Mass, Momentum and Energy Balances in Engineering Analysis
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Lecture – 10
Macroscopic Balances

Welcome. Today, in this lecture, we shall be doing some solutions to some problems on based on the Macroscopic Balances.

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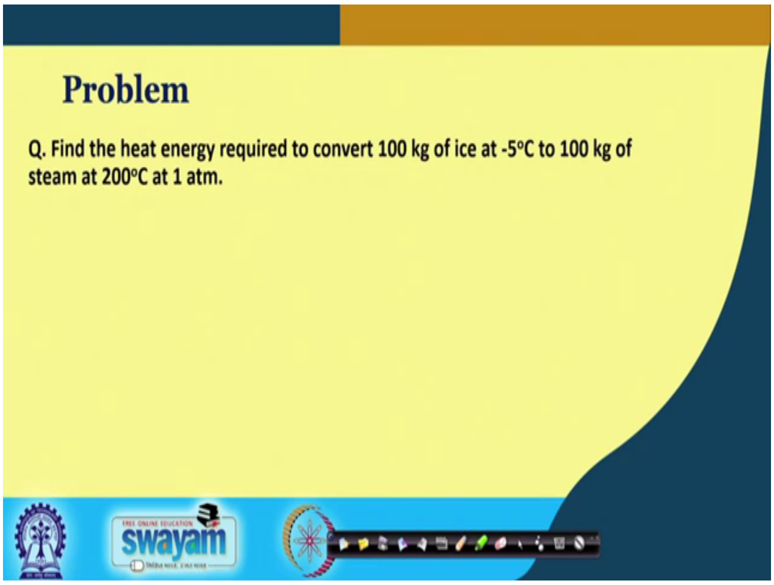

So, first we will take up a problem on the energy requirement to heat up ice to water. This is a problem which you might have done quite regularly in your school days, but we shall be looking into it from the angle of setting up the various equations, because this kinds of equations are always necessary for energy balance problems. So, we will find that the energy requirement for such kind of conversion of phases come, as some kind of source terms in the energy balance equations.

So, here we have to find out the heat energy required to convert 100 kg of ice from minus 5 degree centigrade to 200 kg of steam at 200 degree centigrade and 1 atmosphere. So, first to do this problem, we have to understand the thermodynamics state of ice and the steam.

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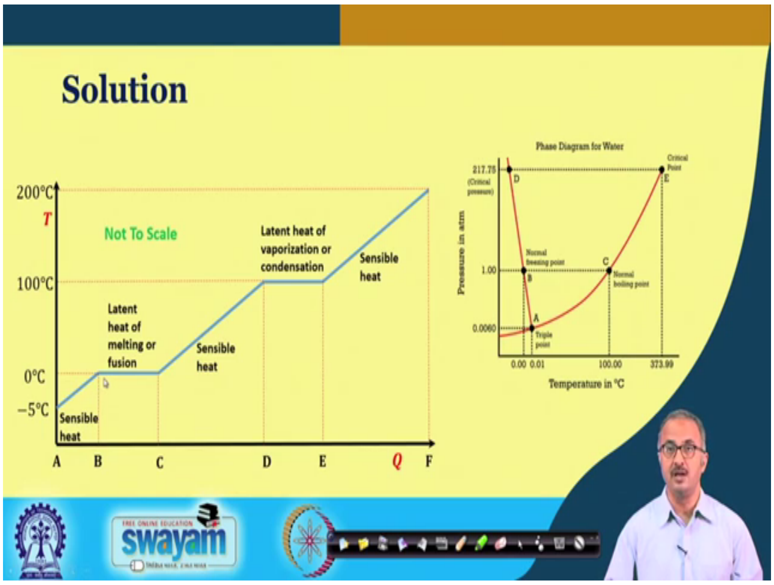

So, here you have to refer to this phase diagram and we know that minus 5 degree centigrade is below the freezing point of water at 1 atmosphere. Similarly, 200 degree centigrade is above the boiling point of water at 1 atmosphere. So, you see that in this particular thermodynamic diagram, we have these three curves, which are denoting the phase change curves. So, this is the condensation curve, this is the fusion curve and this is the sublimation curve.

And here we have triple point, and here we have the critical point, and this is for water. And we know we see that our temperature lies somewhere on the right hand side of this 100 degree centigrade at 1 atmosphere and the temperature of the ice is below the freezing point. That means, here somewhere it is 0 degree centigrade so ice is somewhere here at 1 atmosphere ok. So, we have to go from the solid zone to the superheated vapor zone.

So, in this case, we see that if we plot the temperature versus our heat energy requirement, we see that when we start heating up the ice, it undergoes various changes in its phase. And during each of these changes of the states, what we find that we need different types of the heat energies. So, first the minus 5 degree centigrade ice has to be first heated using sensible heat up to 0 degree centigrade. And perhaps you know sensible heat means that heat that can be sensed by the thermometer through some temperature change ok.

So, during this point from minus 5 degree centigrade up to 0 degree centigrade, we need some heat energy. And then once it reaches 0 degree centigrade the phase transformation will occur. So, during this time there will not be any change of the temperature while the ice will be melting. And in this case, we shall be using the latent heat of melting or fusion. After all the ice melts then what happens, it will now be again further increasing its temperature.

Now, it will be water here. And again we shall be needing the sensible heat to increase the temperature of water from 0 degree centigrade to 100 degree centigrade to get the saturated liquid. And once it goes to saturated liquid, then again we find that phase change will occur at constant temperature and during this period we shall be using the latent heat of vaporization or condensation.

Once all the liquid has got transformed into the vapor, we get the saturated vapor state. And further addition of the heat energy will result in the increase in the temperature of the superheated vapor. And during this particular process, we shall be again using the sensible heat. Now, to find out the total amount of heat energy required, we have to simply add up the heat energy required at each of this processes.

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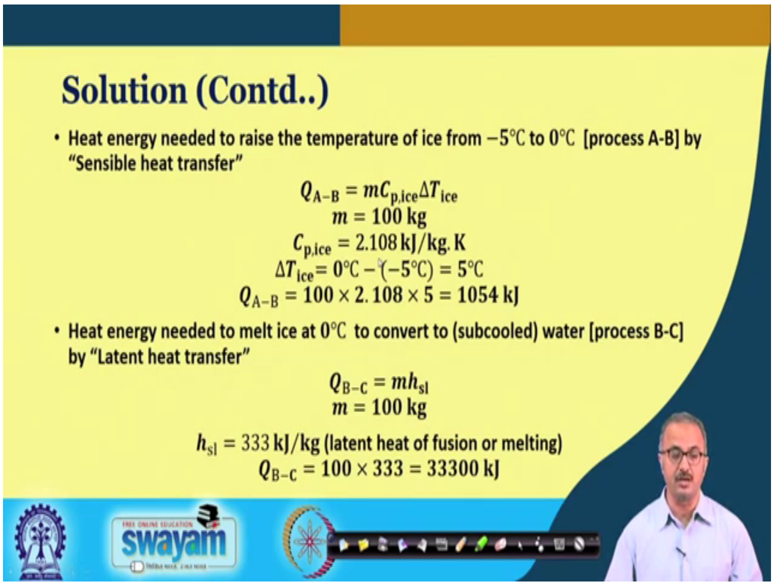

So, let us see how we do it first we see that heat energy required to raise the temperature of ice from minus 5 degree centigrade to 0 degree centigrade by sensible heat transfer this is the process AB in the diagram. So, heated here Q A B that is the heat energy during the process A to B, we have the mass of the ice then C p that is specific heat of the ice, and the delta T ice is the temperature difference of the ice.

So, we put all the values given in the problem and this value we get from some data source. And here we have the delta T ice is the final temperature minus the initial temperature. So, this is 5 degree centigrade. And so, we can find out Q A B by putting this values and this is the value of the sensible heat required to heat up the ice. Next we come to the phase transformation of ice to liquid water. So, here we need the latent heat of vaporization for this we simply need to multiply.

