Mass, Momentum and Energy Balances in Engineering Analysis
Prof. Pavitra Sandilya
Department of Cryogenics Engineering Center
Indian Institute of Engineering, Kharagpur

Lecture – 25
Microscpic Balance Equations – II

Welcome. We started with the Microscopic Balances in the last lecture and in that I explained to you how to find the control volume, and how to start making the microscopic balances by considering a small control volume. And we did the continuity equation that is the mass conservation equation, and we started with the momentum conservation. In this lecture we shall be continuing on that and shall we talking more on this Microscopic Balances.

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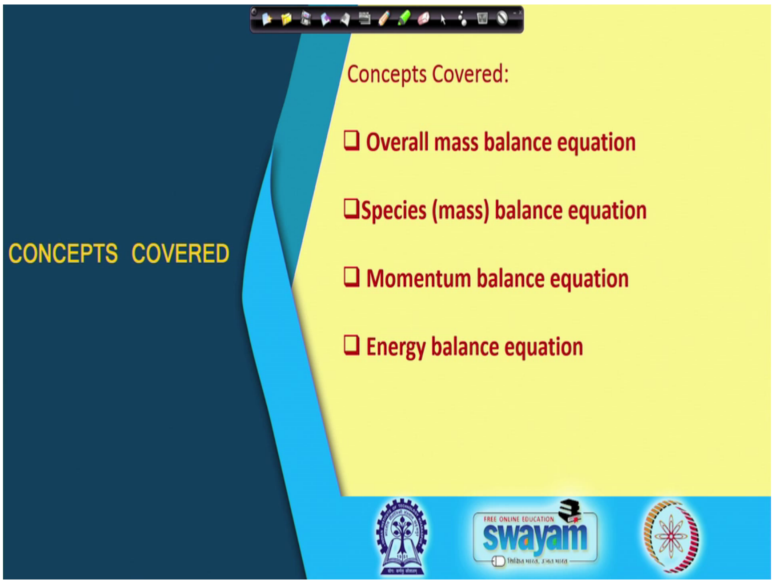

So, today we shall be covering up something on the momentum balance and something on the energy balance.

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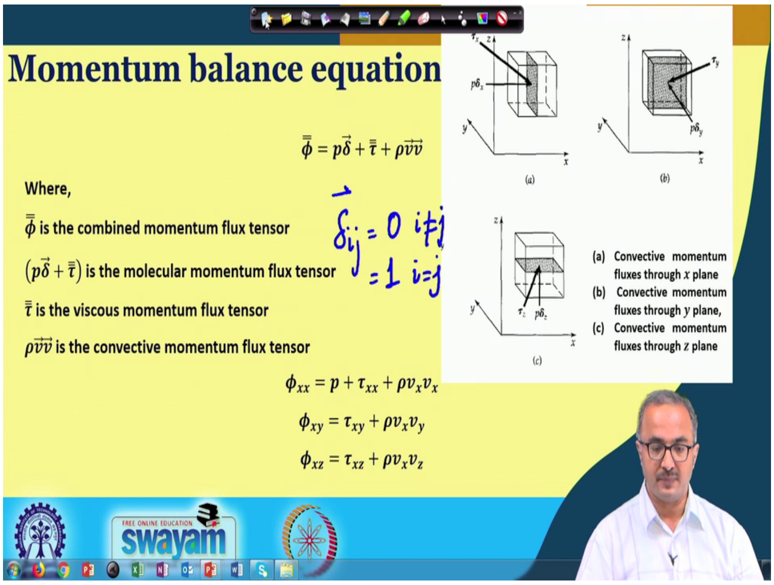

So, here to continue with the momentum balance I talk to you about phi in the last lecture. So, in this lecture I give you the expression for the phi, and tell the physical significance. So, phi is basically the total momentum which is the contribution of 3 components.

So, here you can see that as I told you in the last lecture that these total momentum flux is due to the molecular transport and due to the convective transport. So, in the molecular transport we have the viscous transport and also the contribution due to the pressure and the convective transport is due to the motion of the fluid. And we will have these different components of this tensor and you can see here for the x direction we have phi xx is equal to this pressure plus tau xx plus rho vxx. Similarly, we have psi x y, this is tau xy, rho vx vy and phi xz tau xz plus vx vz.

Now, please understand that there is no pressure term in this two expressions because pressure always adds normally to a surface and these represent the parallel to the surface. So, there is no pressure term in these two expressions. And similarly you can also find out the expressions for phi yx or phi yy or phi yz or phi zx, phi zy and for phi zz.

So, in this way you will find that in the 3 coordinate directions you have this kind of a set of flux fluxes and that will be due to the various types of velocity gradients you are having, and the pressure changes occurring in the various coordinate directions. So, here you can see in this particular figure, what we have tried to demonstrate is how you visualize this kind of momentum transfer due to this flow or the molecular transport. So, that this whole this is a particular control volume in a rectangular coordinate so, this is x, this is y, it is z and this is this with is delta x then delta y and then this is delta z, ok.

