So, that is why we have to be very careful to understand the effects of the various types of phases involved. And please understand that while the process is going on there will be a number of phases may change right. I have given the example of the boiling of the liquid conversely you can also think like this, that is liquid is suppose now flowing through a pipe line.

And imagine that the pipeline is exposed to a very low temperature and that low temperature is lower than the freezing point of the liquid. Then what will you see? That as the liquid is flowing through the pipeline you will find that may be after certain time or after certain after traversing certain length of the pipe line the liquid now get frozen. In that respect also you find that two phases are getting generated from a single phase ok. So, in these method you will find that in our day today life many process are there where it is involving their phase changes ok.

And whenever there is a phase change you will find that you have multiple phases to account for and when you want to know the temperature distribution or constant distribution or the velocity distribution in each of the phases you have to write these momentum balance, the energy balance and the mass balance for each of those phases separately.

So, that you can get the profiles of the concentration velocity and the temperature in each of the phases ok, that is one. Second thing was this many times whenever you are using some kind of reactor we find that in a reactor we are using maybe some catalytic reactor we have and we have some catalysts which are generally in solid phase. And through this catalyst bed we find that the reactants are flowing the reactive may be in the liquid phase or may be in the gaseous phase ok. So, whatever it is you find that in the following and some reaction is happening through this. Now if you want to know the temperature distribution within the catalyst ok. Then in that case you will write in energy balance for the catalysts, for the bed. If you want to know the temperature distribution in the liquid or the gas you have to write and energy balance separately for that.

Also you might it that when you look at the overall microscopic balance like the (Refer Time: 10:21) equation. You will find that many of the terms all the terms may not be important for each of the phases a for example, in the solid phase the if the solid is considered to be remaining static in that case what you find the solid will remain static so it will not be moving. So, any kind of velocity associated with the solid will be neglected; that means there will not be any convective heat flow by the solid ok. So, for the solid it is on the conductive heat flow.

So, when you look at the energy balance equation you will you can easily neglect or the conductive terms and you will be having only the conductive term with you. And apart from this you may have some source term depending on whether there is any kind of energy interaction between the mass within the (Refer Time: 11:17) and outside or between the solid and the gaseous phase ok. So, these kind of sources will come the single you been look at the liquid and gaseous phase in that case you find that you cannot neglect the convective terms.

Now also we know that for the liquids and gases the conductive terms are generally negligible in comparison to the convective terms. So, if that is the case, in that case you again see that you can simply the energy balance equations by simply dropping or the conductive terms and retaining only the convective term ok. So, in this way you see that by considering the kind of phase involved in the particular process you are able to neglect many terms and that is how you are able to simplify a three dimensional equation may be to one dimensional equation ok. Similarly you can do these kind of analysis for many other purposes.

For example, if you want to know that how the some pollutants are getting distributed in the air, many a times you find from the industries from you have might have found chimney and from there you find that some flume gases are coming out. And especially if you pass by some refinery or some fertilizer plant or some other chemical plant you will find that something is coming out the top. And what you find sometimes when you dark coloured, black coloured flumes are coming out. What are basically they are basically the carbons ok. Similarly the many other gases also come out from these industries and they get into the atmosphere. Now may a times you find that we are talked and told about the acid rain etcetera.

So, what are these? These are nothing, but the emission which are happening from these sources, may be industrial sources they are getting trapped by the moisture which are getting frozen in the cloud. And those things like some nitrous oxide or some sulphurous oxide those things are getting trapped and now they are coming back to the earth as what we call the acid rain ok. In that case also when you want to know that how these pollutants are getting distributed in the atmosphere, how they are getting trapped or frozen by the atmospheric moisture; all those things when you want to study, these you can study very well with these balance equations.

And in that case as I have just given you the example if you understand the particular system you can simplify a 3D equation to may be a 2D or 1D equation that is going to help you to get a very good understanding, it is not that by simplification you may not be always getting bad result it is not. So, because you may find that even if you are constrain some 3D problem and you are not simplifying it you may find that ultimately to the result your obtaining after. So, much of effort is no better then what you are getting by solving for a 1D problem ok. So, it is always our intention to see to it that how we can simplify a 3D equation to 1D or 2D ok.