The mass of the ice with the enthalpy of vaporization, enthalpy of melting and in this case this s is signifies the transformation from solid to liquid state. Again putting these values here we find this is the heat energy required to melt the ice to water. And here we can see the latent heat is much more the sensible heat it is always. So, generally whenever there is some moderate amount of temperature change, we find the latent heat contribution is much more than the sensible heat contribution.

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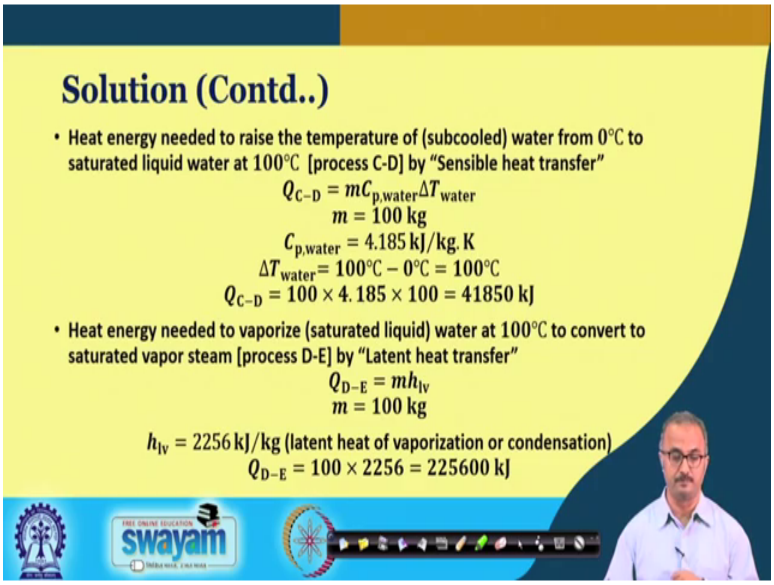

Now, next we come to water. In this case, now the sub cooled water at 0 degree centigrade will be heated up to get the saturated liquid water at 100 degree centigrade at one atmosphere pressure, and here we are using the sensible heat transfer. So, here it is computed by multiplying the mass with a specific heat of the liquid water and the delta T water please note that we are assuming the specific heats to be constant within the temperature range.

So, for ice, for water, for water vapor, we are taking them to be constant ok, but if it is not to be taken constant, then we have to take the integral C p into d t and we should have in that case some correlation for specific heat based on temperature. And we have to integrate the correlation with temperature to get the heat energy required. So, here we are taking them to be constant. So, we are taking value of the specific heat of water here and the temperature difference of the water from 100 degree and 0 degree centigrade. So, we are having this is the sensible heat required to convert water from 0 degree centigrade to 100 degree centigrade.

Now, next the phase transformation of liquid water that is saturated liquid to the saturated vapor.

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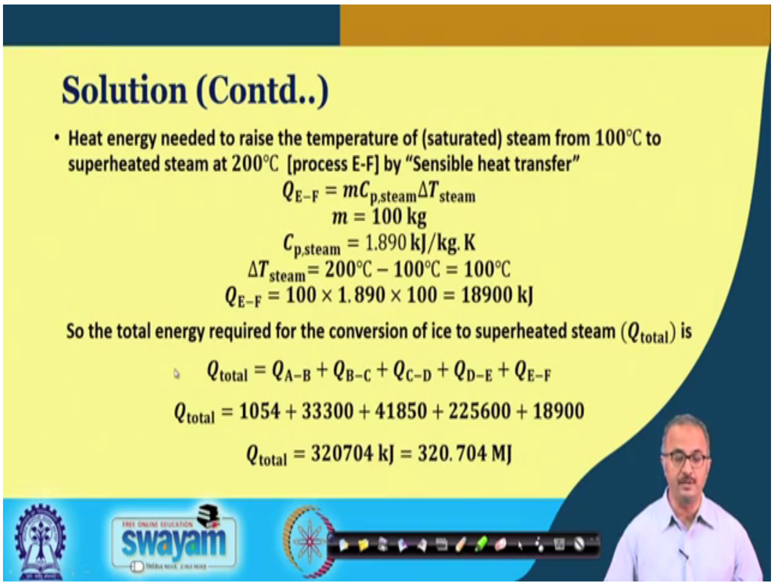

So, here we are again using latent heat of vapor vaporization. So, here h l v signifies the latent heat from liquid to vapor state. So, this is the value of the latent heat. And here we are again getting the amount of heat energy required to convert the saturated liquid to saturated vapor.

After this we go to the raising the temperature of the saturated vapor to superheated vapor at 1 atmosphere to 200 degree centigrade. Here again we put the specific heat of the steam and this is the value. And again we get the delta T steam as 200 minus 100 degree centigrade that is 100 degree centigrade. And again putting the values in this particular formula, we get the sensible heat transfer needed to raise the temperature of the steam.

Now, when we want to know the total amount of heat energy required, we simply sum up the heat energy required in each of the sub processors. And here we get after adding all this heat energy contributions. This is the value of the total heat energy required in kilo joule or in mega joule.

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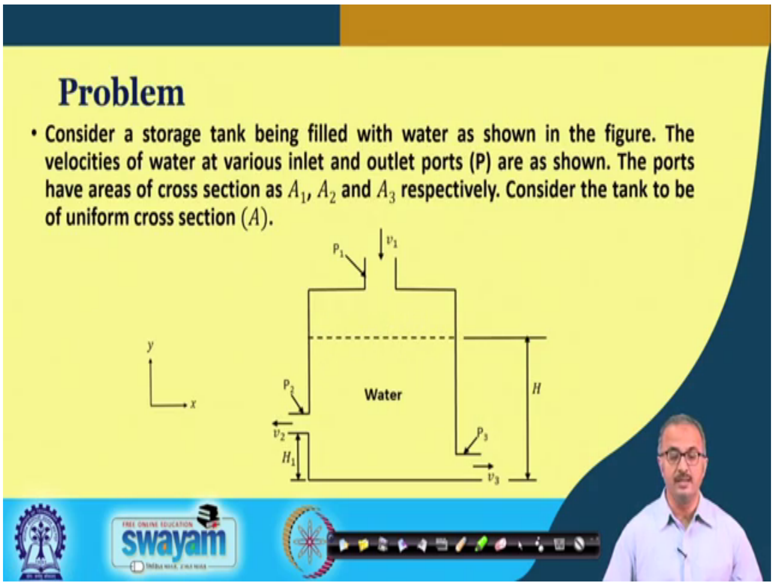

Next we come to a problem where we are we shall see that how to apply the material balance. Now, we are going to the lumped parameter analysis or macroscopic balances. Here we have a problem which says that we have to consider a storage tank being filed with water as shown here. And here we can see that in this particular tank there are several ports one port is for the inlet of water and there are two ports which are the outlet of water now during this flow of this three streams we shall be having some water.

Now, this water depth will be changing depending on this three flow rates ok. Now, once this three flow rates come to a value come to some values, which resistance steady state, we will find that the level of water will also reach constant value ok. So, this is the process description and other things are that we are designating the ports by P. So, this P

1 port, P 2 port, P 3 port, P 1 port is inlet P 2 and P 3 are outlet ports then we have the H 1 that is the height of the port P 2 from the bottom of the tank.

So, and here the liquid height when the bottom of the tank is taken to be H and this v 1 is the velocity of the incoming water v 2 is the velocity of the outgoing water from port two and v 3 is the velocity of the outgoing water from port three. Now, please note that these velocities are average velocity over, the whole cross section; That means we are not considering any distribution of the velocity across the cross section. And the areas of each of the ports are taken to be A 1, A 2, A 3 and the area of cross section of the tank is taken as A.