So, you can see that we in this 3 figures the, they differ the way ah this forces or tendency that we have drawn this particular cross sectional area that is a cut section or to show the yz plane. Here we have shown the xz plane and here we have shown the zx plane, and each of them you can see that in this case the pd x as well as telling you this is acting normally to this and here on this particular plane you can see p delta y is acting normally to this and all this particular plane you can see is p delta it is reacting is normal to the plane. So, that is why you will find that this p term will be appearing only ones out of these three tensors, ok.

And also you see that these kind of the tau is showing the all the shared and they will come parallel to the surface. And this delta is the chronicle delta perhaps you know that this chronicle delta will assume value of either 1 or 0 depending on to the pair of the vector which is pair of directions which are considered, ok. So, delta is this delta will be ij, we write delta ij equal to 0 if i is not equal to j and delta ij is equal to 1 if i equal to j. So, that is the, what we call the chronicle delta. This is 0 if i is not equal to j and this is 1 if i is equal to j, ok.

So, with this kind of notations you can easily see that how we write for this equation. So, this defines that how this particular terms are automatically losing these things.

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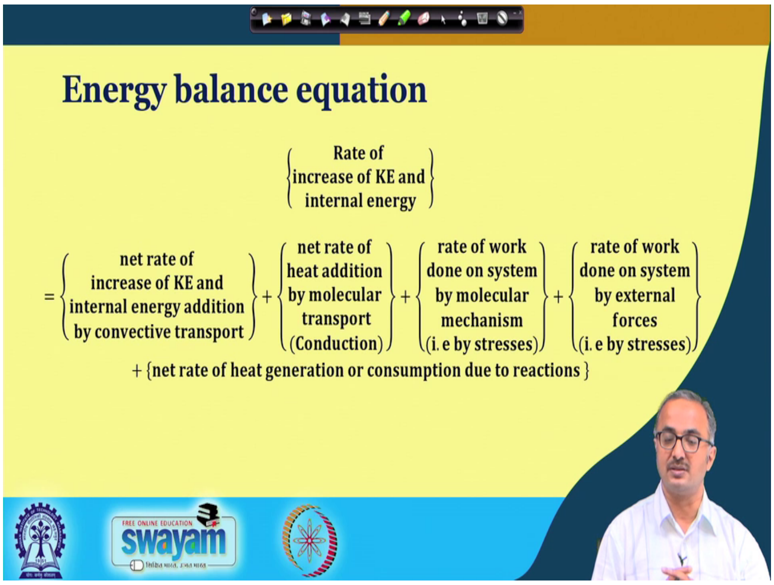

Now, let us go to the energy balance equation, ok. Now, in this energy balance equation it is similar to the one which we have studied for the momentum balance or the mass balance. Here you say that the net change in the energy of a particular system is given by the net of energy coming into the particular system and due to the construction and the net energy addition by the molecular transport or what we call conduction. And then rate of work done by some molecular mechanism like by stresses, and rate of work done by the external forces and net of any heat generation or consumption due to some reactions.

So, when you talk of energy, energy can come in various manners as we learnt earlier that work will also generate some kind of energy change system. And you know that there could be three forms of energy convection, conduction and radiation, and there could be some reactions which involve either production of energy heat energy or the absorption of the heat energy that is some may be exothermic or some maybe endothermic. So, all these kinds of things will generate some kind of energy change in the system, so those things have been taken care of in this particular equation.

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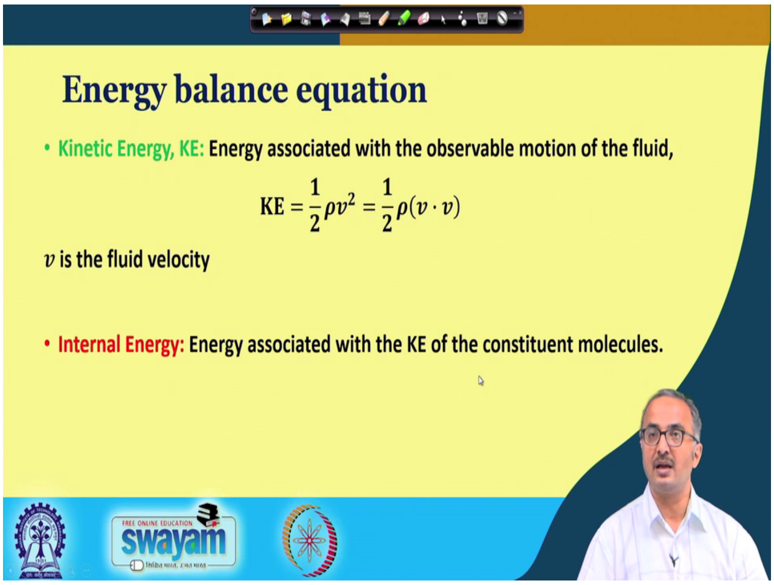

And here I have shown that the kinetic energy is how we put the kinetic energy per unit volume, and then we have the internal energy which is due to the temperature of the system.

