Mass, Momentum and Energy Balances in Engineering Analysis
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Lecture – 18
Solution of Macroscopic Balance Equations

Welcome. In this lecture we shall be doing a few problems on the root finding methods for single equations. So, whatever I have talked to you in the previous lecture we shall be implementing those methods here.

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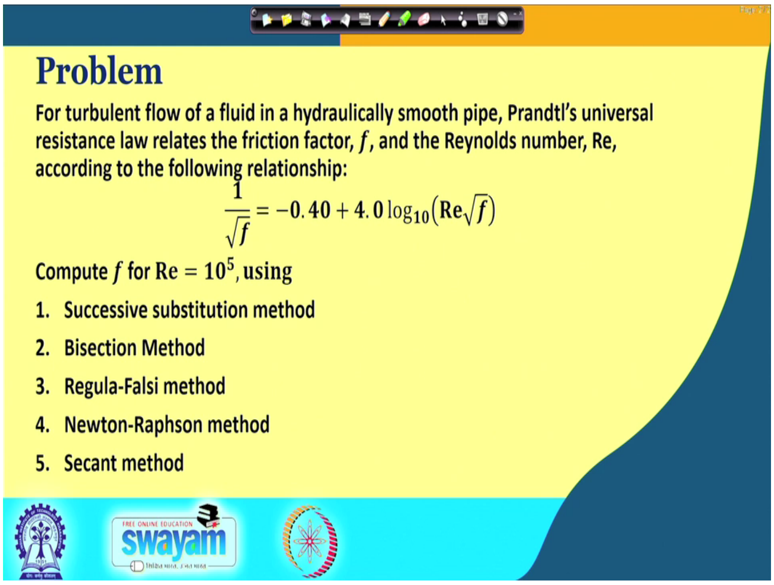

So, first let us go to this particular problem; in this problem we find this kind of problem in many at time in the fluid mechanics. And similar kind of problems you will also find in other engineering applications even in the chemical kinetics. So, here in this particular problem we find that we have a turbulent flow of a fluid in a hydraulically smooth pipe; where the Prandlt's number universal resistance law relates to the friction factor and the Reynolds number according to the following relationship.

Now, before we proceed to the solution let me just briefly some idea about what this particular problem means. Is this that whenever a fluid is following through some pipe line or over a surface due to the its own viscosity and also the flow rate with which it is moving on the surface or in a particular closed pipeline, it will have some kind of resistance to its flow. And these flow resistance may be found out a by knowing the viscosity, and sometimes when the flow is laminar we can find it very easily by in terms of Reynolds number which signifies the type of the flow.

And it type of flow means that; it could be laminar flow, it could be turbulent flow, or it could be somewhere in between these two types of flows. More about this is a nature of the flow you learn in fluid mechanics or fluid dynamics courses. So, this problem pertains to the evaluation of a factor that is we call the friction factor to know how much resistance a fluid would encounter when it is flowing over a surface or in a conduit.

Now, how does this knowledge help us? This knowledge is required by us whenever we are trying to design a system and in this system either we are pushing the particular fluid by using a compressor or using some pump. And depending on the friction factor the resistance to the fluid flow will change, and then we can understand that how much will be the power required for a pump and compressor. So, for all these reasons we often need to evaluate the friction factor. And second use of the knowledge of the friction factor is that is sometime we need to find out the rate of heat transfer or a rate of mass transfer; in terms of heat transfer coefficient or mass transfer coefficient.

Many a times we find that for many systems these two coefficients are not available directly. So, in that case we can find out the values of heat transfer coefficient and mass transfer coefficient from the value of the friction factor using something called analogies. So, these analogies are also thought separately in the courses on transport phenomena. So, in many ways we find the knowledge of the friction factor is often needed to solve the engineering and scientific problems ok.

So, with this brief introduction all the friction factor let us go to the this problem in which we have been given a relationship between the friction factor and the Reynolds number. And we have been told the Reynolds number is 10 to the power 5. Now generally what happens that in a circular pipe there is a critical Reynolds number which signifies the transition of the flow from the laminar to the turbulent.

So, the generally for a circular pipe this critical Reynolds number is about 2000 or 2100 So, beyond this there will some transition zone and very high flow rates in when the Reynolds number is also goes high so we have turbulent flow. So, in this case the Reynolds number value is 10 to the power 5. So, it is quite high and this particular flow is turbulent.

Now, here you see that we have to find the value of the friction factor for the Reynolds number and we have to use various kinds of methods we have studied so far and each of these methods may be use to find out the friction factor for the given Reynolds number. So, let us take each of these methods one by one.

