Mass, Momentum and Energy Balances in Engineering Analysis

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Lecture - 24
Macroscopic Balance –I

Welcome. After learning about the macroscopic balances for energy, momentum and mass and also their solution techniques, we move on to the Microscopic Balances.

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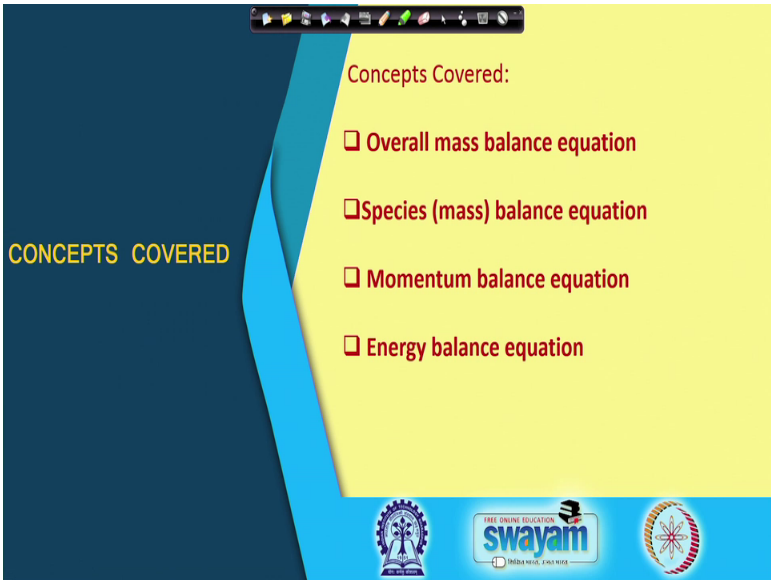

So, in this particular series of lectures what we shall learn about is that, how to make the microscopic balances for mass balance. And, these two represent for overall mass balance and species mass balance and the momentum balance and energy balance.

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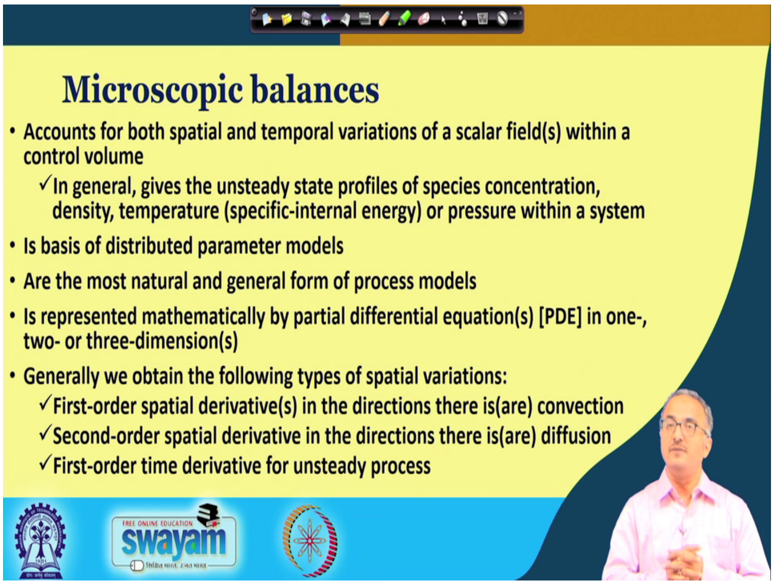

So, first let us look into the significance of microscopic balances. So, these microscopic balance accounts for both spatial and temporal variations of some scalar field within a control volume. This contrasts with what we have done in the macroscopic balances, where we neglect any kind of spatial variation of the scalar fields. So, in general this kind of balances will lead us to an unsteady state profiles of species concentration. So, this is important. It is the unsteady state that is the time variation and profile of species concentration of density, density or temperature or pressure. So, any kind of profile that is distribution of the particular scalar with respect to the location.

So, this will be accounted for by the microscopic balances. And, this kind of balance leads to distributed parameter models. Like macroscopic leads to lumped parameter model, this microscopic leads to distributed parameters models. And, these are the most natural and general form of process models. Because, in this way we are able to take into account the variation of the various properties not only with respect to temperature, but also with respect to the location. And, as you know in our day to day life many of the things keep changing from the location as well as from the time.