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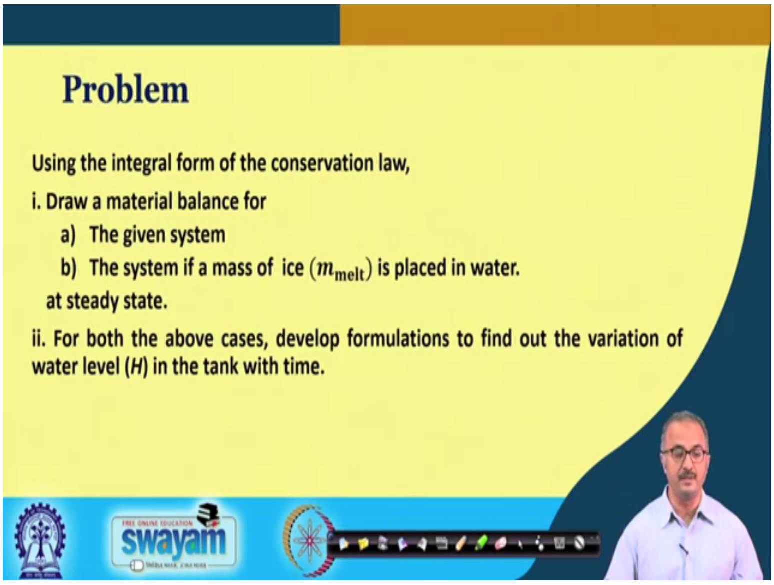

Now, with this we have to do something that we have been ask to use the integral form of the conservation law to draw, the mass balance for the given system and then for the system if a mass of ice is placed in water, and this is at steady state. Now, for both the above cases develop the formulations to find out the variation of water level in the tank with time.

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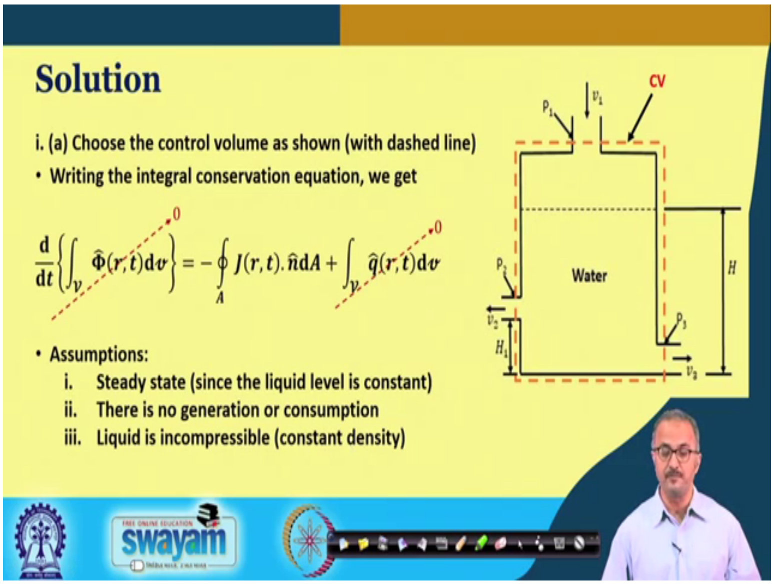

Now, in this case first we come to this particular case that we choose the control volume and the control volume is cutting all the ports ok, so that we can take care of all the inputs and outputs into or from the control volume. Now, we write the mass, the integral conservation equation and this is a generalized one, it can be applied for mass balance, energy balance, momentum balance. Now, first we understand that this is steady state process. So, we take the variation with time to be 0.

And because in this case the liquid level is also constant and there is no generation or consumption. So, we take this term to be 0. So, what we are left with is only the flux term. And here we are taking the liquid to be incompressible so that the density of the liquid will be considered constant along the flow path and at the three ports.

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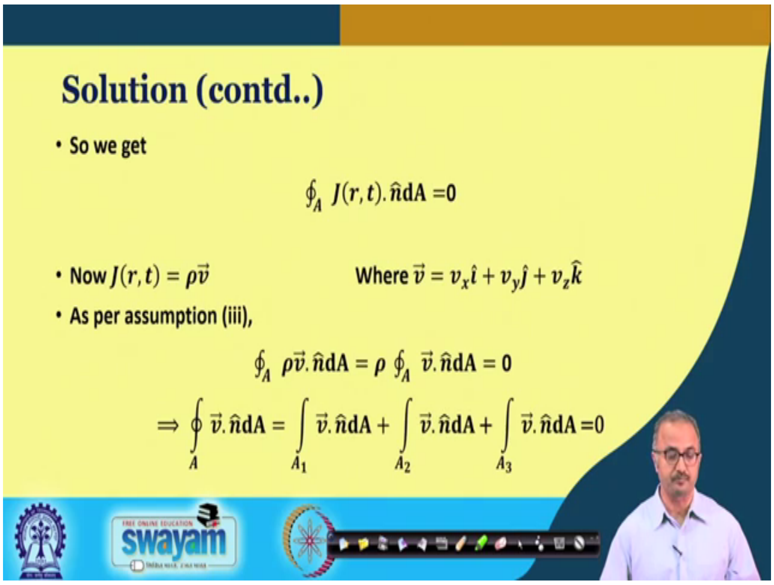

So, we write this particular flux term here. And now we recognize that the flux is the product of the density and the velocity that is the mass flux. Now, suppose you take this density to be kg per meter cube you take the velocity to be meter per second, then this term will be a unit of kg per meter square per second that is flux is the quantity per unit time per unit cross sectional area

So, here we have the velocity is written in its vectorial form this v x, v y, v z are the three components in the three directions and i, j, k are the unit vectors in the x, y, z directions respectively. So, here we are now expanding these terms so first, because the density is constant. So, we can take out the density out of the integral, and then we get this particular term, and this term is now putting at apply to three different ports. Now, we shall come to each of these terms one by one.

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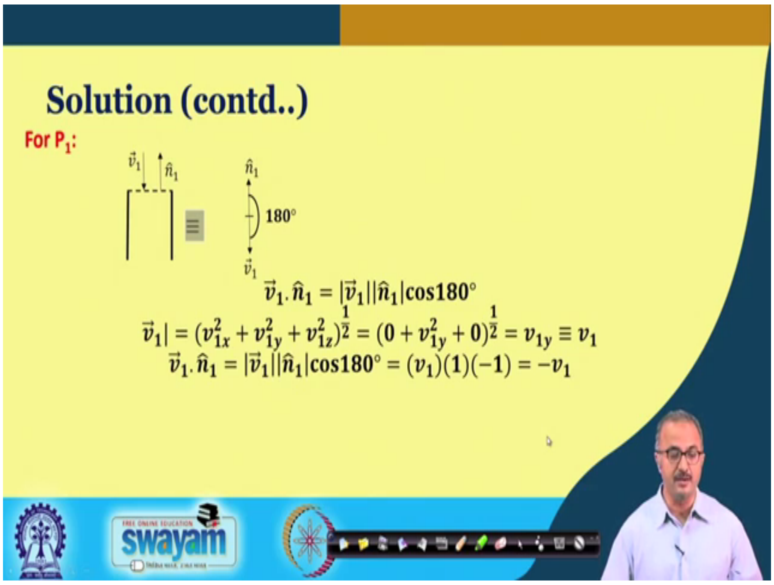

Let us see now for the port one what we see the velocity has this vertically down direction whereas, the normal of this particular surface is out word. So, this velocity direction and these normal directions are opposite top each other, and the angle between these two vectors is 180 degree. So, when we take the dot product, we have to take the magnitudes of each of these two vectors, and the cross of the angle between them and this angle is 180 degree.

Now, we know the magnitude of this vector velocity is v 1 x square plus v 1 y y square and v 1 z square and to the power half. So, in this case we have only the y component and vertical component. So, we write this vy v 1 y and this we take equivalent to v 1 ok. So, now this v 1 dot n is this we put this has v 1 this is at unity and this cross one the degree is minus 1. So, we get minus v 1.