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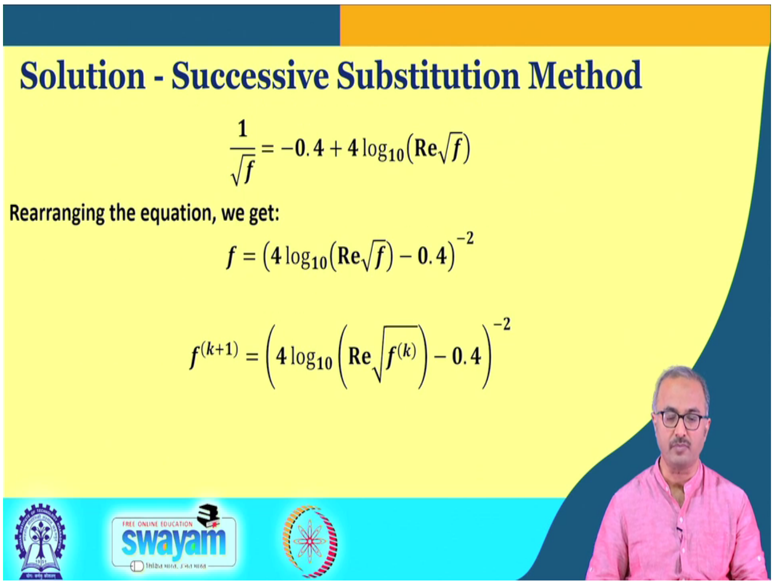

So, first we go with a successive substitution method in this method as we learned earlier that we have to rearrange the equation to get the value of the unknown variable explicitly in terms of the other variables. And this arrangement may be done in a various manners that is there no unique way of representation of the unknown variable from the given equation.

So, here what we shall show you that if we assume one kind of representation how the solution is going to proceed. So, let us see that this is the one representation I have shown you here. Here what we are doing that we are basically taking square on both the sides and then we are taking the reciprocal. Now when you square this whole equation what you find that this under root f becomes f and then you would take a reciprocal of this particular equation ok. So, that is how we find that we are getting this equation.

Now, whenever we are going for this method; what we do? We start with some initial value and from that we keep updating this value of the unknown variable using the equation we have just written. So, here that is why we are writing here that these we take as a unknown and these we take as a value we just coming from our initial guess or from the previous iteration. So, here we are writing if f k plus 1 where k signifies the alteration level and this everything remains same and here on the right hand side we are writing f k. So, that becomes an explicit way of finding the unknown variable.

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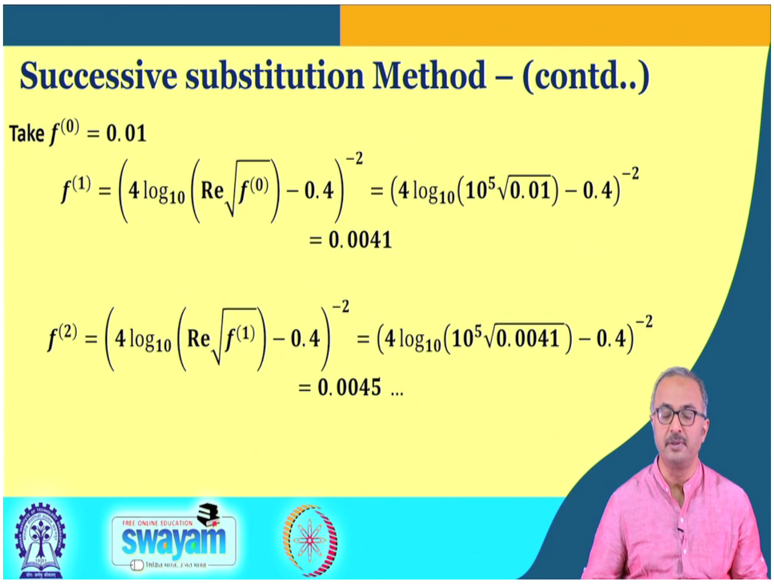

So, implement this what we do we start with some initial guess and that we are putting as f 0 f 0 0. Because we have not started the iteration yet so we are putting it as 0 k equal to 0 ok. So, now, we have f 0 f 0 0 equal to 0.1; what we have to do simply is that put this value on the right side of the equation we just showed. And we this value we get the updated value of the friction factor so we find that this is coming out to 0.0041. So, you can see that from the initial guess one of 0.01 we have come done.

Now, understand this that depending on the initial guess this particular value will also keep changing. Now with this particular value again we can go for updation and we see that this particular value we are obtaining next has we are obtaining by putting the latest value and again we are getting the updated value. Now we can keep proceeding with this kind of updation until convergence and as I mentioned to you in my lecture, that convergence means that the user would specify whether we are approaching the true solution.

That is whatever accurate solution or precise solution that is whether the two consecutive values of the unknown parameters are getting converged or not. So, in this case we can see that 0.01 and 0.0041 they are different, but again it will depend on the user's specified criterion for convergence whether we should continue or whether we should stop our iteration. So, here you can I have put this dots to tell you that this continuation of this method would depend on the users specified convergence criterion.