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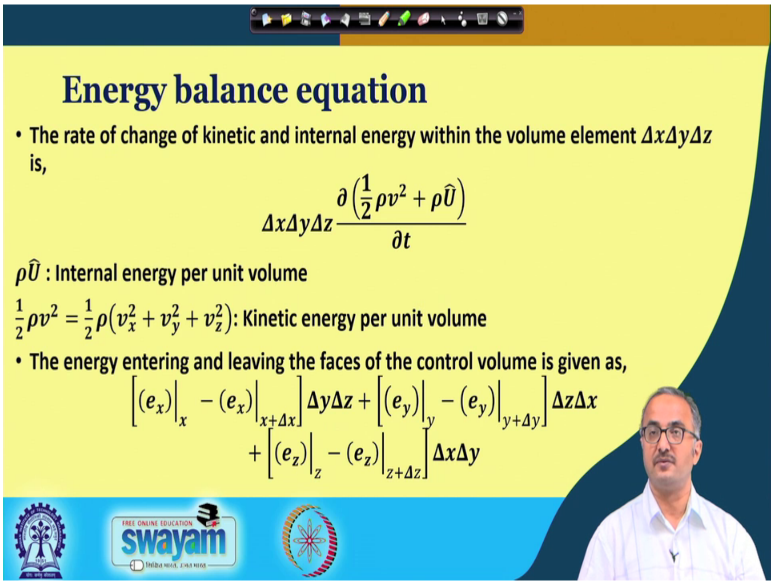

Now, then to find out the total change rate of change of the energy in the system is to multiply because these are the volumes specific values, so we multiply with the total volume. And because this volume is constant, so we can take it out of the differential; so we find that this total is the energy change per unit time of the particular system and then we have this particular term for the left hand side of the conservation equation.

And here I have shown that this rho U cap is showing the internal energy per unit volume and this particular thing is showing the kinetic energy per unit volume of the system. And here we are writing all source of energies which are entering or leaving the system. Now, please understand hereby e, I will show you later that what this e constitutes.

So, e is the effective energy that is either going or coming out of the system. So, here we have written that in the x direction something is going and something is coming out, and into delta y delta z gives is a rate because this things are fluxes; fluxes into the area of cross section gives the rate. Similarly for the y direction this is going in and this is coming out, and z direction this is going in and this is going out. And you can see here that in this case everything is coming under a single equation because it is a scalar form, based on vector it is scalar form, ok. So, we are getting only one equation.

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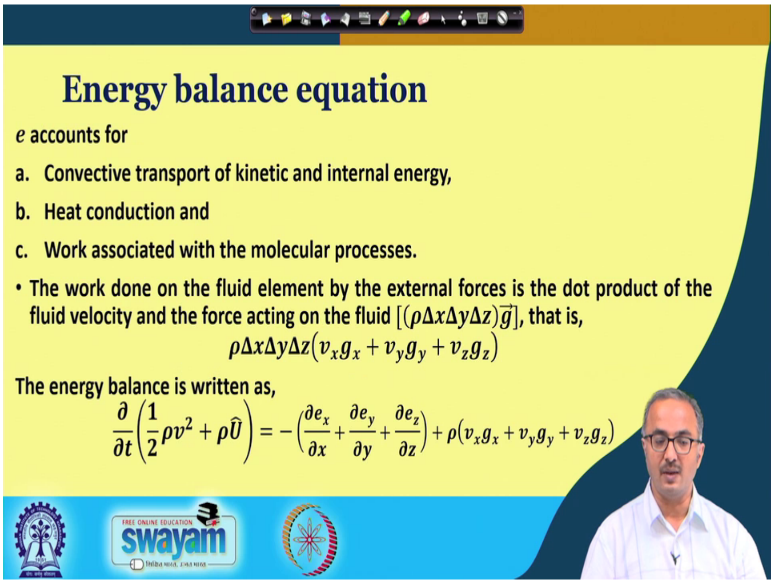

And now as I told you that e is the effective energy of a given stream. So, it has the unit the convective transport of kinetic and internal energy, then heat conduction, and any kind of work associated with the molecular processes. Work for example, that whenever a fluid is flowing, over a surface it has to work against the drag forces, ok. So, that is the kind of, that is that will lead to some kind of energy exchange, ok. There will be energy will be dissipated also do overcome this resistance to the flow, ok. And this kind of things will not be present when we are talking of solid, but under no flow situation this particular energy exchange will not be there. So, you can see that depending on the system to system you will have different types of energy exchanges involved.

So, the work done is given by this particular thing it is a dot product of the fluid velocity and the force acting on the fluid and here by g we do not mean the gravitational force only, we mean any kind of body force on the system and per unit mass. So, this is the mass of the system that is the density into the volume, ok. So, this give the total amount of force and this in dot product with the velocity gives us the work done, ok, Now, after putting all the values we find this is the final expression for the energy balance, ok. And in this you may also add, in this kind of exchange can also add any kind of generation terms due to the reactions.