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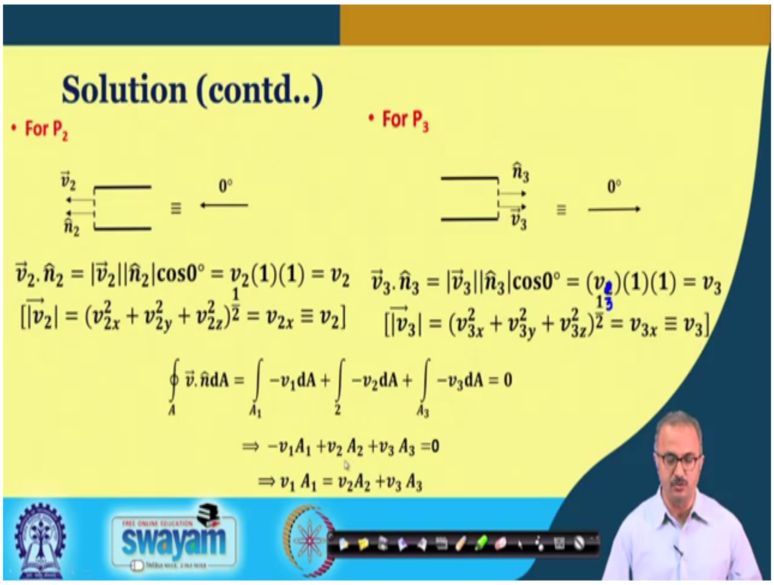

Now, with a similar logic we also see at port two that the direction of the velocity, and the direction of the out word normal at port two are the same so that the angel between them is 0 degree, we apply the same analysis as before. And we find that this v 2 dot n 2 is coming to v 2 x in this case we are taking this as the x direction. So, v 2 x and we are putting that is v 2.

Next for the port three again we find the out word normal and the direction of velocity are in the same direction. So, that the angle between them is 0 degree. And again we find that this is the expression for v 3 dot n 3 and we put this as v 3. Now, please note here this will be v 3 ok. So, with this now we have all the terms of the flux in the equation ready.

Now, we shall be applying this two our previous equation for the mass balance. And now we see that we are putting the respective values minus v 1 A 1 plus v 2 A 2 and plus v 3 A 3; oh this will be plus. Here it will be plus here you see plus here. So, this is how we are getting, the final expression correlate correlating the velocities at the three ports and the areas of cross section.

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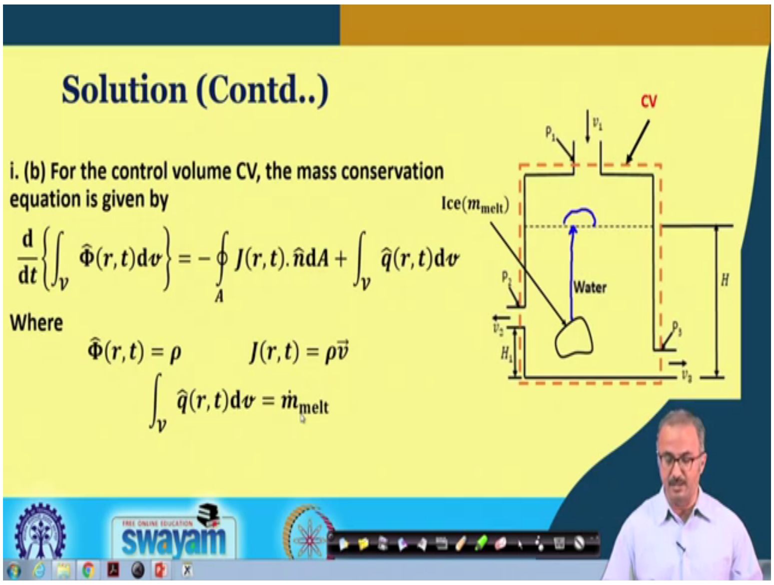

Next we come to the second part of the problem here we have been asked to consider that there will be some ice, which has been put in the water. And this ice will be with time it will be floating here; that means, when once, it goes down then this ice will also come to the surface of the water and we can see that ice will be floating here. So, this is the way for this particular situation again, we have to make the mass conservation law. Now, you see that in this case when the ice is put the ice will start melting. And due to this melting there will be a change in the water level ok. Now, this makes the system unsteady.

So, now this ice the addition of the ice is not at any of the incoming or outgoing port it is within the system. So, this may be taken as some term for the generation of the mass. So, now when we put this we find that we will not be able to put this as 0, because this is unsteady state process, we have to written this and neither, we will be putting this as 0, because this will take care of the melting of the ice. So, here we are writing this that this phi cap is taken as the rho and this j is the rho v again as earlier and this particular term is the amount of ice in the system.

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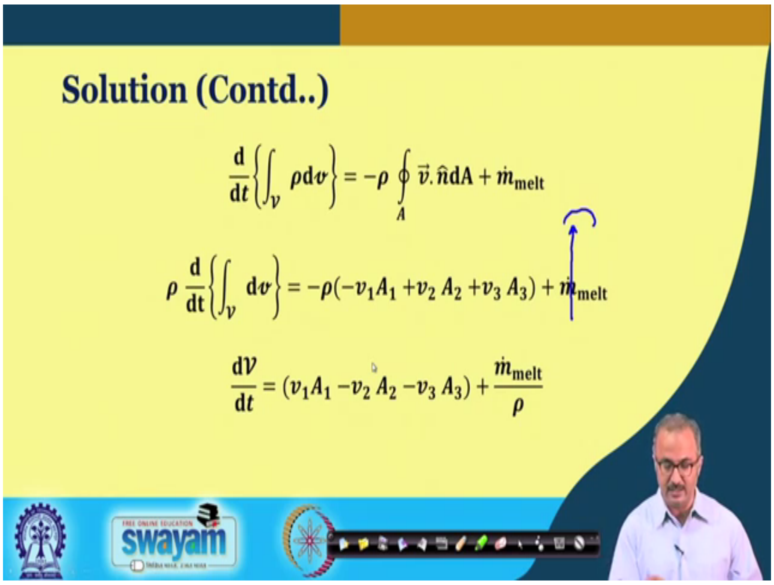

Now, we see that this rho dv is this rho dv, we are doing on the left hand side and we are putting this v dot n and this is put a melt. And again whatever we have done earlier we are put with that we find that this is the rho is taken out of this differential, because it is constant. So, this is giving us the total volume of the system, and this is giving all these, all these terms for each of this flux terms and this is melt. And then after rearrangement we arrive at this particular equation to find out how the volume would change, because of the inflow and outflow and due to the melting of the ice.

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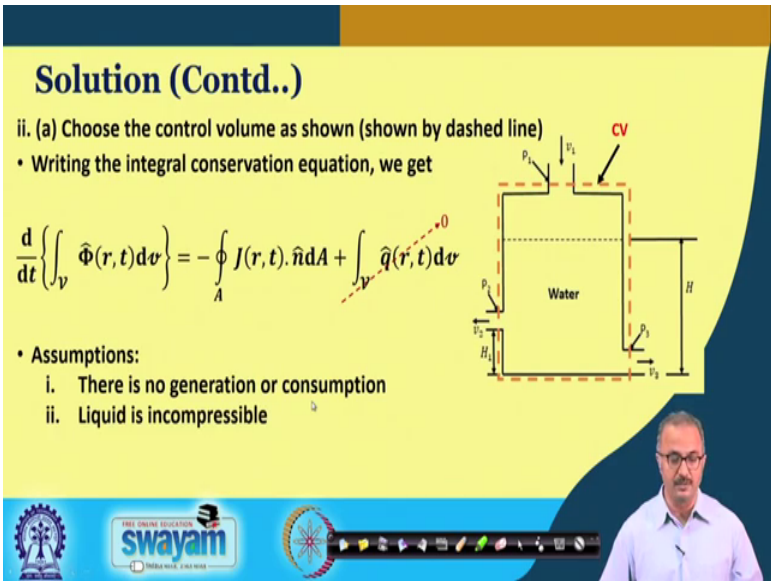

Because now we have to find out the change in the height of the system for the next part of the question: so first we take this particular thing now here we are not taking any kind of steady assumption. So, there is only we are taking that there is no generation or consumption ok.