For example, even you can put this thing the population. Population you can find that at various locations, the populations are different. And, also at a given location you see that population may change from time to time. The population may be of any species maybe some plants maybe of some human beings may be of some other animals.

So, this is one of the ways you can also implement all these balance equations ok. Similarly, you can also find that in a given reactor the species concentration in the reactor is changing from various position and also due to reaction happening the concentrates are also changing with respect to time. So, these kinds of systems are there around us for various types of purposes. And, generally whenever you are accounting for all the variations with respect to time and as well as space, then we will end up with some partial differential equations which is again in contrast with the ordinary differential equations we were having under the lumped parameters the systems or the macroscopic balances.

So, now we shall be during with the partial differential equations and these partial differential equations may be written in general for the three dimensions. As you know that the space is having the three dimensions in rectangular coordinate we have x, y, z for example. So, you can write in three dimensions which is very general. But, sometimes when you make such kind of three dimensional equations you find that they become very difficult to solve ok. Then you make some kind of assumptions which are relevant for a particular system. So, that you can reduce the dimensionality of the problem and you can have two dimensional or even one dimensional equation.

So, even if we have we have the reducing dimensions, but this dimension is reducing in the space not in the time. So, you still have the partial differential equations because, in one dimensional it will be having a temperature and one of the coordinates ok. So, you will be still having the partial differential equations whenever you are going for the macroscopic balances in a dynamic state.

Now, we you find that you have different types of derivatives in such kind of microscopic balance equation. Now, generally what you find the first order derivative spatial derivative will be there whenever we have convection. So, any kind of convective term will lead to first order spatial derivative; that means, with the location. And, we have generally second order spatial derivative in the directions where there is diffusion ok. So, we shall see that why we get this diffusional thing as in two dimensions we shall see later. But, as of now we understand that we get second order spatial derivatives we whenever we have some diffusion problem.

So, looking at a particular equation you can identify that which of these two are there whether we have convective term or we have a diffusive term ok. And, understand this convective term or diffusive term in all kinds of phenomenon like in mass transfer, in momentum transfer and in heat transfer. And, and generally we have first order time derivative and this comes when we have unsteady state or dynamic processes. So, when we have steady state then there is no dependency on the time.

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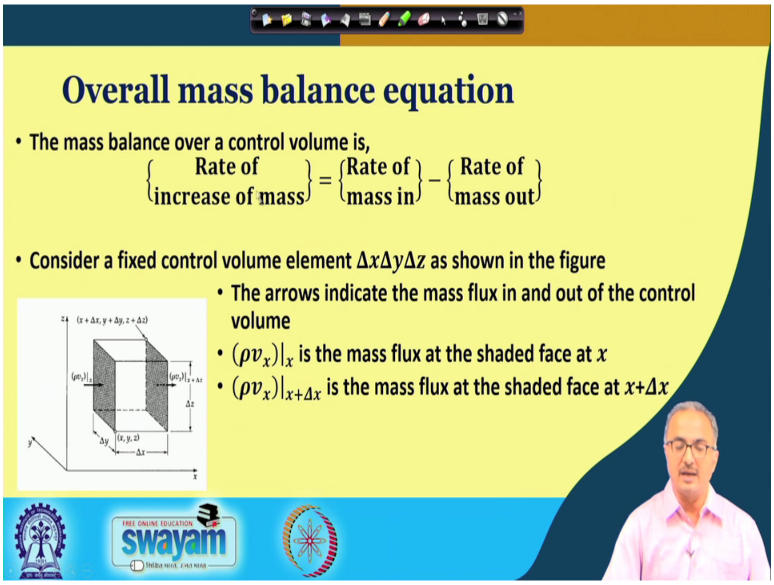

So, first let us start with the overall mass balance equation. So, in this case first we apply the conservation law and for doing that, first we have to identify some control volume ok. So, here we are doing it in the Cartesian coordinate for ease of our understanding. So, we when we write the conservation, now you find the rate of increase of mass is equal to rate of mass in minus rate of mass out because, mass cannot be created or destroyed. So, there is no question of having any kind of generation or the consumption term in the overall mass balance ok.

So, this you must be careful about. So, consider fixed volume. So, hear what we are doing in a Cartesian coordinate we are choosing this x direction over here, positive x direction. This is the positive y direction and it is a positive z directions. And, in this we are choosing a cuboid you can say that in this we have this x direction we are choosing a small length of delta x and then we have delta y and we have delta z ok. And, the total volume of this one is delta x into delta y into delta z.