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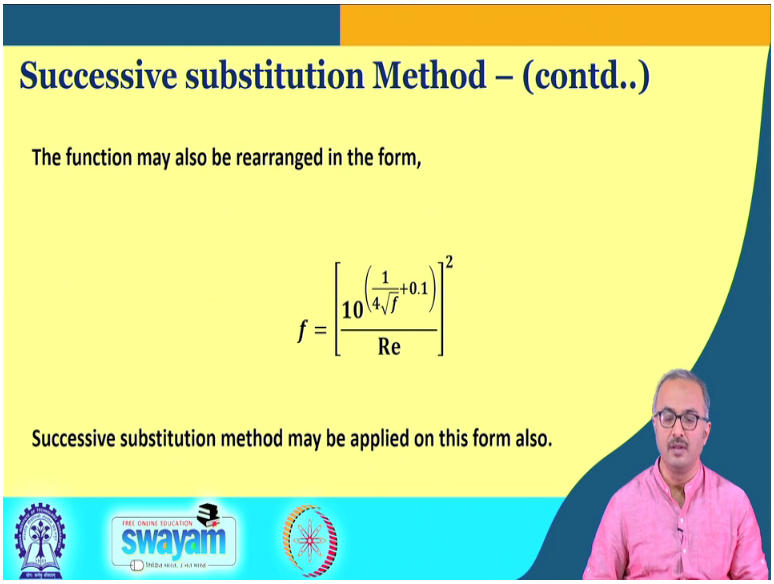

Now, another way somebody can say that we may get the value of f from the same equation is this. Now you can see the equation patterned as changed, now in this case you may also you get the value of f. Now it will again depend that whenever you are starting with some new value from the initial guess value whether this new pattern of the equation would give us convergence or not. So, you may check these different types of these representations in terms of the convergence.

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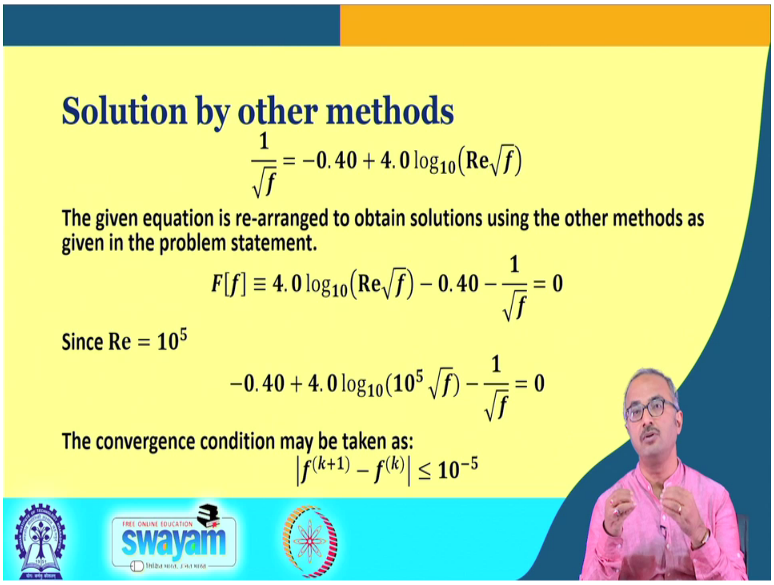

So, here you can see that we are putting this method for some other methods like successive substitution verse that here we just needed to rearrange the equation to get explicitly and equation for the unknown variable. Now when we go to the other methods as given in the problems we find that that kind of rearrangement is not necessary, but now we have to put this particular equality in terms of an equation. So, we are given this particular equality now what we do we put in terms of equation so that what we need it do that we take everything on one side.

So, in this case what we have done we have defined another new function ok; that is in terms of the friction factor. And we have taken this particular term on the right hand side you may might as well take the right hand side on the left hand side so it does not change the solution So, let us define this new function capital f as function of small f friction factor like this and these we are putting as 0 and we are putting the value of the Reynolds number. So, we are having this particular equation and suppose the convergence criterion is given like this ok. This is your arbitrary; you may choose some other convergence criterion.

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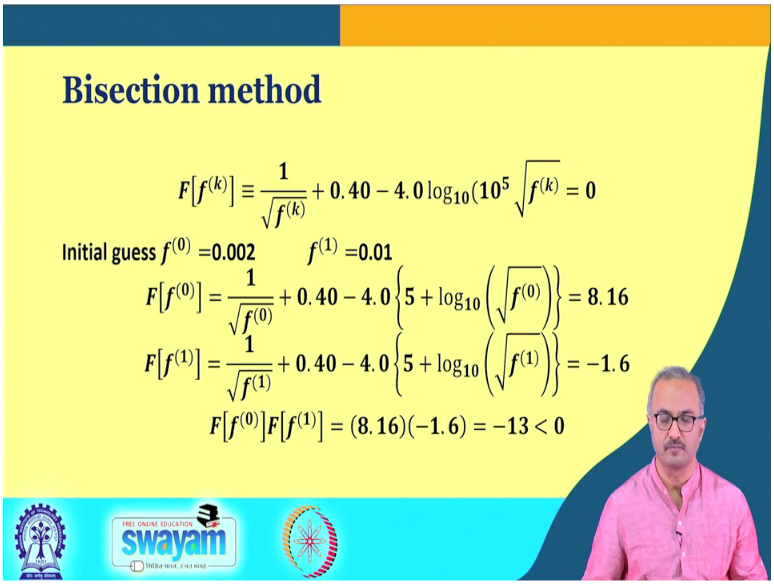

Now, let us go to bisection search, and as we learned earlier this particular method needs the root to be bracketed for the initial guess values. And we know that this particular method needs two guess values initially; and that particular guess values they will define the domain within which the particular solution would lie. And we can keep chopping of into half this particular interval to get in to the solution. So, what we do here that we choose these two values arbitrarily and what we see that we check the values of the function.