Here I am not touching upon the unstatistic part or statistic part, because that will depend only on the whether the process is studied or un studied. So, that is on the big concern for us only this in modeling this 3D the simplifications becomes by not having the 3D to 2D ok. So that why I explain that how the phases of the system are important factors in having some a simplification in the model equations.

Next comes the flow conditions, now flow conditions means that what kind of velocity field we know one are envisaging ok. Now it may be so that in generally you find that whenever is a flow is happening we know that velocity is a three-dimensional field ok. But many a times under situations will be there like for example, if there is a flow through a pipe and it is fully developed ok. And if we assume that there is a proper mixing and there is no axial transfer of anything any solute or something then we can might us will say that at a given cross section of the pipeline, there is only one velocity ok. So, that sometimes we call that velocity as a superficial velocity ok.

So, in that case if we can assume that these velocity is uniform over the whole cross section then you can see that as the fluid is moving there is only one velocity which matters to us. And this kind of the thing we call it the plug flow. Plug flow you can imagine it is something like a flow which you find in a syringe system, like whenever you are having the injection syringes ok. You can see that the you are put the plunger inside to drive out the particular in injectant ok. Now that is the kind of thing that is if the whole fluid is without any kind of flow distribution any kind of actual distribution the whole fluid is going like a plug ok. And if we can assume it is a plug flow then what happens that our whole 3D momentum balance equations reduced to only 1D momentum balance equation ok.

So, that is the important when and very important simplification we are doing. Another you simplification may be done in some cylindrical coordinate; in that what happens that many times that in cylindrical coordinate we can assume that there is axial symmetry. And what I mean by that axial symmetry? It means that suppose you have a cylinder with a axis at the from a center at the cylinder ok. Axial symmetry means that that if you move along the azimuthal direction that is theta direction in a cylindrical coordinate any particular value of say temperature or velocity or something we will not be changing if you kind rotate along a given radius and a given action position.

This is important for you to know that you have to keep suppose r theta z we are talking about at a given r and at a given z, if you change theta like this you will find that the particular value is not changing ok. This we call axial symmetry; that means, it is symmetrical about the axis. So, if you can assume that symmetry then what you find that you are able to neglect any derivative with respect to theta ok. So, this also coming by the kind of the flow conditions you have. So, you are able to make that kind of simplifications which understand this may not be true all the time you are some and very very careful whenever you are making such kind of simplifications you should be very careful. And the last one is the mixing condition.

Now what is mixing? Mixing means that we are trying to have we in mixing we have at least two species and these two species will be kind of going into each other that is very crudely we can define mixing ok. Now suppose we in our day to day life we are mixing. So, many things for example, you are mixing sugar in the milk, sugar in the tea sugar in the coffee. So, this is mixing problem when we are cooking something we are mixing the spices in our dishes ok. So, we in our day today life we are having so many examples of mixing even we are mixing for heat transfer we are mixing say hot water and cold water so that we can use it properly.

Because if it is too hot then we may not be able to use it, used too cold also we may not be able to use it. So, many a times we mix a hot water with cold water to come into bring to a temperature that can be we can handle ok. So, the all these are examples of mixing, now what does mixing do? Whenever there is mixing, mixing tries to break any kind of profile of temperature or velocity or concentration. Without mixing suppose as I if you get this well if you put some sugar in milk and if you do not stir it; that means, you are not mixing it then what will happen that you will find that it will take some time and how do you know that it will take some time for mixing.

Because if you test the milk from the top surface you will find that you are not it is not testing sweet ok. But if you give enough time you will find that from the top surface if you take some samples out the sweetness is increasing, it means that the sugar has started dissolving in the milk ok. So, this initially; that means, what happened that if you are not stirring it we can say there will be a gradient of the sweetness and the sweetness is nothing, but an indication of the concentration of the sugar or the glucose in the particular system in a milk ok. So, you have to look in that fashion.

So, sweetness is nothing, but the kind of expression of the concentration of the because you cannot see sugar, you cannot see the solid, but you can taste them to figure out the whether the concentration is more or less ok. Now without stirring you find that suppose you could take a sample right near from the sugar, sugar from where is sitting at the bottom of the particular container. You would find that the sweetness near the sugar lump is more than the sweetness near the surface of the particular container. That will tell you that there is a gradient of the concentration of the sugar molecules within the particular milk ok.