Now, let us choose that if suppose this is x; that means, this point will be representing x plus delta x. If this is y then this is be y plus delta y and sorry, if is z this is z, z plus z delta z and if this is y and is this is y then this is y plus delta y. So, that is how we are finding that we have put the coordinates. So, this particular coordinate will represent which is farthest from this represent x plus delta x into y plus delta y into z plus delta z. And thereby, you can also find out the coordinates of the other points ok.

Now, let us see that how do we look at the mass balance. We assume let us look for one dimension then you can extrapolate for the other dimensions. So, here we find this at x this at this particular plane and this plane is which plane, y z plane. So, in this y z frame the x from x direction you are finding some mass is going in and some mass is going out. And how much is this mass going in, that is equal to the row into the v x. So, these are rate at which the mass is going in and row when you are you are putting with row; that means, you are basically talking of mass per unit volume.

So, because we are talking about is only volume metric scale we are talking. So, it has see that in this unit volume how much mass is going in through this particular area ok. And, in the other side we find that the some amount of mass is coming and we are designating this mass going is as row v x into x. And, this we are talking about row v x into x plus delta x ok. So, this is the mass going in and this is the mass coming out in the x direction.

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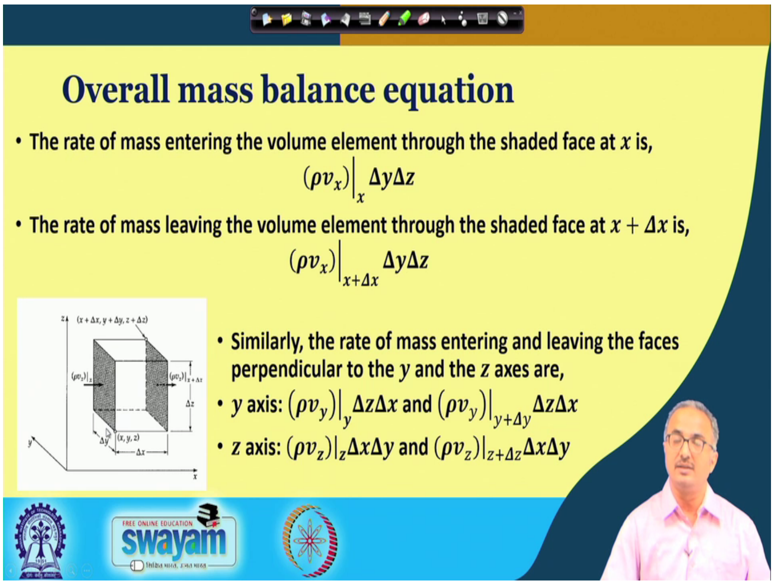

Similar thing we can write for the y direction and for the z directions. Now, you see that once you write all these expressions, what you find that this you this row v x is the mass flux ok. And, when you multiply with the area you get the rate of mass going in ok.

So, that is why you find the you find that we are also multiplying with this particular area. So, in case of v x we are multiplying by delta by delta z, in case of y where we are multiplying by delta z delta x. In case of z direction mass transfer we are multiplying by delta x delta y ok. So, this now gives us the rate of mass going in and coming out of the particular control volume.

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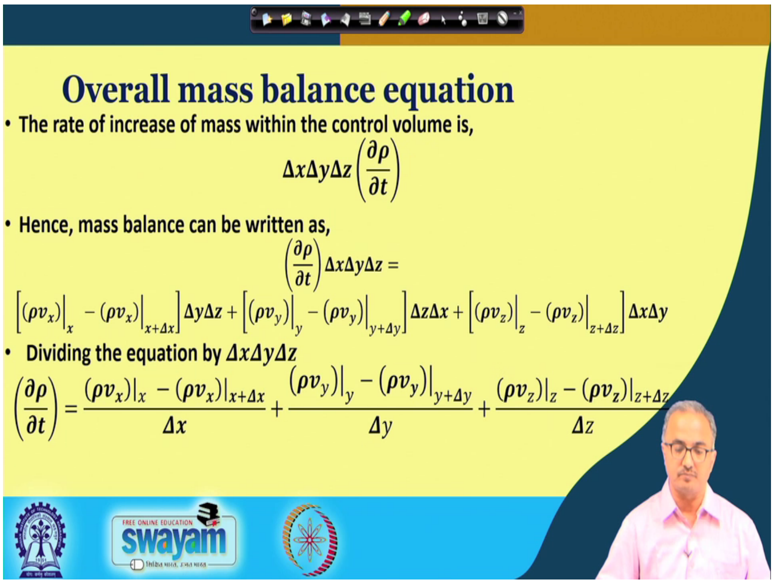