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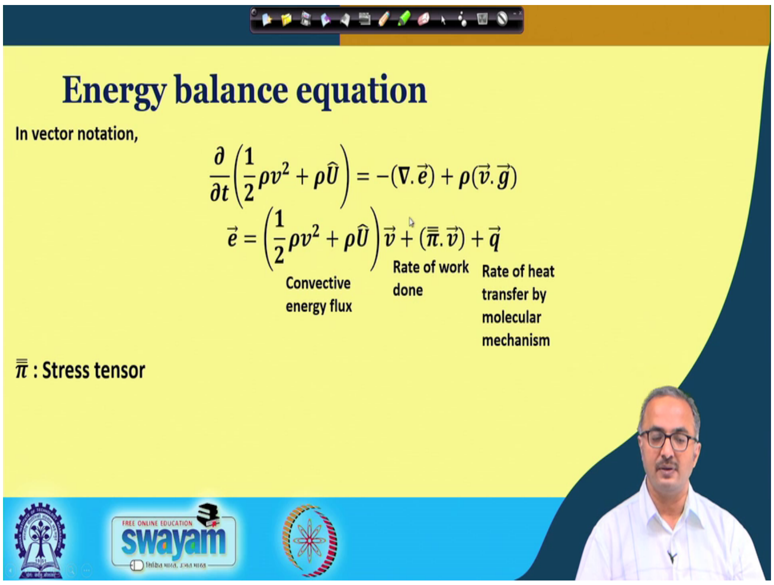

So, after doing if you write that equation in the vector notation you will find that you have this kind of an equation, the scalar equation ok; this is scalar equation. So, in this form you will find that a scalar equation, but in a vector notation they sell that. So, here you find that we write in vector notation. Vector notation means we are using the operator like nabla here, ok. So, in the vector notation this, the equations which we have just shown can be written in this form, ok.

And you see that these particular forms, whenever we are putting in this nabla form this become independent of the coordinates. So, even though we derive all the equations in the Cartesian coordinate for ease of our understanding, but all these equations can be represented equivalently in any other coordinate systems. And in this you can see that e has 3 basic contributions, one is the convective energy flux then these rates of work done and this is the rate of heat transfer by molecular mechanism, and this phi is the stress tensor. More detail can be found in any book on the transport phenomena, and I have given some references at the end of this lecture.

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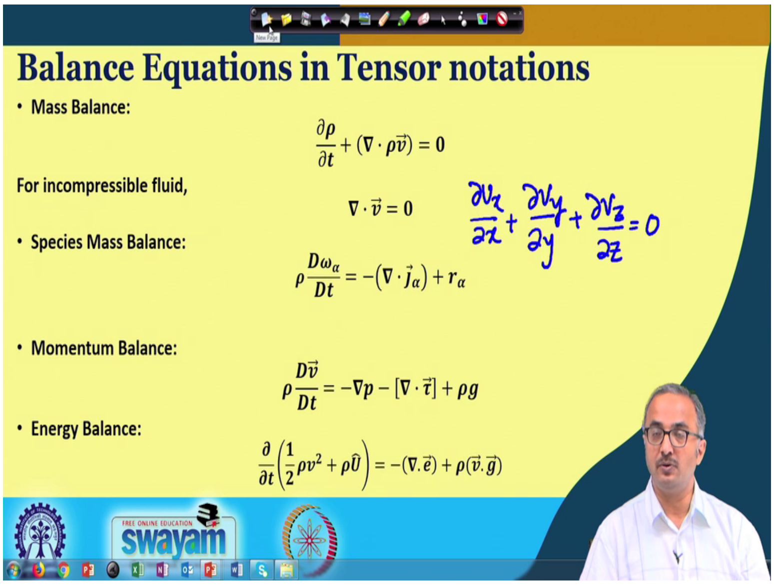

Now, after deriving all the equations for the mass balance, the energy balance and the momentum balance, I just reached out is the simplified form of these equations in this particular slide. And first we start with the mass balance. So, these the mass balance which considers no, only it represents the general like compressible fluid, ok. That means, if this particular density is within the deferential.

Now, in case of incompressible fluid we find the density to be constant. So, this particular thing goes to 0 and so we will be living with only delta dot v because this rho will also come out of this nabla operator. So, we shall be having only this nabla dot v, ok. And you can see this particular thing means in the if you write in the Cartesian coordinate it will coming like dou vx by dou x plus du vy by dou y plus dou vz by dou z equal to 0. So, this is what you will get for the incompressible flow overall continuity, ok.