So, we put was this value and then this value and check the values of the function. Now, what we find here that when we take the product of this two values its coming to be negative. Even without taking a product you can also by observation also you can check that if the two function values are coming out to be negative; that means, that the root must be lying within the zone. So, it is just a coincidence that for this particular range we have selected arbitrarily the actual root is getting bracketed between these two values.

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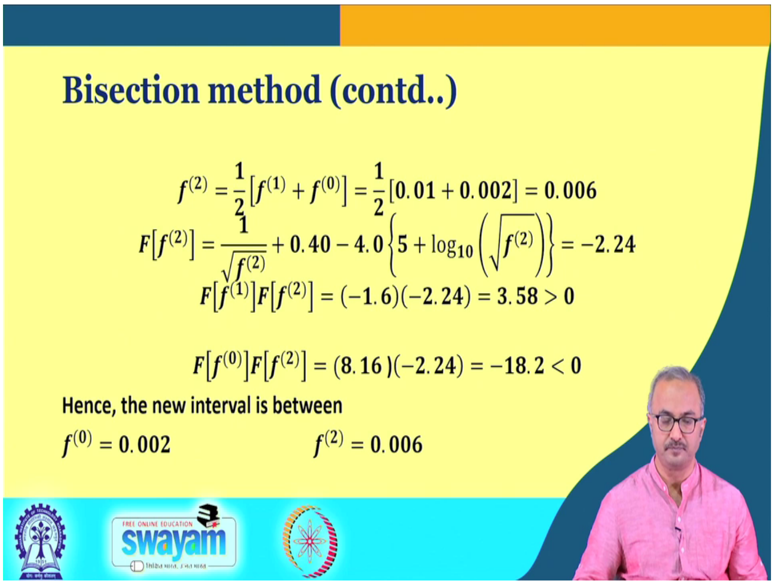

Now, what we are doing is that we are now to find the new values of the root we chop of the initial range. So, what we do we bisect; that means, we are dividing by 2. So, we take the initial two values bisect it and we get a new value of f. And now what we do we find again the function value for this new value f and we find this is coming out to be negative. It shows that now the our region of search has now shrunk and we when we take these product between f 1 and f 2 and we find it is coming to 0 and when we take for a f ff 0 and f ff 2 we find it coming to negative. It means that now the region as shrunk between f naught and f 2 and f naught is 0.002 and f 2 is 0.006. Now new search domain is 0.002 and 006.

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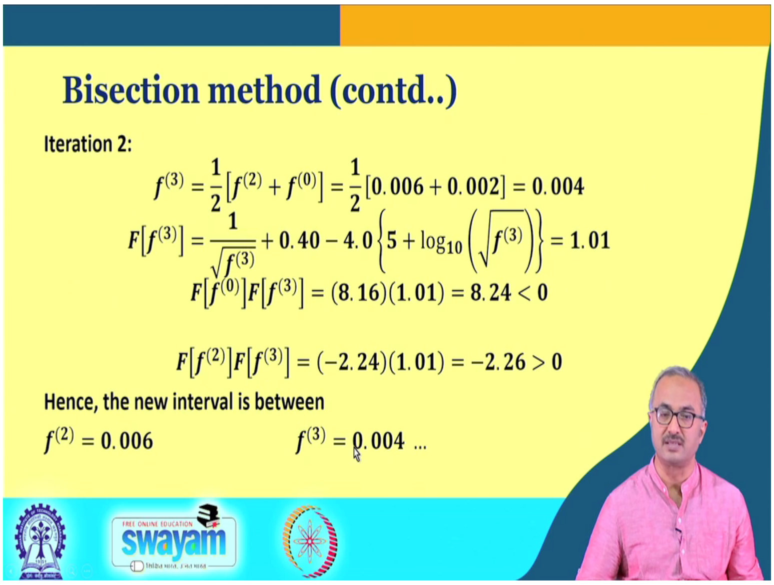

Now, with this what we do again we bisect it to get another estimate. So, here we find bisection we are getting 0.004 we find the function value at this new unknown variable and we find this is coming to 1.01. We again take the product between the function values for these payers. So, f 0 f 0 0 into f 3 is coming positive and f 2 into f 3 is coming negative. It means the new search zone should be between f 2 and f 3 and now this is f 2 and this is f 3. And now you can see that you can now carry out this particular iteration and keep on shrinking the domain of your search.

Now, from this particular thing you see that it seems that the domain of the search is now moving closer and closer and the next value next search will be 0.004 plus 0.006 divided by to that is 0.005. Now again you have to take the function value to that point and you can see that how we are slowly and slowly able to shrink the domain our search for the root.