Now, you see that at what rate the mass is changing within the control volume. So, again what we are doing? We are doing that how the density is changing that is dou rho by dou t row into the volume is the mass change ok. Now, because these particular delta x delta by delta z are remaining constant; so, we are taking it out of the differential. So, we are left with that density with the differential with respect to time.

So, this total thing gives us the rate of change of mass within the control volume. So, this rate of change of mass in the control volume as from the conservation law is given by whatever mass is going into. So this mass, this mass and this mass along with the respective areas of the cross sections. So, this mass are going in and this things are (Refer Time: 10:55) coming out of the control volume.

So, we rearrange this expressions and we find that from the Taylor series you can easily find that how you will represent all these things. So, this particular thing you are getting as dou v x by dou x ok. If you take the limit of delta x tending to 0 delta y tending to 0 and delta z trending to 0, then all this difference equations will lead to differential form ok.

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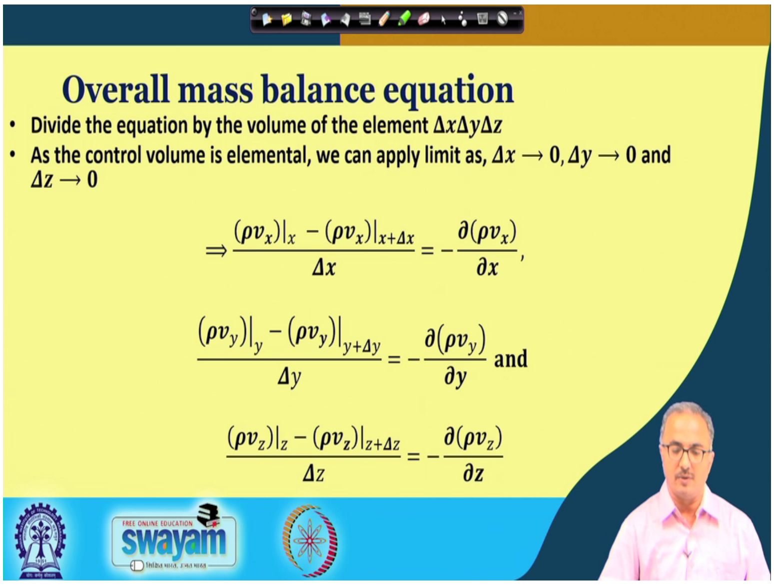

So, that is what we are doing that because, they are very small you are finding that these all this difference equations are leading to differential equation ok. So, we are writing all the differential equation like this for the three directions.

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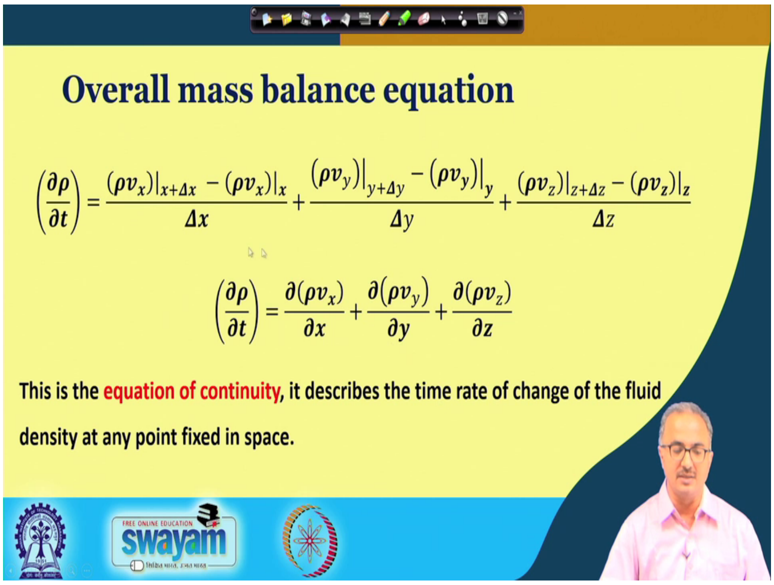

Once you put this now, again you apply this in this expression and you find you are getting this particular expression ok. So, this thing we call the equation of continuity. Continuity means that mass is continuously distributed and there is no gap between the mass; that means, there is no hollow space between the masses now, that is why its continuities; continuously going in or going out. So, this is the basically the mass conservation for the overall system.