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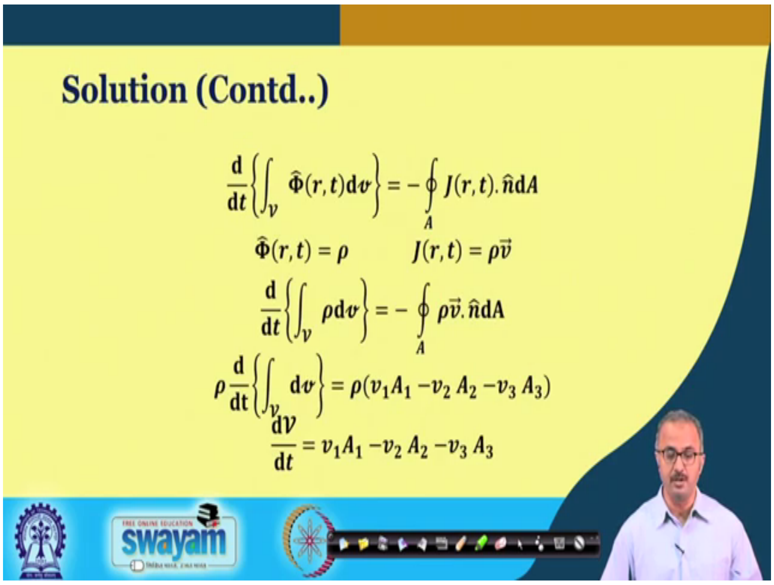

So, with this and this liquid is in compressible. So, again we put all these values over here and without generation term and again, we see that all these things we have done earlier repeating all those exercises, we arrive at this particular expression without any generation term ok.

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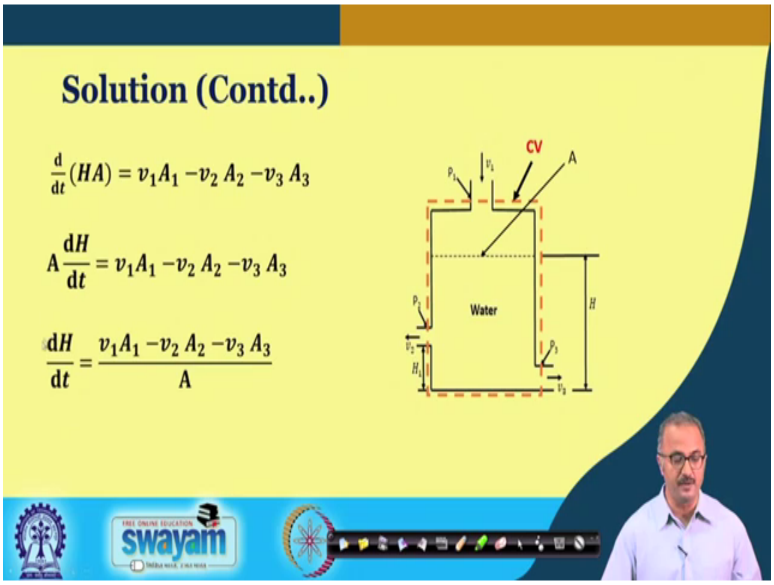

And now to find out a change in the height of the liquid inside the tank what we do we put the volume of the liquid inside the tank as HA. And we take out the H out of this differential, because it is taken to be constant. And we arrived at this particular expression, which will give us how the height of the liquid inside the tank would change due to this various influxes and out fluxes ok. So, here this thing is finally here. So, here we rearrange this term to get the change in the height of the liquid with time.

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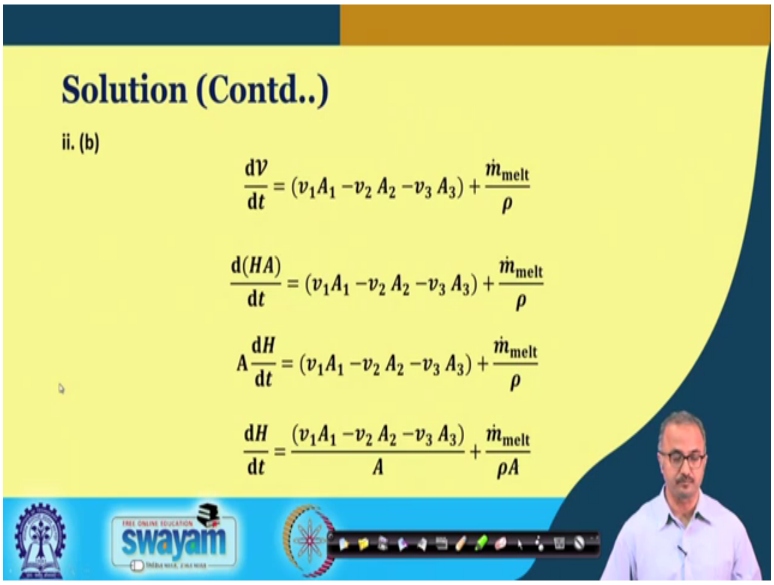

For the next problem, we have the melting ice. So now, again we write for this v we put HA and A is taken out of differential. And we finally, get this expression. Now, you can see this expression is a slight modification of the earlier expression. And in this case if you put the m melt as 0, then we shall be having the same expression as earlier. And this is some kind of an internal check whenever you are writing this balances, you should see that for some asymptotic cases you are getting the same result as without the particular some term ok. So, here we find that if we take this to be 0, we are getting the same term as earlier. So, this balance seems to be alright.

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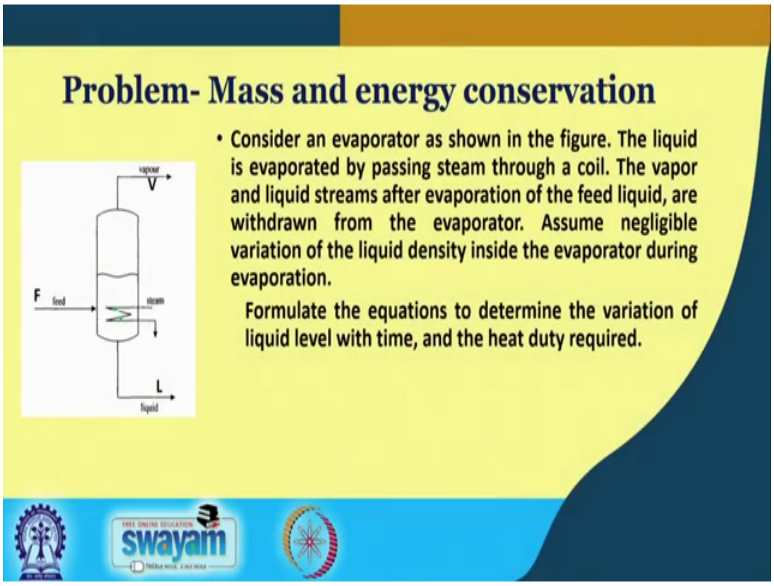

Now, we shall consider another problem and this will be involving both the mass and energy conservation laws. So, the problem is like this that we have an evaporator and evaporator is a device. In which we evaporate a liquid and may be used for concentrating some kind of juices in industries. So, here we have the evaporator in which we have some feed that is going into the system and after evaporation the vapor is taken out from the system from the top.

And a liquid is taken out from the system from the bottom and to do the evaporation some steam is being used. So, we have been asked that you have to assume the density of the liquid to be almost invariant during the evaporation and we have been asked to formulate the equations to determine the variation of liquid level with time and the heat duty required. And this particular steam is getting flowed through a pipe or may be a coil.