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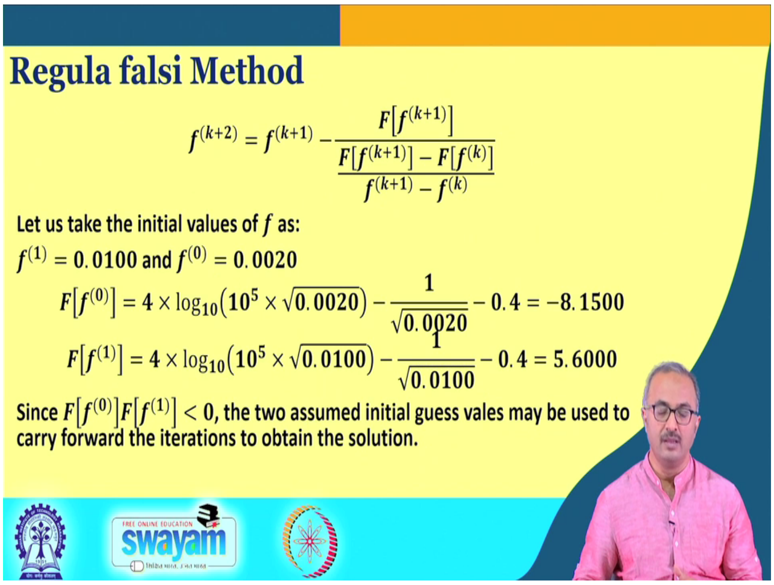

Next we come to another bracketing method that is the Regula Falsi Method and in this case we are taking some slope value here and here also we need the evaluation of function values at two points. So, here we need also to start with two initial guesses. So, we are what we are doing here we are taking the initial guesses same as we have done for the bisection search So, we have to just simply find out the function values which we are doing it here at point f 0 f 0 0 point f 1 1 and here at the function values. And we can see that f 0 f 0 0 and f 1 1 coming less than 0. So, we see that this f 0 f 0 0 and f 1 1 are able to bracket the roots ok. Now after bracketing the root it becomes now easy to implement this particular method.

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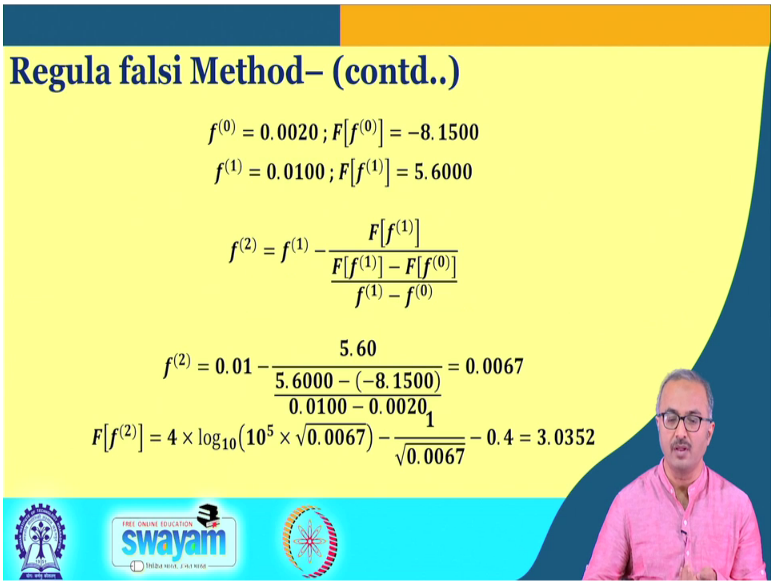

So, we find the function values at this two assumed values, and we are finding this function values and we have put this values here. And we generate the value of the next updated value for the friction factor. and this is coming out to this. Now again we evaluate the function value with this new friction factor and we find that this is the value of the particular function. Now here you can easily see that if you multiply this with this particular ff 1 you will get a positive value if you multiply this ff 2 with ff 0 you will get a negative value. Now that means, that the new search domain should between this f 0 f 0 0 that is 0.002 and the f 2 that is means 0.0067.

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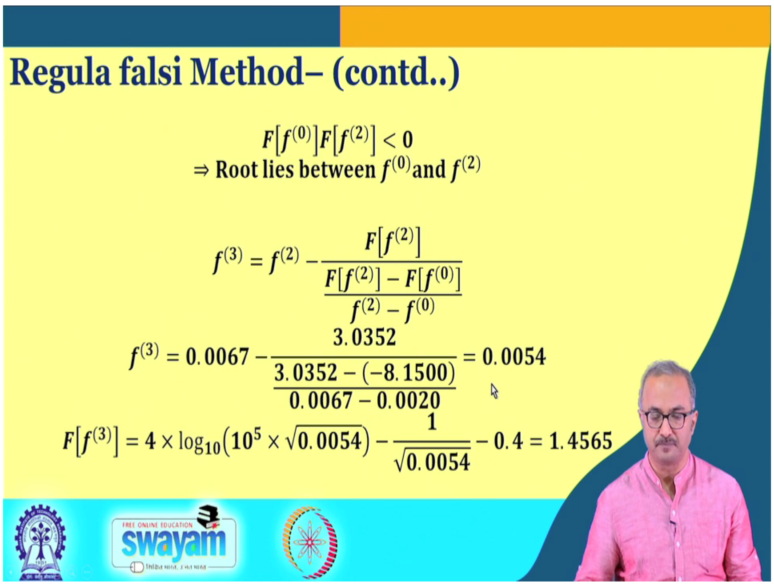

So, with this what we do we find the next updated value; that is f 3; and again we find that f 3 is coming 0.0054 and at this f 3 we are finding the function value and this is coming to positive value.