Now, this same thing may be taken for to find out the species distribution. Because, in many systems we find the there could be some kind of reaction, some kind of generation or consumption of a species ok. In that case overall mass balance will not give us the whole picture because; it will not tell us how the species concentrations are varying within the system. For that we need to do the species balance.

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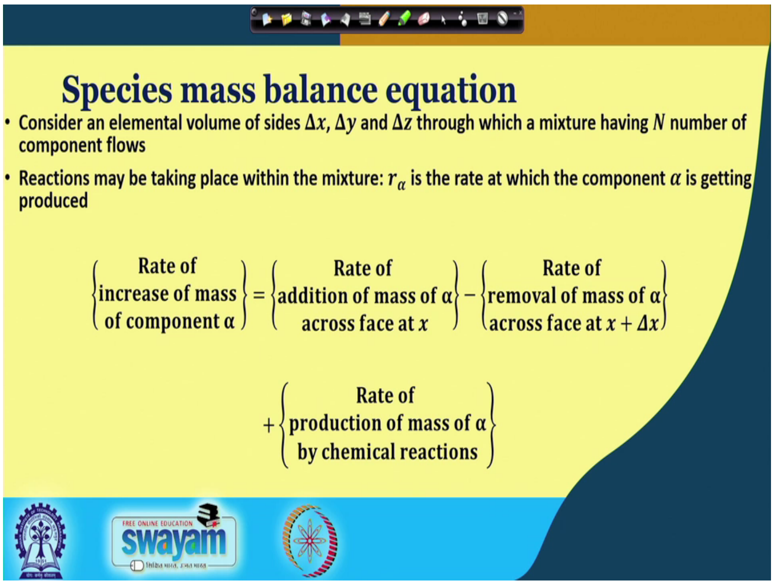

Now, let us go to the species mass balance equation. In this case as we have done previously again we are doing the mass balance over the same control volume with these delta x delta y delta z at the width in the three direction for the control volume. And, here we are considering that there is some kind of reaction going on with respect to a species alpha.

Now, we apply the mass conservation species by species. So, alpha is an arbitrary species we are talking about. So, when we write the conservation law we find that we have the rate of mass change; here we are called increase in the species alpha. And, then this equal to the rate of mass that is coming inside the control volume of the particular species and rate at which the particular marks is getting removed from the control volume and this is the rate of production of mass of alpha.

So, in this case we are talking about some kind of reactions going on. So, we are talking about this production. Now, please understand it may be production or it may be consumption. So, if you find a value negative production you take it as conception of species. Now, species when you look at then a species may be created or destroyed ok. So, this is unlike the case when we talk of the overall mass which is neither created nor destroyed.

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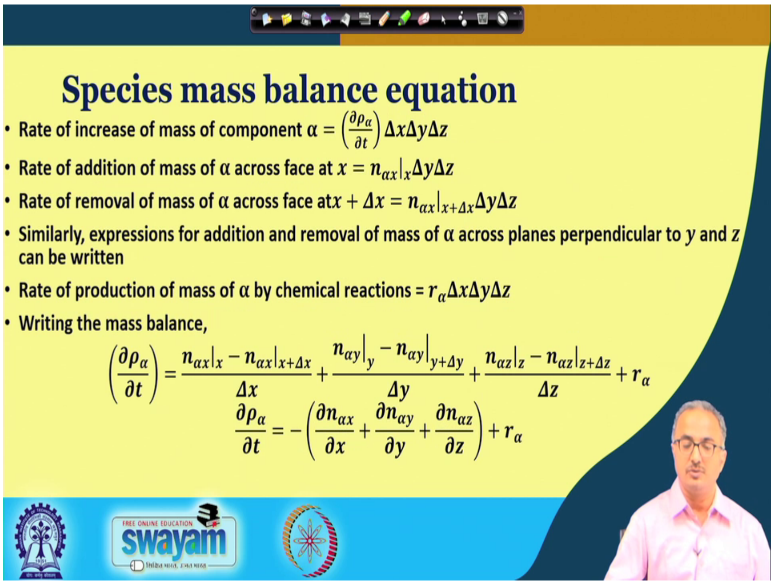

So, writing all these expressions now, let us see that, what is the rate of increase of the species alpha within the control volume again because, of volume is constant. So, we are taking out the volume of the differential and this is the density of the species. Now, addition of the mass cross alpha is given by some kind of flux.