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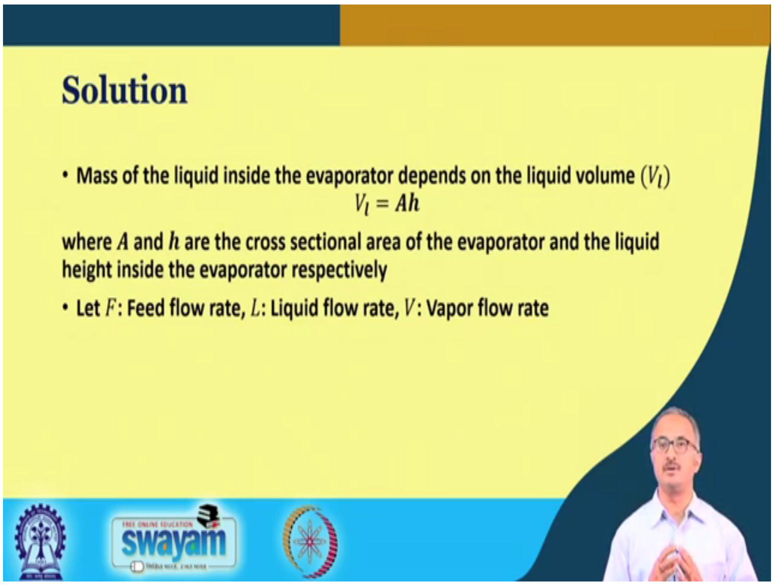

So, that means, that steam is not coming in direct contact with the liquid inside the evaporators so that as per the mass balance of the liquid there is no effect of the mass of the steam on to the mass of the evaporator system ok. But the liquid inside the evaporator is interacting with the steam through, the heat energy transferred from the steam to the liquid.

Now, first you understand that the height of the liquid decides the mass of the liquid inside the system. And the mass is the product of the volume of the liquid and the density. So, here we write the volume here and here we are putting all the flow rates like this ok. So, liquid flow rate for an L, V vapor flow rate is given by V and feed flow rate by F.

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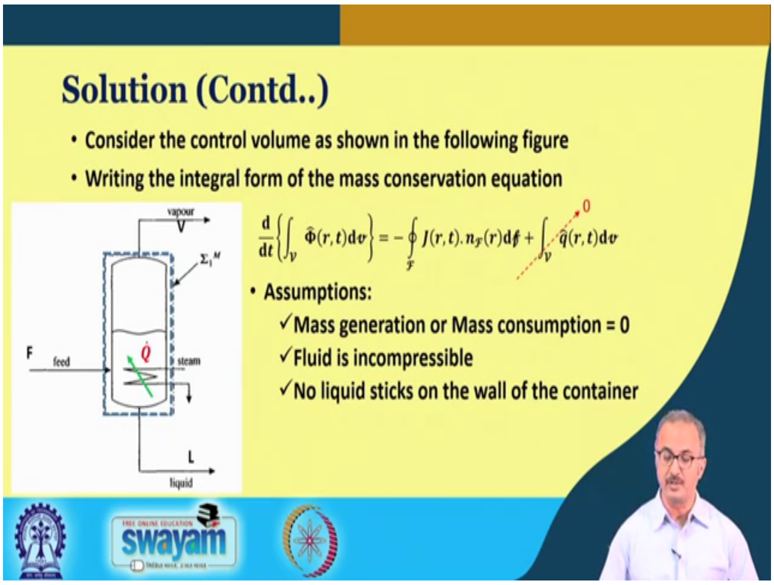

Now, we take the control volume around the whole reactor and for this we first see the mass conservation here we are finding that no mass generation. So, we take this as 0 and then this flow rate it is incompressible to. So, that density will be taken to be constant later, and we assume that there is no liquid sticking to the wall of the evaporator.

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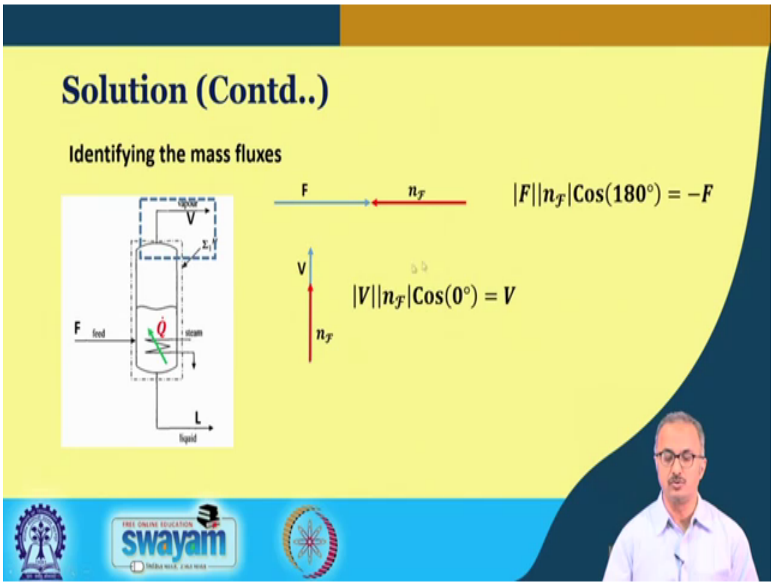

Now, as we have done earlier we find out that flux terms for the feed, we see that the feed the direction of the feed, and the outgoing normal are opposite to each other. So, the angle between them is 180 degree and so we put these magnitudes of the feed, and the unit normal and we get the flux to be minus F. Similarly, we when we go for the vapor we find that this outgoing normal and the vapor of the same direction; so that the flux value its coming to V.

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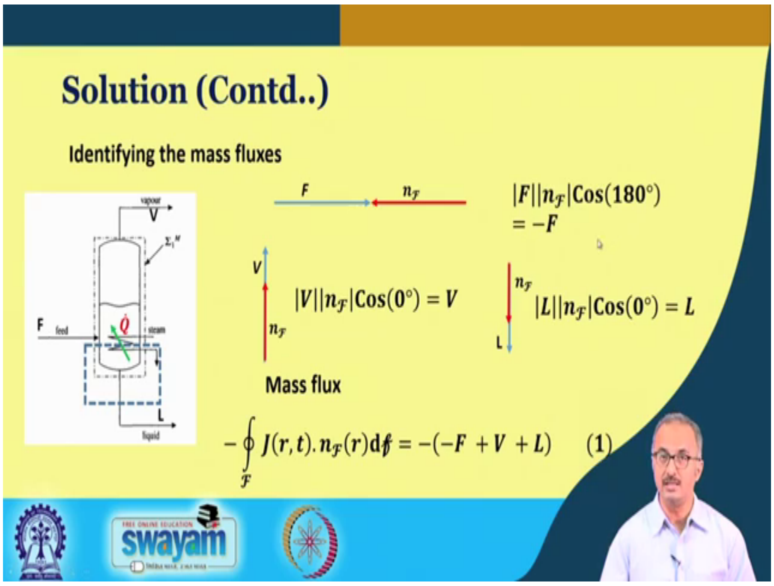

Similarly, for the liquid side the same thing I mean as vapor, we find the flux of the liquid its coming to L. Now, the mass flux cannot be written by taking the sign into consideration. So, this is a minus F plus V plus L and with the negative outside so it becomes F minus V minus L.

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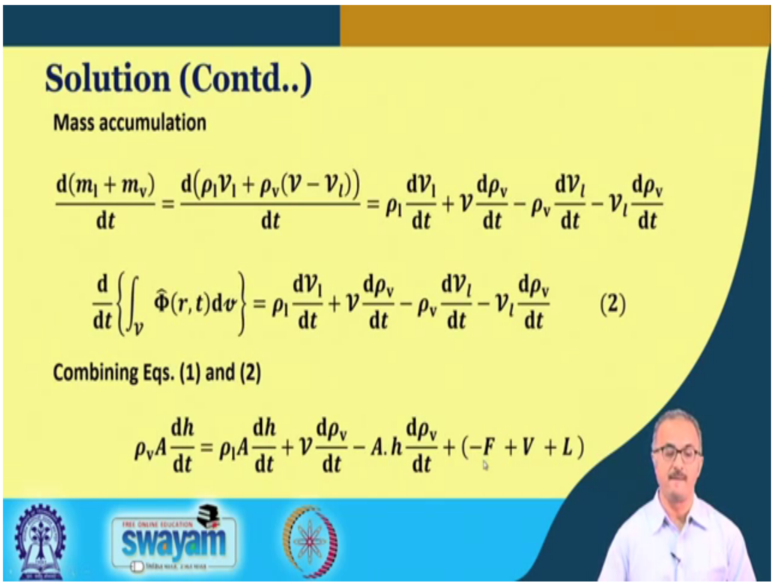

Now, coming to the mass accumulation term, we see that the mass within the evaporator comprises two components, one is the mass of the liquid, and the mass of the vapor. And if we put the mass of the liquid in terms of the density, and the volume of the liquid and a mass of the vapor as a density of the vapor and volume of the vapor. Now, you see the volume of the vapor is the total volume of the container minus the volume of the liquid. So, so that we are avoiding m an extra variable that is the volume of the vapor. Now, if we put this here in the accumulation term and then we expand it, we find this is the particular terms we shall be getting.