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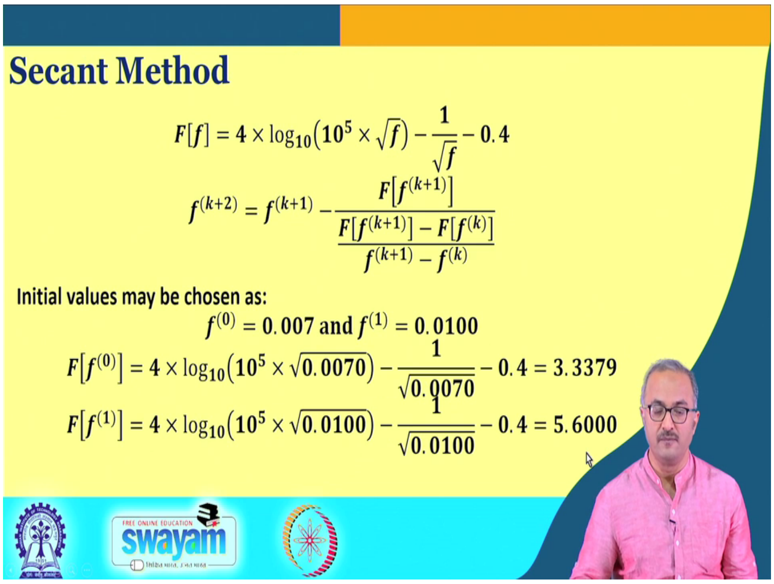

So, as we are done earlier we again take the product and we can keep on finding the new search domain. Next we come to another method that is similar to the Regula Falsi method, but this differs in the way that in this case we are not bothered about bracketing the root. So, it is open way of finding the root. So, here what we are doing that the formula remains the same.

Now, what we are doing that we have just changed our initial guess values. So, instead of taking it 0.002 we are taking it to be 0.007 and 0.01. And when we find the values of the functions at this two guess values what we find that they are both positive. But here we do not bother to check for any kind of sign of the products we simply go with these two values and find out the next value of the function.

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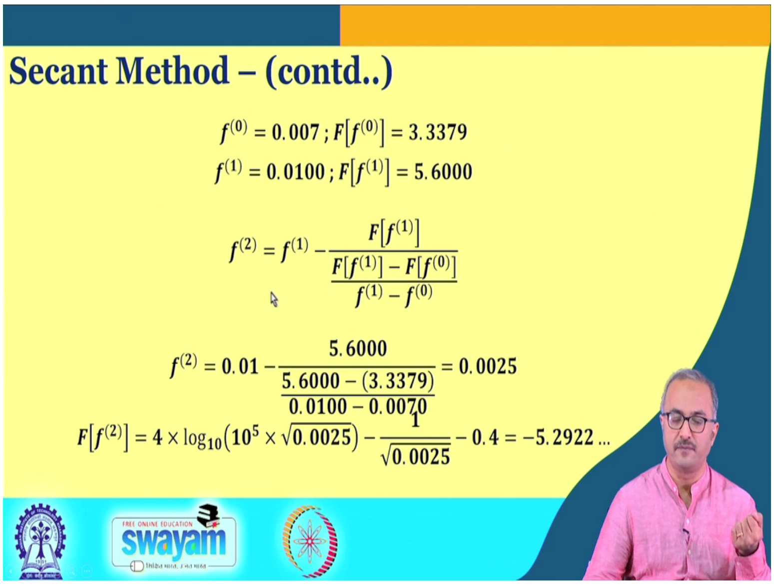

So, here we have that we find that with a function value this new value is coming out to 0.0025. And as we have discussed earlier that depending on the nature of the function we can also see the nature of function whether it is increasing or decreasing. We can check that f 0 f 0 is decreasing and f f 1 is more. That means, the seems that we if we go from a f 1 to f 0 f 0 we will be able to go towards the root ok, because root means that if this capital f value should be 0. ok.

So, since we are going towards 0 in this direction and not the other direction; that means what next for next we should choose this one and the new value for our search purpose ok. That is how we keep on eliminating these values of the unknown.