So, this flux is given as n alpha x; that means, flux of species alpha in the x direction at the x point and this is multiply by the area of cross section. So, that we get the rate and then we go to the other end that is x plus delta x, we find a similar expressions like this ok. So, this much is going in at the in the extraction and is a coming out at the x direction and similar things we can also write for the other two directions in y and z ok.

And now let us go to the reaction term which is causing the production or the consumption of the species alpha and generally the reaction terms are again giving in terms of volume. So, let us see that this particular rate of reaction is per unit volume of the particular reactor you can say. Then we are simply multiplying with this volume of the this small control volume and we are getting the rate at which the species is getting generated or consumed.

Now, putting this all these things together we find that we are landing up with this kind of expression ok. And, then again with taking the limit for delta x delta y delta z to 0 we find that these difference equations are getting into differential forms. And, we get this particular expression for the species balance ok.

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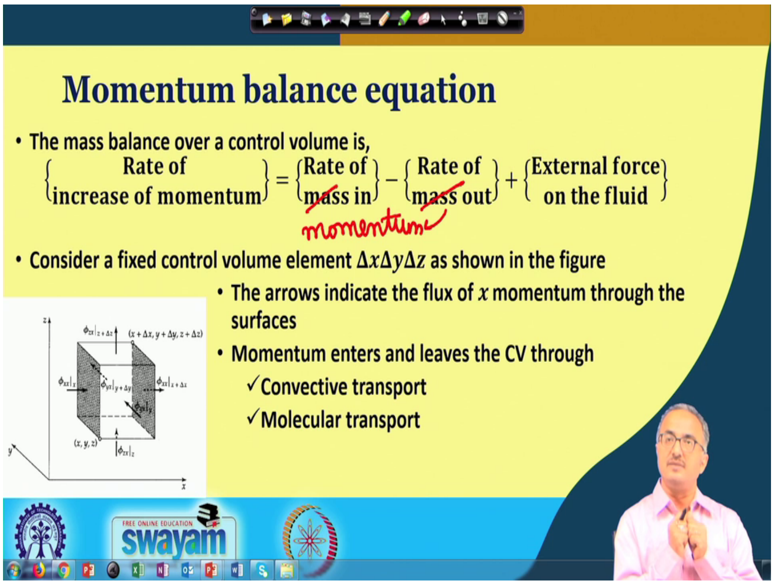

Now, let us go to the momentum balance. Now, in this moment balance again we consider the same control volume as before. Here we are writing the rate of increase of momentum in the control volume is equal to rate of momentum is not be momentum not mass ok. So, rate of momentum that is going in the control volume and so, this will be momentum. So, this rate this how much momentum is going in the control volume and how much momentum is coming out of the control volume and, you know the rate of change of the momentum is nothing, but the force ok. So, that is why we are writing other than this these forces are external forces ok.

So, we are writing all these external forces club together. These forces could be due to many things: it could be gravity force, it could be electrical force, it could be magnetic force, it could be viscous force; so there are various types of forces which will be there in this. So, we are those forces are put in this a club together in this particular terms ok. So, let us now just look at these expressions individually. So, we see that the momentum may be transferred due to either conductive transfer or by molecular transport. Because, when a momentum is going on then what we find that the molecular molecules or the particle themselves are exchanging the momentum between them, because of the differences in their velocities that is one thing.

That means, a faster particle will try to take a speed up a slower moving particle; on the other hand a slower particle will try to retard a faster moving particle. That is how they are, we say that they are exchanging there momentum that implies the molecular level. Or, the convective means when the overall system is moving as a convective ok. In that case also we will find that there could be some changes in the momentum across the cross section of the particular system. And, there again we find there will be slower moving and faster moving layers of the fluid will be there and they are also exchanging the momentum. So, its convective transfers momentum. So, all these things should be considered for the momentum balance.