Now, this is the after putting this terms and I am putting the flux we will be having this particular expression to find the variation of the liquid height with time inside the evaporator. However, if you look at this particular expression you see that this expression is not a very straightforward expression, it is a differential equation in terms of h. And here we have one term we will be having the dh by dt, another term will be having just the height of the liquid, and rest of the terms will not be having the h, but this will need the knowledge of the variation of the vapor density with time. And rest of the things can be solved using some standard solver for differential equations.

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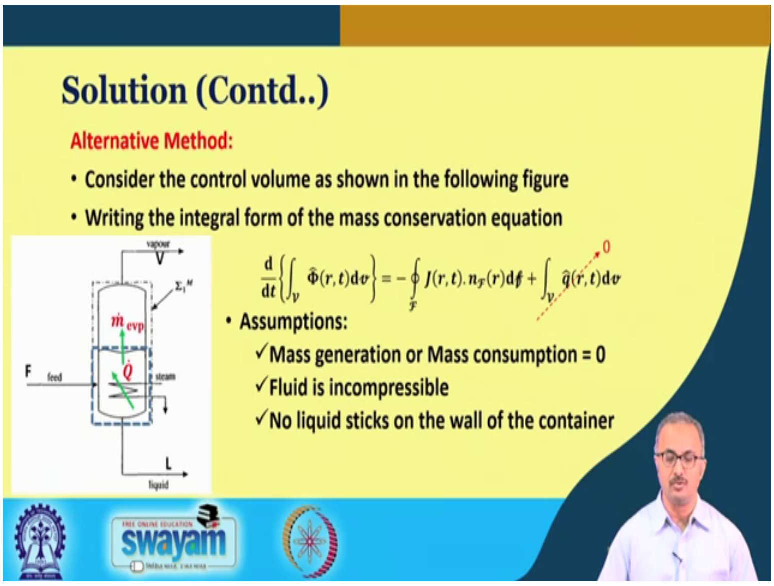

Now, another formulation may be made to simplify this problem a bit and that is like this thus instead of taking the whole evaporator as the control volume, we can just focus on the liquid side as the control volume. Let us see that what advantage, we are deriving by this particular alteration in the selection of the control volume. So, again we write the mass balance equation for this new control volume. And here in this case, we see that the there is no generation or consumption of the mass the fluid is incompressible and then the no liquid is taking to the container.

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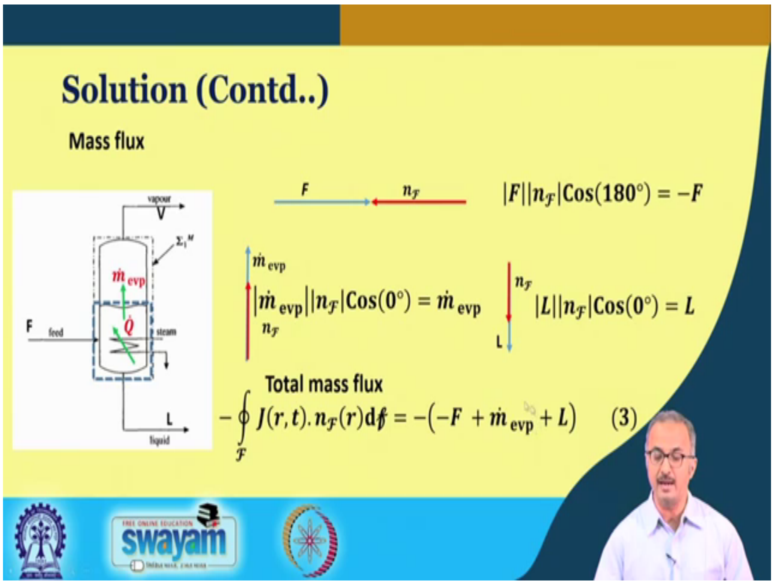

Now, again we write for this particular control volume the flux terms this same as earlier one. And this is also the same as the earlier one only new addition is coming due to this particular term and what is this term. This term is signifying the amount of vapor generated from the liquid and this vapor is going out from the liquid phase to the vapor phase. And since this is cutting this particular control surface; so this is taken as a flux term and not as any kind of generation term. So now, with this new flux term I am what we can see that the net flux will be the contribution of the feed flow rate, the liquid flow rate, and the amount of evaporation taking place.

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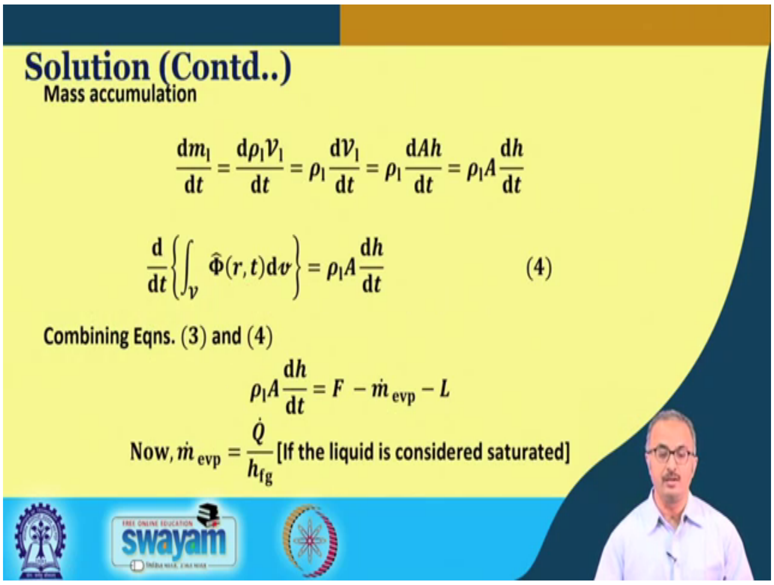

Now, again we write this amount of liquid change with time in the evaporator as this dm l by dt. So, within this particular control volume, this is a change in the mass of the particular system. So, this is again written in terms of density, and the volume of the liquid. And here we are writing this particular volume and this volume is replaced by the product of the height of the liquid and a area of cross section, this area of cross section is taken out of the differential. So, we get this dh by dt into rho L into A.

Now, after putting this value and combining it with the flux term, we get this particular expression. Now, you see this expression looks much simpler than the earlier expression for the change in the liquid height with time. And the advantage we are getting is like this that if we assume that the liquid is a saturated liquid, then whatever mass is getting generated due to evaporation can be easily computed from this expression. In this we have the amount of heat liberated by the steam divided by the enthalpy of vaporization of the liquid.

So, this will we are able to simplify this particular expression only thing, we have to see is this that if this assumption is not made, then we cannot write this expression. Because, the amount of heat liberated by the steam will also be used to heat up any kind of sub crewed liquid present in the system up to the saturated condition, and only after that the evaporation term can be added.

So, without going into those details even though we can take those fetch into account easily and only thing will be this the solution will be grid will get more involved other than this rest of the things can be done easily.