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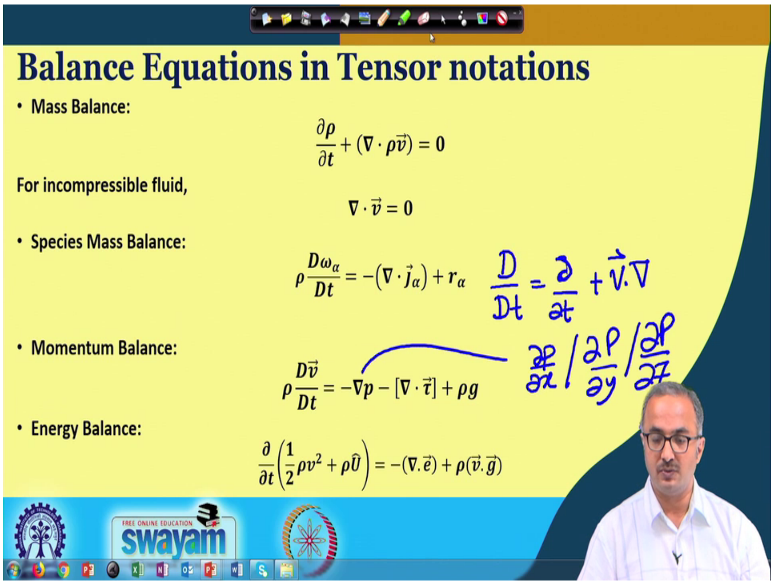

Next we come to the species continuity. In a pieces continuity we also derived this particular equation and here we have the total derivative this d by Dt is the total derivative, this is operator you can say, this will be dou by dou t plus v dot nabla, ok. So, this is the operator d by Dt, the total derivative and it is a partial derivative, ok.

So, in this total derivative we have the contribution for the change at any given location with time and this gives due to the convective flow, ok. So, this is the total derivative. So, in this case we can see that if you write if you expand this one you can replace this d by Dt by this particular terms over here, and you can see you will get the same equation as we have derived earlier and this particular term gives the contribution of any kind of reaction, ok. And this gives any kind of diffusive term that is involved in the species transfer, ok. So, that means, this particular derivative includes the constructive term and this includes the diffusive term.

Next we come to the momentum balance as we have just derived; again with some simplifications we find that this can be written like this. Here we have the change in the velocity and again you see that d by Dt is same as dou by dou t, this I am writing again. So, this this d by Dt is again dou by dou t plus we have v vector dot nabla. So, here again can you find that this kind of equation you will find. And here you can see that here we have the pressure gradient and you know the gradient or you will get this particular thing will be coming as dou p by dou x, ok. Like, by dou p by dou x or dou p dou y or dou p dou z this things, this 3 things will be there for this. And this is the as that surface interaction of stresses and it is the body force, ok.

So, understand this g is always taken as body force, but not to mean only the gravitational, but any kind of body force acting on the body. And lastly we derived also the energy balance equation and this is the form we derive, ok. Now, what I shall do is this, that I shall just give you the representation of all these equations in the three coordinate systems.

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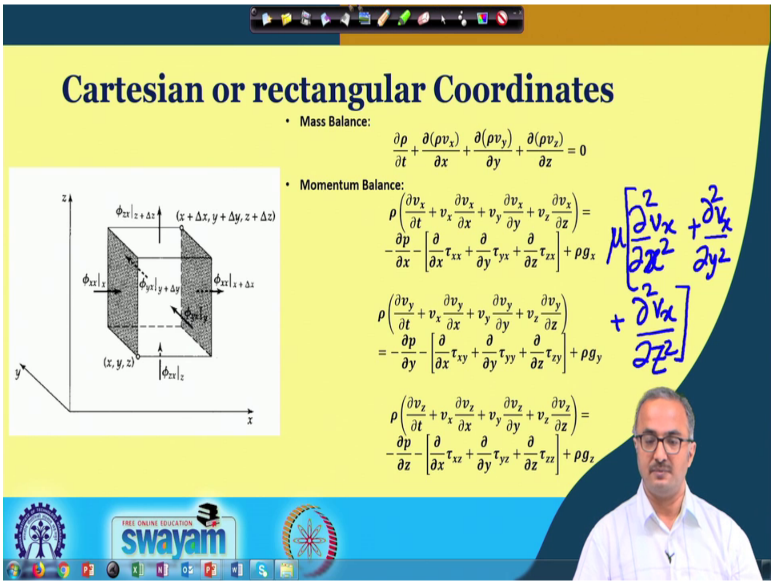

So, first let us start with the Cartesian or rectangular coordinates; and here we have the rectangular control volume and this I explain to you earlier and for this particular system we derive the equations also. So, here you see the generalized equation for the mass balance. And here I have just expanded that capital D by capital Dt term, so here you have found this dou vx by dou t plus vx into dou vx by dou x plus dy into dou vx by dou y plus vz into vx by dou z, ok. So, this particular term is the initial term or due to the convectional term. So, if you find it is this no flow, so these terms will be going to 0, ok.

Now, I am not displaying all these things because they look the similar, ok. And next I go to, one thing I must tell you about the tau xx, the stress term for example, if you are talking of the Newtonian fluid again this stress term will depend on the type of the fluid, ok; Newtonian fluid, Newton fluid. So, suppose you talk of Newtonian fluid. So, this will give rise to this kind of expressions, you will get mu because you will get this kind of, this kind of expression you will get, ok. So, this for Newtonian fluid you will get this kind of expressions or similar things you can write also for the y direction and also for the z directions, ok. So, this is special case. So, what I have shown you is some general case.