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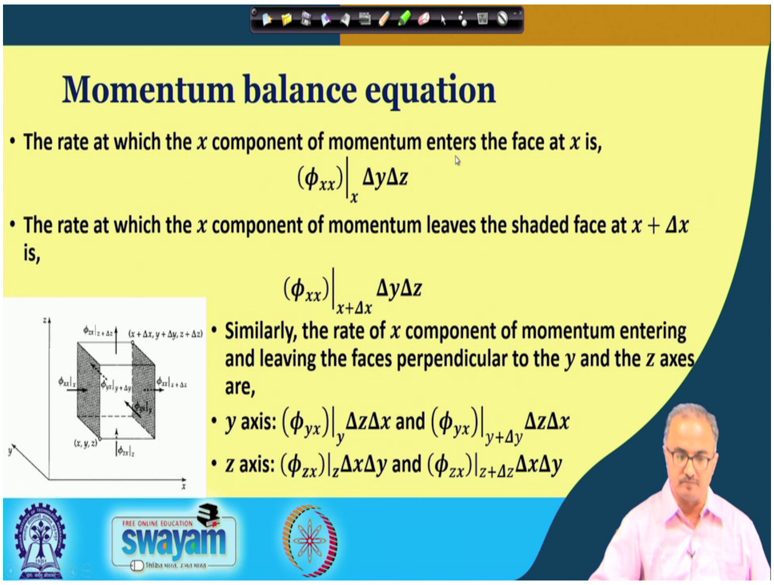

So, moving on we write for the for example, we write the momentum flux. I will talk about this momentum flux later as of now you take as this the momentum flux in the x direction due to the x velocity ok. So, that is why we are talking phi x x and this is at x position. And, this is the multiplication by the area of cross section to get the rate of momentum going in. And, similarly we are getting the rate at which momentum is coming out of the system. Similar things may be also written for the momentum which is going in y in direction and this is for the z direction.

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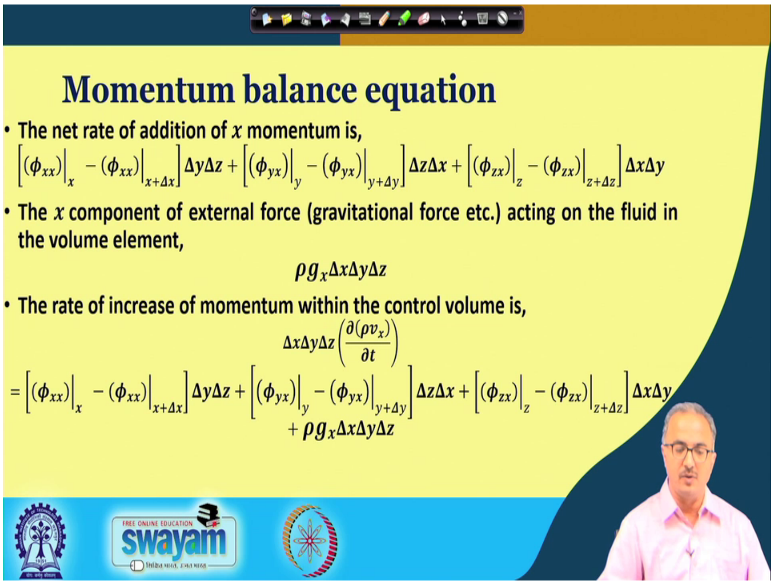

After putting this, what we do that, we go back to the, our equation. Here we have the equation and we are writing as a in the momentum going in and these are the term which are momentum coming out of the system. And, these forces let us say that we take the body force as a gravitational force. So, this is we are writing in terms of some g, but this may be understand that this represent in general a body force. It could be anything gravity is one of the body forces ok. So, this we are writing in terms of g and this particular thing he is also per unit volume.

So, this row into g this point unit volume because for example, mg the total force n by v into g is the density into given that is per unit volume of the particular body. So, that is row g is the per unit volume and we are multiplying by the volume to get the mass, mass into the particle body force. So, this we are putting as the total force term and the rate of increase of momentum can be given by this that this is the row v x is the momentum in the x direction, this per unit volume and this is into volume. We are finding rate of momentum increase in the x direction.