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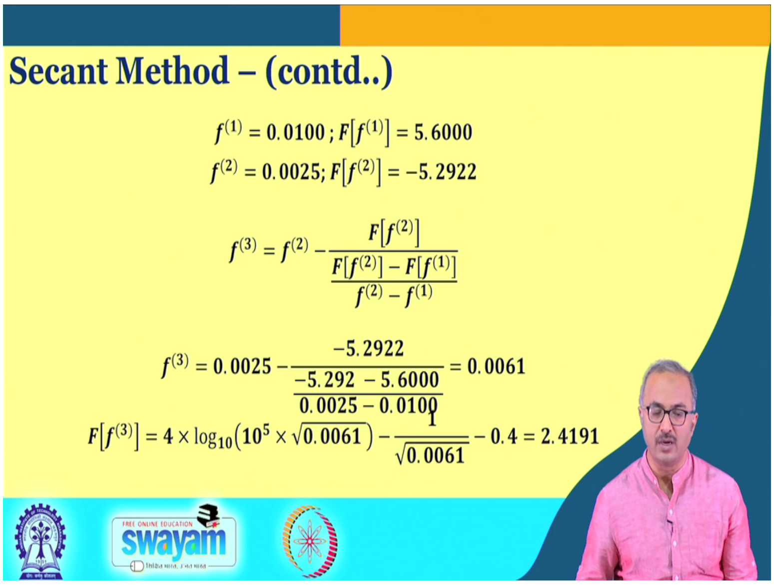

So, what we can do now? Now you can go with this kind of things and we find that we keep finding the values of this new value and again keep checking that in which direction you are going to go and you keep repeat doing these things and slowly and slowly you also find that you are able to approach the root.

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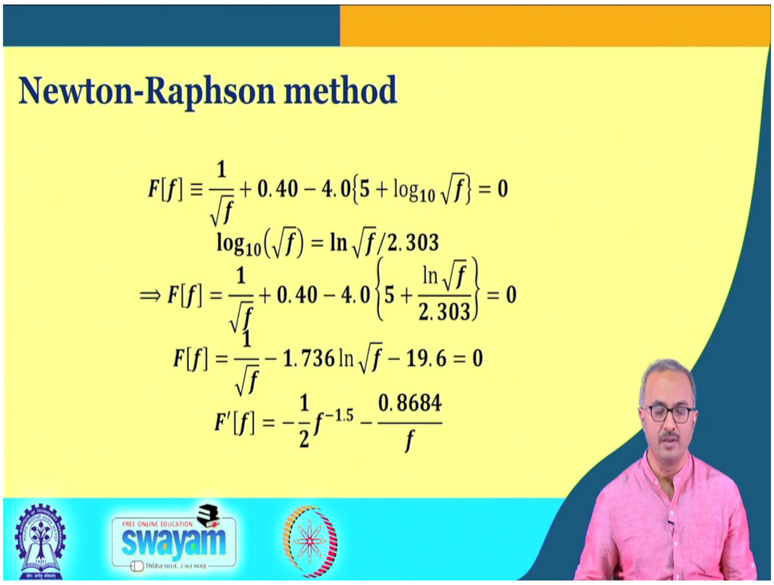

Next we come to a very effective method that is Newton Raphson and as I told you this method works the best among all the methods provided the initial guess value is near by the actual root. It will not be working well if we are much too away from the actual root it may also diverge or it may so oscillatory nature in convergence so that we will not work out. So, here is what you do we put these function value here and as we learned earlier this particular methods needs the evaluation of the derivative of the function. So, in this case we shall take the derivative of f with respect to the friction factor this f prime f shows this is d f by d f that is; the derivative.

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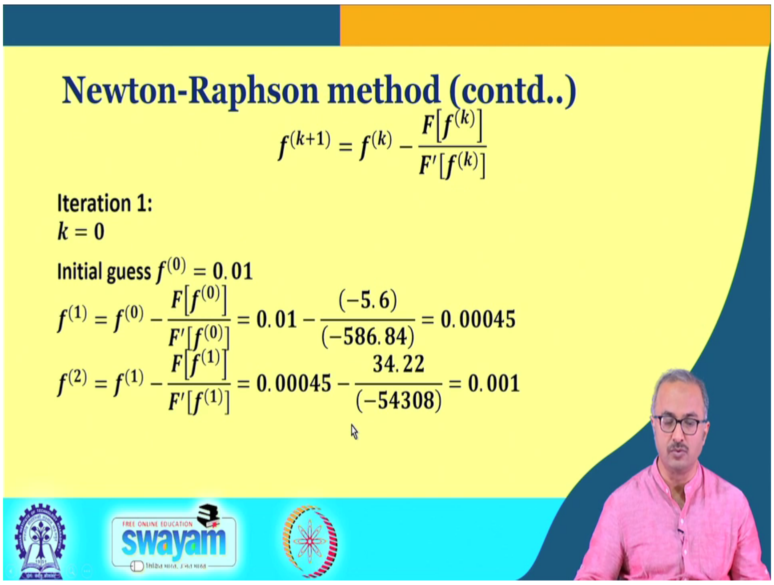

So, once we put this particular expression for the derivative, what we now do that we go to this recurrence formula for Newton Rapshon method, and again we start with a guess value for k equal to 0 and let us take it arbitrarily to be 0.01 ok. Now with this 0.01 we find out the next value and after iteration one this coming out to this particular value ok. And then what we do we take this new value for to get the next updated value and this is coming out to this.