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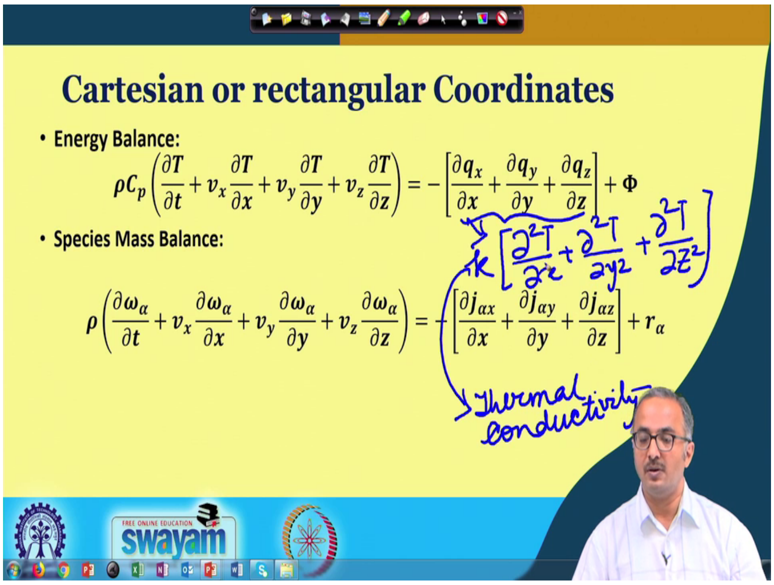

Next we come to the energy balance equation. Here we have the energy balance equation you can see that here again under this particular thing will give us the conductive balances. So, we can write if we assume that the thermal conductivity is constant then this will give raise to, in terms of temperature this will give raise to whole thing dou 2 t by dou y square plus dou 2 t dou z square and everything this is the thermal conductivity, ok. So, this is how we write it, ok.

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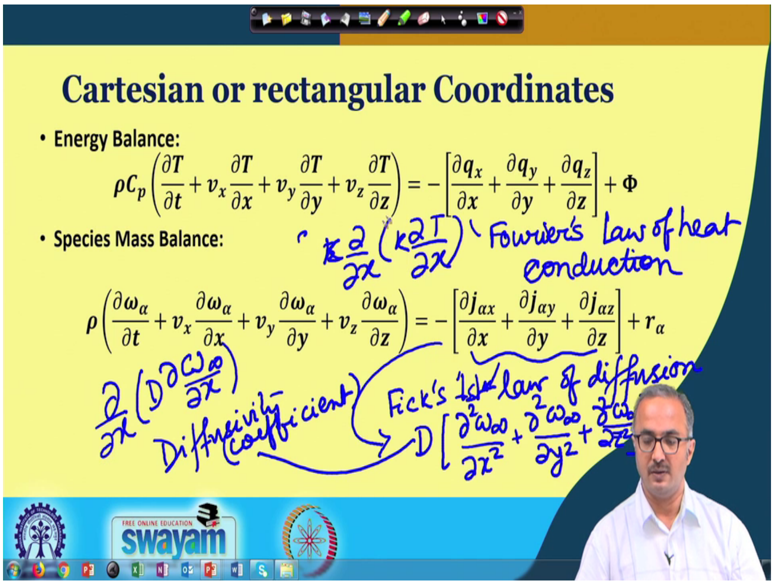

Similarly, we have these expressions for the species balance. Again if you look at these expressions without any derivation of these expressions, we can say that there is something called like in this case we were using the Fourier's law of heat conduction, Fourier's law of heat conduction to write that expression in terms of the thermal conductivity.

Similarly these things can be written using the Fick's first law first law of diffusion, which says that all these things I am not going to detail of these derivations it will simply give us diffusivity coefficient, dou 2 say c alpha or here we talking writing omega alpha, it will be omega alpha by dou x square plus dou 2 omega alpha dou y square dou 2 omega alpha by dou z squared, ok. So, this is the diffusivity coefficient or simply diffusivity, you can percentage for diffusion coefficient or diffusion coefficient or simply diffusivity, ok.

Now, understand this that if in this case if this D is not constant it varies with x or k is not constant with varies with x then what we write that instead of writing this, we write it as dou by dou x d dou w alpha by dou x. Similarly this also for, if it is not constant then we write it as dou sorry dou by dou x into k into dou t by dou x, ok. So, to account for the variation of the diffusivity and the thermal conductivity with location, ok. So, this is how we write.