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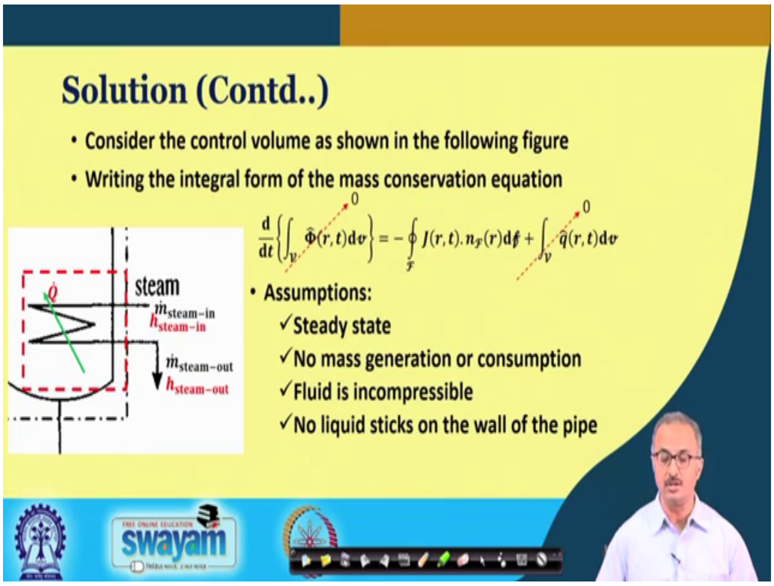

Next, we try to find out how this particular heat duty of the steam can be evaluated. For this, we chose another control volume that will be across this particular pipeline. So, here again we see that this is the um an integral energy integral balance. Here we again put all the assumptions as before, and we can take out all these things that in this case we are assuming that steady state within the pipeline. So, there is no accumulation, and there is no generation term. And we also assume that there is no liquid it is sticking to the pipeline wall.

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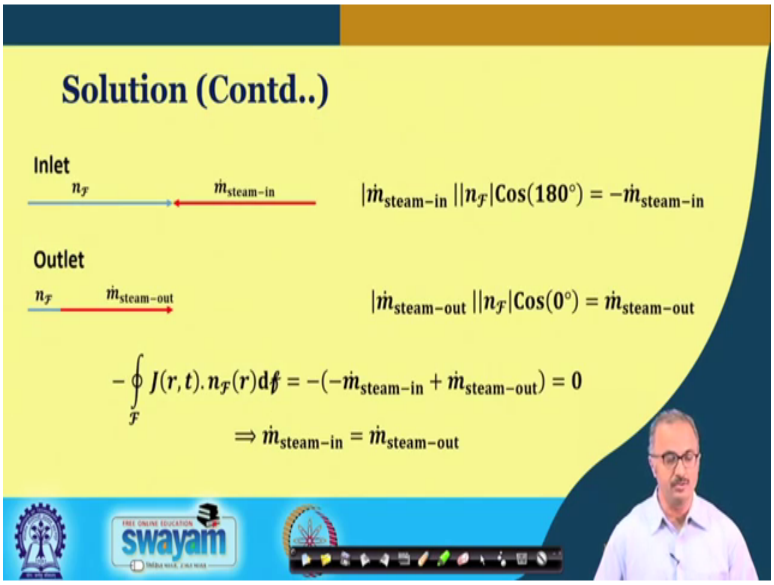

Now, here you see that we can like earlier, we can find out the flux values and this is associated with the incoming steam this is associated with the outgoing steam. And you can see that there will be changes in the sign of these two fluxes at the in inlet and the outlet. And then the net flux is this particular expression. And from this expression, we find the amount of steam going in is equal to the amounts of steam coming out of the system.

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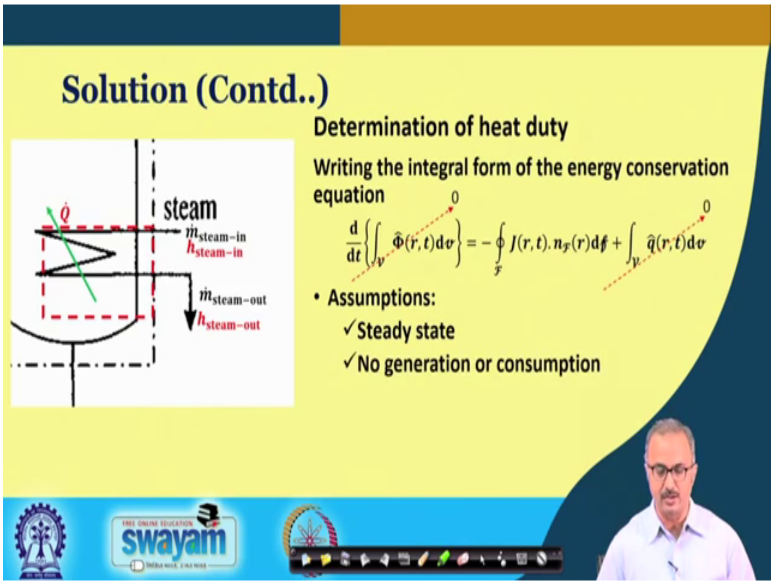

Now, for the energy balance what we do that we again write this in terms of the enthalpies with the steady state assumption and no generation term. And we will find that for the fluxes, we are getting this is the enthalpy that is going into the system with the steam, and this is the enthalpy that is coming out of with the steam.

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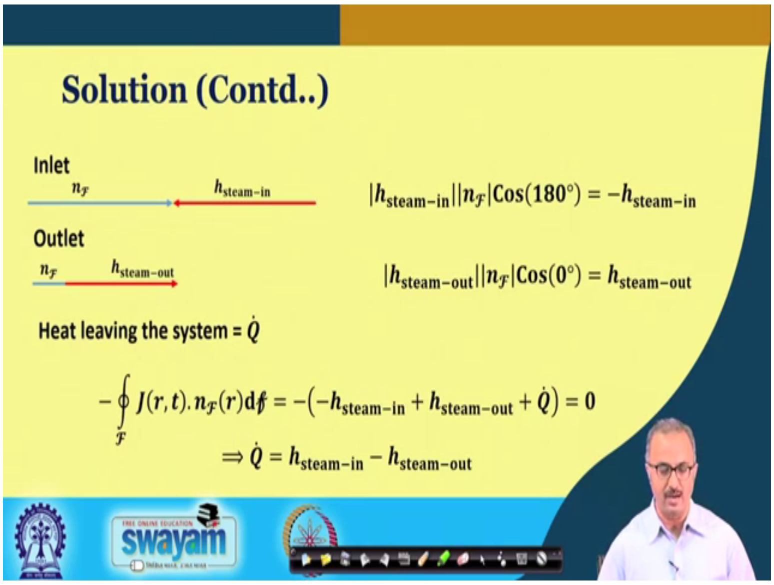

Now, this particular expression that is the heat liberated by the steam is also taken in the flux term, because it is crossing the control surface. So, this particular heat is should not be taken as any kind of generation, but should be taken as a heat flux. Now, once we have written this particular expression at steady state. We find that the amount of heat liberated from going from the steam to the liquid is the difference in the enthalpies of the incoming steam and the outgoing steam.

So, this is the way you solve for the amount of heat liberated by the steam.

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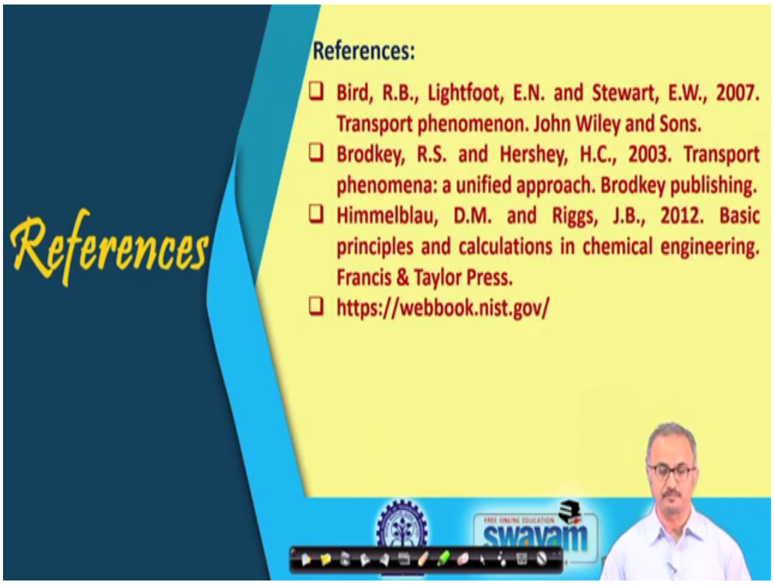

More details can be found out in these particular references. And we shall be doing some more problems from this.

Thank you.