Now, you can see that its seems to be fluctuating that from 0.01 it went to 0.00045 and again it moved upto 0.001. And you may carry out this particular method and you may find that there could be some kind of oscillations; that means, lead to divergence or it may also lead to the solution. But it is not going in a very smooth manner ok, but now if you look at the previous method bisection such etcetera. You will find that this kind of nature is coming for Newton Raphson that may give us some wrong notion as if Newton Raphson is not an effective method. But then you must remember that this method works well when we have the initial guess value good.

So, with these particular guess value it seems that this is not a very a good guess value. Then what we do we take another guess value 0.007, now when we take this particular value now we can see that you use this value how fast we are getting towards some kind of converging value. Now you see that 0.007 it was brought down to 0.0035 then it is going to 0.0043 and then it is moving to 0.0045 we are almost reaching the convergence with this particular method. So, you can see that this Newton Raphson method highly depends on the initial guess values.

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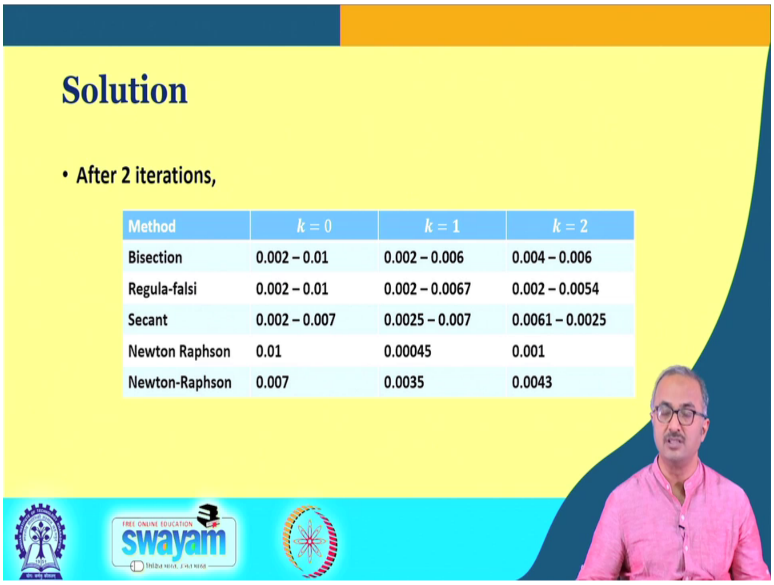

Summarizing, all these methods you can see now that how these methods work I have shown you the various methods and the iteration level here.

And you can see for the bisection search how this region of our search is decreasing it is till long way to go and understand this that Newton Raphson seems to be converging towards this 0.0043 value now with respect to this particular value you can see that all these methods are moving definitely towards the solution with shrinking domain of search.

But it seems that their search domain is shrinking a bit too slow and much slower than the; rate at which the Newton Raphson is able to move towards the solution with a good guess value. So, this particular table clearly tells us that the efficacy of the Newton Raphson method. And if the Newton Raphson method is to be implemented we may not be able to do it right away; what we can do then is this we can check any of these methods which all of them seem to be going towards a mode and mode shrinking zone of search.

So, that with one of these methods we can first generate the right domain of search of the root and then we may move on to Newton Raphson once we have located the search zone or we have found the direction in which the solution would should be taken. So, we may take that we may use some kind of hybrid way of finding the root of the solution for any kind of non-linear algebraic equations.

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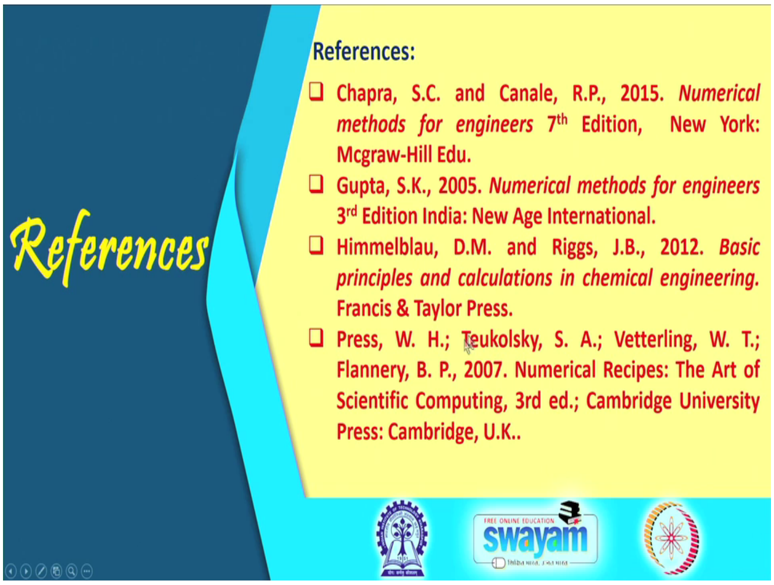

More of this may be studied from these particular references.

Thank you.