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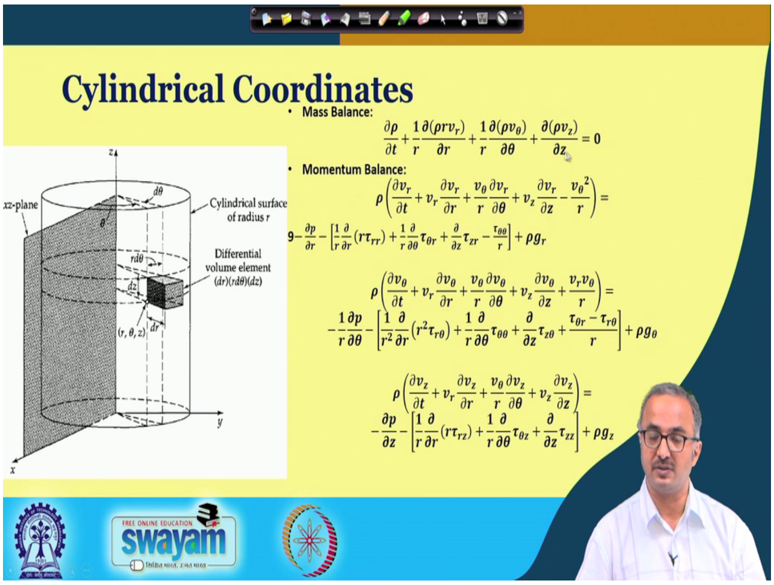

Now, coming to the equations for the other coordinates here we have shown the cylindrical coordinate and in this figure you can see how we represent a cylindrical slice or shell. Here you can see that we are taking these axis from the axis and here you see that there is a, this is the r direct towards this z remains the z, but theta now sees the, what is the in the rotation how much rotation we have to give. So, we can see theta can rotate, and to 360 degree, it can rotate from this you can go 360 like in this particular circle.

So, you can see here that this small slice this is a length is delta z and the small slice is dr, and this particular thing this particular width is rd theta, ok. And you can see here this particular point is r theta z; that means, r is the distance from the central axis, theta is a rotation we have to make from this axis, and z is the distance from this particular plane, ok. So, that is how we locate and that is how and we are moving this is this particular taking a d theta circulation. So, this particular thing is the d theta, ok. So, we are getting this is how we are getting this particular country volume and the volume the control volume is dr into r t theta into the dz, ok.

So, you can the derive it or we can also I also gave in my earlier lecture the transformation of one system of coordinate to another; in nabla form I showed you the different representation of nabla. So, with those you can easily find out these are these expression for the mass balance, these are for the momentum balance, and similarly we have this for the energy balance and this is for the species balance.

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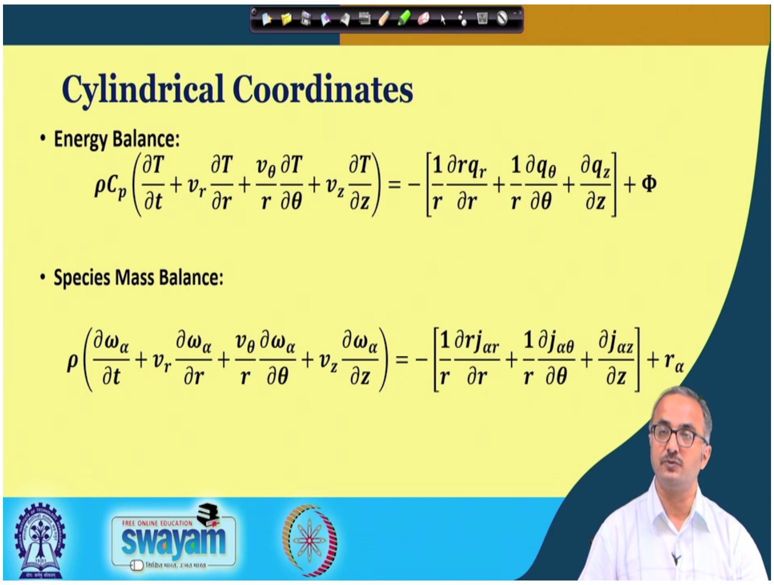

So, please mind it the whatever I have shown you for the momentum balance for the stresses, the right hand side this conductive term and the diffusive term the things to remain the same similar laws will be applied, only thing is this they will be changing the coordinate system.

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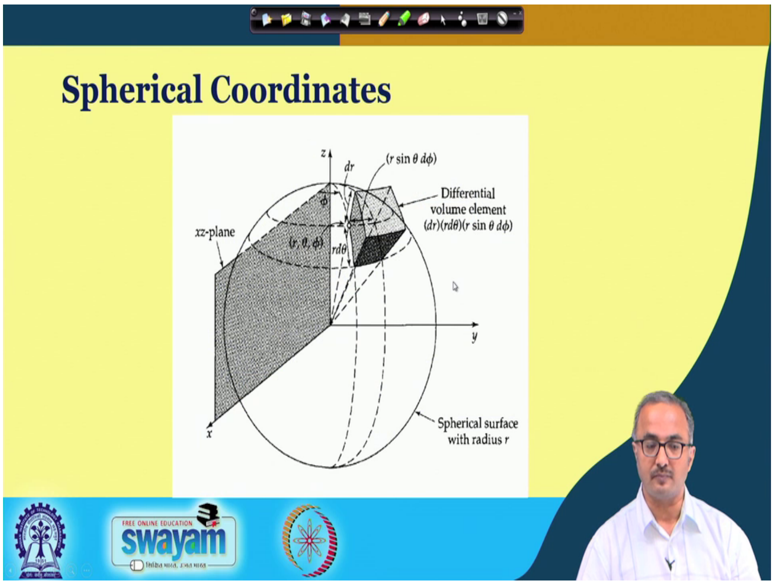