Now when you stir it what happens you know for you from your day today observation that as you stir it with spoon what happens you find that you are able to mix the sugar in the milk very fast. And what you find that if you now you do not bother really to where to taste it from you can taste from any depth of the milk and you will find the sweetness is the same ok. Whether you take from the bottom of the container of on the surface of the container you find the sweetness is the same. What it indicates? It indicates the stirring has held in mixing of the sugar with the milk and that has brought the whole system at an uniform sweetness that is uniform concentration of the sugar.

Now when you have this kind of situation where mixing is proper mixing is there; that means, you can neglect any kind of gradient within the system ok. Now this I have given the example from the point of view the species or mass balance ok. Similar thing you can also extend for energy balance, as I have told you the given a example of mixing hot water with cold water ok. You can might also takes some cold liquid and in from a tap you just have some hot water coming; you will find that it is taking longer time for the system to reach uniform temperature. But on the other end you stir this whole system we will find the system is coming to a uniform temperature earlier ok.

So, all these are examples of the rule of mixing in breaking the profile or the gradient of the properties and bringing the whole system at a single value ok. Now what is its effect on the modeling. Now, when you have this kind of system when you have proper mixing you can neglect all the distributions of the properties like; velocity, temperature, concentration of species, with respect to the coordinate directions. That means, for example, you can say there is no gradient in x direction y direction z direction. So, you can see that how easily if you look at again the (Refer Time: 24:25) equation you find all the dou by dou x dou by dou y dou by dou z all these terms are going to 0.

So that means, that is the effect of the mixing ok. Now you understand one thing that I am saying that dou by dou x dou by dou y dou by it does not mean that the velocity will not be there it will be there but it will be having a constant value. It is not, I am not making the velocity value to be 0, I am just making their gradient to be 0. So, understand the difference between the gradient and the absolute value of the particular property ok. So, gradient 0 does not mean that the value of the particular property or the variable is coming to 0.

Only thing is this by having this mixing assumption we are able to get rid of all kind of spatial derivatives of the system and that is how we are able to simplify the whole system. So, with this particular this modeling assumptions I hope that you have got fairly good idea that; how you can simplify a 3D equation to a maybe 2D or maybe to 1D system and that is going to help you out in simplifying your solution for the system ok.

In our next lecture we shall be now taking some examples and show you how we apply all this assumption for our many commonly observed phenomena.

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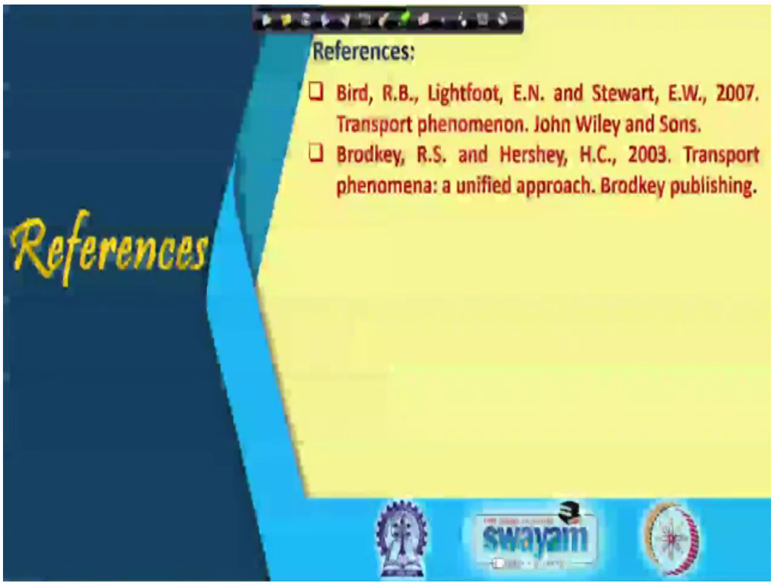

So, these are the references which you can refer to get more idea about all the aspect that have touched.

Thank you.