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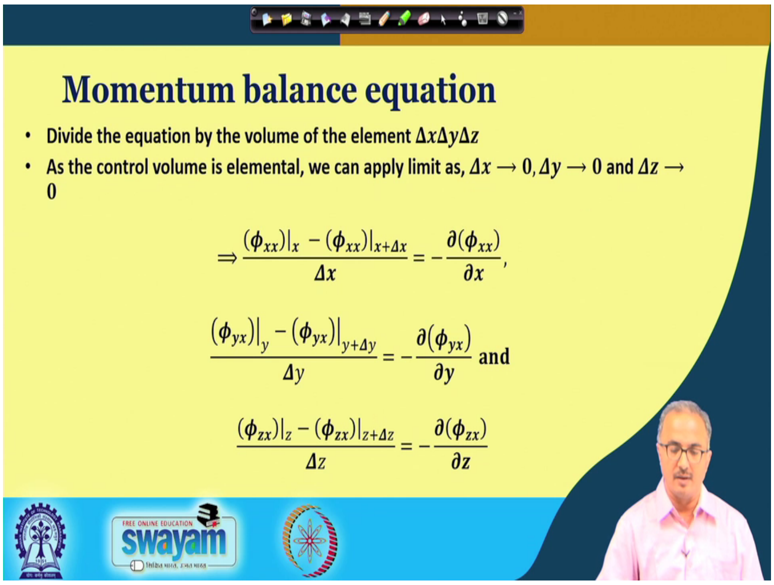

Now, again putting all of these things together, we get this particular expression and again what we do, we divide by delta x delta y delta z as we were doing earlier. And, then take these limits to 0 and what we find that, all this difference equations are being reduced to differential forms over here ok. And, only thing you note that in each of these cases the subscript is changing. This is xx, this is yx and this is zx.

It means that what is the change in the momentum in the x direction due to the x component of the velocity. This means what is the change in the momentum in the y direction due to the change in the x component of the velocity. And, this means now what is the change in the moment in the z directions due to x component of the velocity. So, everything is happening due to the x component of the velocity in the three directions.

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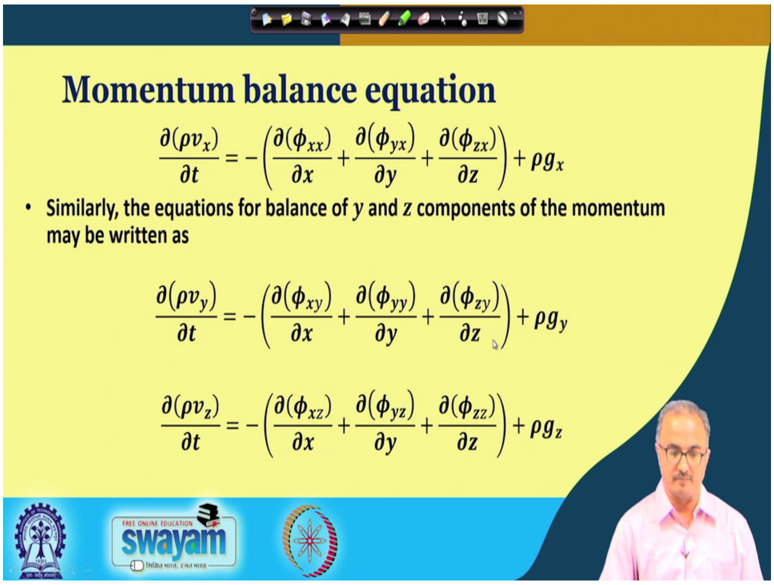

And, now we put all these expressions over here for the three directions over here and we get for each direction, we can get you can easily drive for the other direction, for the other velocities you will get the body forces also in the three directions. And, these are the momentum changes and this is the within the control volume how the things are changing. So, now you see that unlike in case of the mass balance where, we had only one equation for species balance we had only one equation in momentum balance we now cannot have one equation; because momentum is a vector.

So, whenever we are writing momentum equation we have to write them in three direction separately of course, you can club them together in a vectroial form. But, whenever we are writing in a scalar form these have to be written in a in a separately in three equations. So, that is why I am not putting them together as of now, treating each of them in scalar form ok. Later on we shall see how we can club them together and have the vectorial forms. And, whenever you have vectors you find the there are some other things come into picture, tensors will also come into picture about which I will talk in our later lectures.

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So, these are the two books which you can refer to know more about these microscopic balance equations. And, I shall be continuing this in my future lectures.

Thank you.