Next we go to the spherical coordinate. In this we consider phase sphere, and as you know spherical coordinate represented by one distance and two angles. And here you can see that a particular sphere is spherical slice is considered, and here you can see that how this various types of width are considered. Now, this is the theta direction, ok; that means, this theta will go like this, and this is the phi direction as you can see, ok.

So, you can see that easily you can find from geometry that what is the volume of this particular shell and this volume will come out to dr rd theta, r sin theta d phi, ok.

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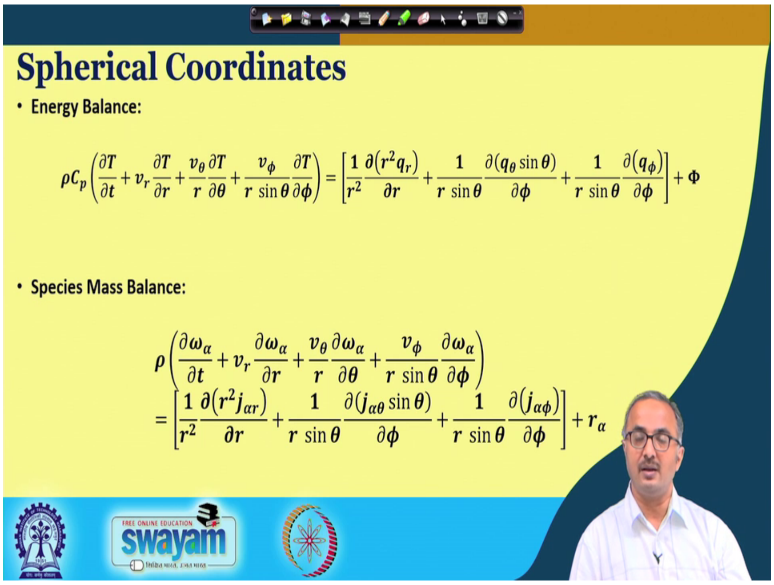

And with this particular thing we can find out the expressions again for the mass balance over here, this is a momentum balance and this is the energy balance and the species balance. Now, you can see that as we go to from the Cartesian coordinate to cylindrical or spherical the equations become quite complicated. However, please understand that you will find that at times it becomes easier to work with the cylindrical and spherical coordinates depending on the shape of the particular system.

And you will find that even though apparently they look complicated, but in real terms whenever you are going to be make the balances you will find that many of these terms go to 0 if you are providing this, choosing the appropriate coordinate system. And many a times you will find that in the cylindrical coordinate some of the terms go to 0 which will not go to 0. If you stick to the rectangular coordinate by looking at his apparent simplicity. So, do not; whenever you are choosing the particular coordinate system choose it carefully so that your solution becomes easier and you can choose, you can reduce the complexity of the equation.

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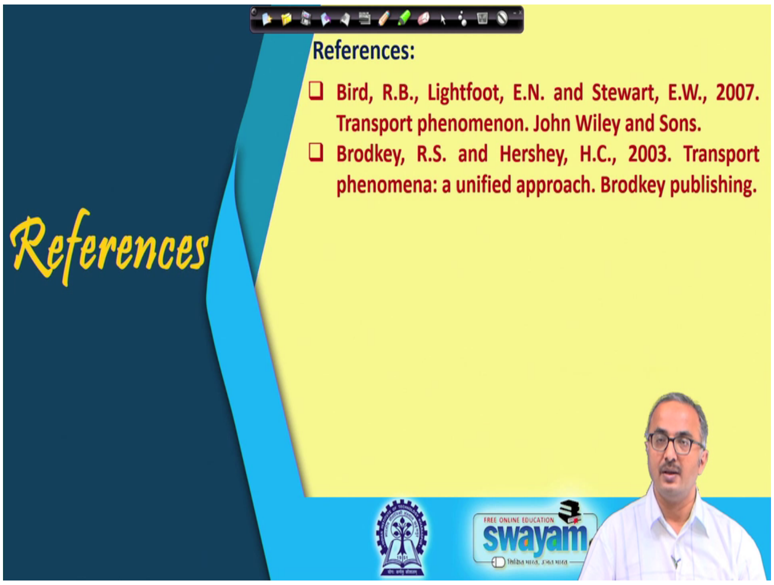

So, these are the two references which will give you a very good analysis and theoretical concept on all these momentum balance mass balance and energy balance.

Thank you.